The impact of transportation optimisation on assembly line feeding

Ebenezer Olatunde Adenipekun\textsuperscript{a,b,}\textsuperscript{*}, Veronique Limère\textsuperscript{a,b}, Nico André Schmid\textsuperscript{c}

\textsuperscript{a} Department of Business Informatics and Operations Management, Ghent University

\textsuperscript{b} Industrial Systems Engineering (SySE)-Flanders Make@Ghent University, Groef Karel de Goedelaan 5, Kortrijk, 8500, Belgium

\textsuperscript{c} IESEG School of Management, Univ. Lille, CNRS, UMR 9221 - LEM - Lille Economie Management, Lille F-59000, France

\textbf{A R T I C L E   I N F O}

\textbf{Article history:}
Received 21 December 2020
Accepted 8 September 2021
Available online 12 September 2021

\textbf{Keywords:}
Assembly system
Production
Decision making
Optimisation
Valid inequalities

\textbf{A B S T R A C T}

In the era of mass customisation, feeding parts to mixed-model assembly lines has proven to be a complex task since customers increasingly demand personalised end products. Consequently, the number of parts required at a single assembly line is sharply increasing. On the one hand, part supply must be done with the aim of avoiding excessive logistical handling activities while managing space at the border of line carefully. Hence, different line feeding policies can be exploited. On the other hand, shortages in parts supply, which may result in line stoppage, must be avoided. To this end, different vehicle types such as forklifts, automated guided vehicles and tow trains must be orchestrated carefully. This study is the first to propose a mixed integer programming model that efficiently assigns each part at the same time to a feeding policy and a vehicle type, with the goal to minimise total feeding costs. To accurately quantify costs, the model selects specific routes and determines the fleet size of every vehicle type used. The model is complemented by valid inequalities and validated by solving artificial problem instances. Within the analysis, we demonstrate that optimal selection of vehicle types is superior to heuristic approaches and show that this optimisation-based approach is around 8% cheaper than the industrial standard.

© 2021 The Authors. Published by Elsevier Ltd.

This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/)

1. Introduction

In recent times, assembly systems need to be more flexible, adaptable and agile in order to cope with the rising trend of mass customisation [4]. Challenges arising from this trend, like frequent product changes and smaller demands for individual products, can be mitigated through co-platforming [1] as well as design and re-configuration of modular assembly systems [28]. In this work, we specifically focus on the space management challenges that arise from mass customised assembly systems. Due to the high number of variant parts to be fed to the assembly operator, the available space at the border of line (BoL), i.e., the area designated for part storage at every station, is often too limited. This can be remedied by incorporating and combining different feeding policies for parts supply to the BoL. Feeding policies are generally characterised by part presentation at assembly stations. For instance, some parts may be presented on pallets, whereas other parts can be presented in boxes or kits. A kit contains various parts and its variants for a single product, similar to LEGO-kits. Parts presented in pallets accrue low feeding cost though they take more space at the BoL. In contrast, parts presented in kits or repacked into boxes are characterised by higher feeding costs due to additional handling effort in preparing them but less space is occupied at the BoL. Research shifted from descriptive models to optimisation-based models to address this trade-off in greater accuracy [27].

Formally, the assembly line feeding problem (ALPP) entails distinctive assignments of each part to a feeding policy to minimise feeding costs (see Schmid and Limère [32] for more details on the following definitions). Feeding policies, such as line stock, boxed supply (also referred to as kanban), sequencing, stationary and travelling kitting, describe how parts are presented at the BoL. Furthermore, each feeding policy is characterised by its specific execution of the four major processes underlying line feeding: part replenishment, preparation, transportation, and usage of parts at the assembly stations. These processes are represented in Fig. 1. In line stock, pallets or boxes filled with homogeneous parts (also known as SKUs) are delivered to the BoL from the warehouse. Line stock remains the only feeding policy where parts are not required to be repackaged in the preparation area before delivery to the BoL; parts assigned to any other feeding policy undergo repackaging. Similar to line stock, containers are filled with homogeneous parts in boxed supply. However, these containers are smaller than those used for line stock occurring that multiple boxes
can be held in a single flow rack. In sequencing, various variants of the same part family—a group of functionally equivalent parts that vary due to some characteristic, such as colour or quality—are arranged into a single container in the order of demand. This order is defined by the production sequence on the assembly line. In kitting, parts from different part families needed for a single end product are collected and placed in mobile containers or racks, called kits. While stationary kits are presented to different stations at the BoL, travelling kits are linked to a designated end product and are either placed directly on the assembly line or are attached to the product and travel very closely with it. This way, they traverse multiple stations at which parts are taken out of the kit.

The replenishment of parts needs to be carried out between the warehouse and the supermarket. It involves refilling the preparation area with parts from the warehouse. Replenishment is mainly concerned with pallets but may also include smaller boxes. The preparation process which takes place at the supermarket concerns repackaging of parts into other load carriers. As the dashed lines indicate in Fig. 1, replenishment and preparation are optional processes, which must be carried out for all line feeding policies except line stocking. Afterwards, the designated vehicle type conveys load carriers to the BoL, which is defined as the transportation process. Hence, transportation operation for preprocessed part is performed between the supermarket and the line, whereas, when parts are line stocked, this operation is done between the warehouse and the line. Finally, the usage process is concerned with the assembly operators picking parts from the load carriers at the BoL.

Various vehicle types such as forklifts, automated guided vehicles (AGVs) and tow trains can be utilised for parts supply according to different feeding policies. Although current ALFP models consider the transportation process when determining an optimal line feeding assignment, transportation costs are modelled approximately. More specifically, existing models assume preassigned vehicle types for the different flows and predetermined routes [32]. However, in practice, these decisions may be coupled to the selection of feeding policies. It is therefore necessary to accurately model vehicle type assignment and route determination in the ALFP in order to better estimate costs and make overall cost-optimal decisions. This extends previous academic insights in the ALFP and leads to additional cost gains for practice. In summary, this research aims to fill the current knowledge gap by integrating aspects of logistics design and assembly line feeding decision making.

We provide a tactical decision-making tool that minimises overall costs of the feeding system. It consists of operational and investment cost associated with the different vehicle types and logistical handling costs for preparation and usage. Fig. 1 depicts the scope of the modelling framework over the four processes considered in this research; replenishment process, the prepa-

![Diagram of ALFP processes](image)

**Fig. 1.** An overview of the four processes involved in ALFPs.

The motivation to investigate ALFPs arose with the seminal paper of Bozer and McGinnis [10] who proposed an early-stage descriptive model for choosing between line stocking and kitting. Different criteria such as material handling, space requirements and work in process (WIP) formed the basis for comparing the two feeding policies. This comparison resulted in a decision for the application of a single feeding policy. Some of the ideas for future research suggested here have been explored by other researchers in the meantime. As mentioned in the introduction, there has been a shift from descriptive to optimisation-based models which we will address in this section.

Battini et al. [7] provided a framework to efficiently decide between three feeding policies in an assembly system, namely line stocking (pallet to workstation), stationary kitting (trolley to workstation), and travelling kitting (kit to assembly line). They further gave an industrial application. Battini et al. [8] subsequently integrated the decision to either centralise or decentralise storage areas into their earlier framework [7], while minimising storage and feeding cost. In another study, Battini et al. [9] designed a descriptive model that incorporated decisions regarding the transportation mode selection for shop floor transportation.

Caputo and Pelagagge [12,13] adapted the model by Bozer and McGinnis [10]. Their studies focused on line stocking, kitting and boxed supply; parts were grouped utilising an ABC methodology to simplify the assignment of parts to feeding policies. They provided production managers with useful decision tools while heuristically selecting feeding policies for parts. Caputo et al. [16] analysed a descriptive cost model for deciding between line stocking and boxed supply in a single-model assembly line. Their findings can be summarised by stating that neither feeding policy is superior to the other. They, however, encouraged the usage of hybrid feeding policies in order to minimise total feeding costs. Caputo et al. [17,19] explored the impact of part features in an assembly system building on their previous work [13]. In addition, they provided heuristic rules to select a feeding policy for each part conveniently. Caputo et al. [18] developed a quantitative model to assess
the probability of errors and error correction costs in parts feeding systems.

Faccio [22] provided a decision making procedure supporting the choice between boxed supply and kitting in a mixed model assembly line. Production mix variation and model variety are included in the factors that provided a framework for selecting the most suitable feeding policy. In another research, Faccio et al. [23] introduced a descriptive model for selecting the optimal feeding policy between boxed supply, stationary and travelling kitting in an assembly system. They applied the model to five industrial cases from different sectors by simulating necessary parameters. The outcome served as a rule that guided their decision between the two feeding policies.

Sali et al. [31] presented a descriptive model for choosing between line stocking, sequencing and kitting in the automotive sector. Criteria such as part characteristics, diversity in part family, and bill of material (BOM) coefficients were identified by the authors to reach sound decisions.

A first approach towards mixed integer linear programming models (MILP) for the ALFP has been introduced by Limère et al. [24] presenting an optimisation model structure utilising a SysML graphical representation. Based on this structure, Limère et al. [25], [27] propose the first mathematical optimisation model to evaluate part assignments to either of two feeding policies, kitting and line stocking while minimising the total feeding costs. Optimal hybrid policies, i.e., a mix of both policies based on part and case characteristics, are found to be more favourable compared to the exclusive usage of either feeding policy. Relevant part and case characteristics were extensively investigated in Limère et al. [25]. Later, variable operator walking distances were incorporated into the model [26]. Analysis demonstrates that the initial model underestimated costs as space constraints at the line and shorter walking distances increase kitting assignments.

Caputo et al. [11] proposed a genetic algorithm for selecting a suitable feeding policy for components while minimising the total cost. Next, Caputo et al. [14,15] developed integer linear programming models for the optimal assignment of components to feeding policies. The three studies propose decision-making tools to select the best feeding policy for individual components out of line stocking, kitting or boxed supply. Caputo et al. [15] further presented factors to consider when evaluating alternative feeding policies by comparing the respective performances of each method on a quantitative basis. WIP, holding cost, equipment, workforce requirements, and intensity of containers flows were included in their consideration.

Sali and Sahin [30] developed an optimisation model that assigned each component to the most efficient feeding policy; considered feeding policies include line stocking, travelling kitting and sequencing. Authors modelled in-house transportation in a way that milk runs are made at an interval of pre-defined taks. They applied their model to a case study and gained insight into the trade-off to consider when deciding the best feeding policy for each individual component. The trade-off comprises the kit container’s capacity and the space available at the BoL.

Sternatz [35] and Battini et al. [5,6] integrated the ALFP and assembly line balancing problem by developing mixed integer programming models tackling both problems simultaneously. While Sternatz [35] was the first to integrate the ALFP and assembly line balancing, the approach is also the only one to provide a heuristic solution method for large instances. In contrast, Battini et al. [5,6] used MIP models only but additionally incorporated ergonomics considerations while minimising the number of workers in the system.

Schmid et al. [33,34] proposed a mixed integer programming model to determine the optimal feeding policy in a mixed model assembly line while minimising overall operating costs. They are the first to include all five feeding policies discussed above in an optimisation model. Furthermore, they investigated the effect of borrowing space, i.e., assigning some space from one station to an adjacent station. Lastly, Schmid et al. [34] incorporated the decision to assign parts not only to feeding policies but also to discrete locations at the BoL.

Baller et al. [3] extended existing mixed integer programming models in the literature incorporating the five feeding policies discussed in this research for a multi-model assembly line. For some feeding policies, multiple container types are distinguished. They apply their model to a case study at an automotive company.

Hitherto, researchers have addressed the assembly line feeding problem from different perspectives. These include descriptive and optimisation-based approaches with varying degrees of decisions w.r.t. the feeding policies or cost calculations considered. Those efforts have been summarised and classified in [32] which showed that limited efforts have been made towards the integration of logistics design and assembly line feeding. Table 1 gives an overview of the literature based on the feeding policies considered with the corresponding vehicle types used for their transportation process. It can be inferred from the table that vehicle types such as forklifts and tow trains are generally pre-assigned to feeding policies. While Battini et al. [9] incorporated vehicle type decisions into their studies, authors did not distinguish cost between different line feeding policies. This work discusses heuristic selection rules through a descriptive model. Similar to Battini et al. [9], Nourmohammadi et al. [29] included vehicle type decisions into their model, however, they did not consider the ALFP. Other studies such as Emde and Baysen [20] and Emde et al. [21] proposed optimisation-based models to address the decision of routing, loading and scheduling of vehicles. However, those do not consider line feeding. While the scope of these studies is somewhat operational, incorporating the routing problem in ALFP in this paper is more tactical as it concerns long term decisions such as layout and fleet aspects.

3. Modelling ALFP with vehicle type selection

In this section, a mixed integer linear programming model for the assembly line feeding problem is proposed. The model additionally incorporates the assignment of parts to vehicle types while assigning parts to feeding policies. In this regard, we anticipate to minimise overall operating and investment costs. In the model, every part i is assigned to a feeding policy p and a vehicle type m. The three vehicle types under consideration are AGVs (m = A), forklifts (m = F) and tow trains (m = T). The five line feeding policies are line stocking (p = 1), boxed supply (p = 2), sequencing (p = 3), stationary kitting (p = 4) and travelling kitting (p = 5). In this framework, we consider two transportation flows namely: (i) the flow between the warehouse and the assembly line w = 1 and (ii) the flow between the supermarket and the assembly line w = 2. To support readability, we split the model into a mathematical model formulation, presented in Section 3.1 and the calculation of costs used in the objective function in Section 3.2. The assumptions are listed in the following:

- Part demand and operation times are assumed to be deterministic.
- Forklifts are designated to execute the replenishment process.
- Parts are assumed to be placed in pallets at the warehouse initially.
- Not more than one stationary kit and travelling kit can be used at once in an assembly station and line respectively (see Constraints (6) and (8)).
- The space at the BoL is fixed for every assembly station.
3.1. Mathematical model formulation

In this section the mathematical model is presented. For the sake of clarity, the objective function, general constraints and three groups of supporting constraints are delineated. These supporting constraints cover route determination, line-sided storage as well as fleet size determination. The notation used throughout the model formulation is described in Table 2. Big-M values will be specified for each constraint set in Section 4.1.

3.1.1. Objective function

Minimize:

\[
\sum_{m \in M} \sum_{p \in P} \sum_{r \in R} c_{rpm}x_{rpm} + \sum_{f \in F} \sum_{i \in I} \sum_{a \in A} c_{fm}x_{3im} + \sum_{m \in M} c_{m}t_{m} + H \sum_{m \in M} c_{am}u_{am} + H \sum_{m \in M} c_{am}u_{am} \leq H \sum_{m \in M} c_{am}u_{am} \tag{1}
\]

The objective function (Equation 1) aims to minimise seven kinds of costs: (i) cost of feeding part \(i\) with feeding policy \(p\) through vehicle type \(m\), denoted by \(c_{rpm}\). This cost covers all processes, namely replenishment, preparation, transportation, and usage cost. However, this transportation cost only includes transportation cost for line stocked and boxed parts supplied delivered by forklift; (ii) cost of transporting all parts assigned to an AGV or a tow train between the warehouse or the supermarket and the line, denoted by \(c_{fm}\); (iii) cost of transporting sequenced parts by forklift \(c_{fm}\); transportation cost of kitted parts by a forklift for (iv) stationary kits \(c_{ms}\) and (v) travelling kits \(c_{m}\); (vi) forklift acquisition cost \(c_{am}\) for replenishment purpose; (vii) acquisition cost for every vehicle of type \(m\) used for transporting parts to the BoL, denoted by \(c_{am}\). Costs for transportation with forklifts are calculated differently from tow trains and AGVs since the former serve only one station at a time whereas the latter are used to serve routes consisting of multiple stations. The total cost is calculated in \(\epsilon\) and considers the whole planning horizon consisting of \(H\) taksks.

3.1.2. General constraints

\[
\sum_{m \in M} \sum_{p \in P} x_{rpm} = 1 \quad \forall i \in I \tag{2}
\]

\[
x_{rpm} = x_{jpm} \quad \forall f \in F, i, j \in I_f : i + 1 = j, p \in P, m \in M \tag{3}
\]

\[
x_{jpm} = 0 \quad \forall a \in A, i \in I : r_{ia} > r_{a2}, m \in M \tag{4}
\]

\[
\frac{n_{f}^{2}}{r_{ad}} \sum_{a \in A} \frac{r_{am}}{w_{a}} x_{ia} \leq k_{ms} \quad \forall s \in S, m \in M, a \in A \tag{5}
\]

\[
\sum_{m \in M} k_{ms} \leq 1 \quad \forall s \in S \tag{6}
\]

\[
\sum_{m \in M} t_{m} \leq 1 \quad \forall m \in M, a \in A \tag{7}
\]

\[
x_{rpm} \in [0, 1] \quad \forall i \in I, p \in P, m \in M \tag{9}
\]

\[
u_{rpm} \in [0, 1] \quad \forall p \in P, m \in M', t \in T, r \in R, w \in W \tag{10}
\]

\[
k_{ms} \in [0, 1] \quad \forall m \in M, s \in S \tag{11}
\]
### Table 2
Definition of notations used in the model.

<table>
<thead>
<tr>
<th>Sets</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{A} )</td>
<td>Set of part attributes (indexed ( a ))</td>
</tr>
<tr>
<td>( \mathcal{F} )</td>
<td>Set of part families (indexed ( f ))</td>
</tr>
<tr>
<td>( \mathcal{I}_s )</td>
<td>Set of part families at station ( s ) (indexed ( i ))</td>
</tr>
<tr>
<td>( \mathcal{I}_f )</td>
<td>Set of parts in family ( f ) (indexed ( i ))</td>
</tr>
<tr>
<td>( \mathcal{M} )</td>
<td>Set of vehicle types (indexed ( m ))</td>
</tr>
<tr>
<td>( \mathcal{P} )</td>
<td>Set of line feeding policies (indexed ( p ))</td>
</tr>
<tr>
<td>( \mathcal{R} )</td>
<td>Set of routes (indexed ( r ))</td>
</tr>
<tr>
<td>( \mathcal{S} )</td>
<td>Set of assembly stations (indexed ( s ))</td>
</tr>
<tr>
<td>( \mathcal{T} )</td>
<td>Set of taks (indexed ( t ))</td>
</tr>
<tr>
<td>( \mathcal{V} )</td>
<td>Set of transportation flows (indexed ( w ))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{fs} )</td>
<td>Number of facings for part family ( f ) assigned to sequencing at station ( s )</td>
</tr>
<tr>
<td>( f_s )</td>
<td>Number of facings for stationary kitting at station ( s )</td>
</tr>
<tr>
<td>( r_s )</td>
<td>Number of racks for boxed supply in station ( s )</td>
</tr>
<tr>
<td>( w_{mns} )</td>
<td>Number of vehicles for each type ( m ) used in transportation flow ( w )</td>
</tr>
<tr>
<td>( w_{pmrw} )</td>
<td>Continuous variable that decides the number of vehicles of type ( m ) for transporting parts assigned to policy ( p ) in flow ( w ) over route ( r )</td>
</tr>
<tr>
<td>( \kappa_{pm} )</td>
<td>Binary variable that assigns part ( i ) to feeding policy ( p ) and vehicle type ( m )</td>
</tr>
<tr>
<td>( d )</td>
<td>Product demand (equal to the number of taks in the planning horizon)</td>
</tr>
<tr>
<td>( d_s^p )</td>
<td>Two-way distance between supermarket and station ( s )</td>
</tr>
<tr>
<td>( d_p )</td>
<td>Depth of rack used for parts assigned to feeding policy ( p )</td>
</tr>
<tr>
<td>( H )</td>
<td>Planning horizon</td>
</tr>
<tr>
<td>( L_p )</td>
<td>Length of facings for feeding policy ( p )</td>
</tr>
<tr>
<td>( L_s )</td>
<td>Length of an assembly station ( s ) at the Bol</td>
</tr>
<tr>
<td>( m_i )</td>
<td>Number of units of part ( i ) in one end product</td>
</tr>
<tr>
<td>( n_f )</td>
<td>Number of parts in a part family ( f ) that fit into a sequenced container</td>
</tr>
<tr>
<td>( n_{pm} )</td>
<td>Number of bins or containers of feeding policy ( p ) that fit into a vehicle type ( m )</td>
</tr>
<tr>
<td>( n_t )</td>
<td>Number of travelling kits that fit into a kit container</td>
</tr>
<tr>
<td>( r_{ap} )</td>
<td>Resource availability of attribute ( a ) (weight or volume) for a container of policy ( p )</td>
</tr>
<tr>
<td>( \nu_m )</td>
<td>Velocity of vehicle type ( m )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Time in one takt</td>
</tr>
<tr>
<td>( \lambda_i )</td>
<td>Demand for part ( i )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{pm} )</td>
<td>Costs of providing part ( i ) with feeding policy ( p ) and vehicle type ( m ) to the Bol</td>
</tr>
<tr>
<td>( c_m )</td>
<td>Transportation cost for part family ( f ) and vehicle type ( m )</td>
</tr>
<tr>
<td>( c_{pmw} )</td>
<td>Milk run transportation cost for deliveries made with vehicle type ( m ) when transporting parts assigned to policy ( p ) in flow ( w ) over route ( r ) with frequency ( t )</td>
</tr>
<tr>
<td>( d^{ws} )</td>
<td>Two-way distance between warehouse and supermarket</td>
</tr>
<tr>
<td>( d_s^r )</td>
<td>Two-way distance between warehouse and station ( s )</td>
</tr>
<tr>
<td>( d_i )</td>
<td>Family of part ( i )</td>
</tr>
<tr>
<td>( h_s )</td>
<td>Number of shelves in a rack used for boxed supply containers</td>
</tr>
<tr>
<td>( l_{pm} )</td>
<td>Loading and unloading time for a feeding policy ( p ) container on vehicle type ( m )</td>
</tr>
<tr>
<td>( M )</td>
<td>Sufficiently large number</td>
</tr>
<tr>
<td>( m_{rtw} )</td>
<td>Milk run length for route ( r ) in transportation flow ( w )</td>
</tr>
<tr>
<td>( n_{ip} )</td>
<td>Number of parts ( i ) that fit into a bin or container of feeding policy ( p )</td>
</tr>
<tr>
<td>( n_{pmk} )</td>
<td>Number of stationary kits that fit into a kit container</td>
</tr>
<tr>
<td>( q_p )</td>
<td>Minimum number of facings allocated to containers for feeding policy ( p ) at the Bol.</td>
</tr>
<tr>
<td>( r_m )</td>
<td>Resource demand of attribute ( a ) (weight or volume) for a unit of part ( i )</td>
</tr>
<tr>
<td>( w_s )</td>
<td>Number of boxes that fit on one shelf of a rack used for boxed supply containers</td>
</tr>
<tr>
<td>( \lambda_f )</td>
<td>Demand for part family ( f )</td>
</tr>
<tr>
<td>( \mu_{m} )</td>
<td>Utilisation rate of vehicle type ( m )</td>
</tr>
</tbody>
</table>


\[ t_m \in \{0, 1\} \quad \forall m \in \mathcal{M} \tag{12} \]

\[ u_{pmrw} \in \mathbb{R}^+ \quad \forall p \in \mathcal{P}, m \in \mathcal{M}', r \in \mathcal{R}, w \in \mathcal{W} \tag{13} \]

\[ u^w_m \in \mathbb{N}_0 \quad \forall m \in \mathcal{M} \quad \tag{14} \]

\[ u_m \in \mathbb{N}_0 \quad \forall m \in \mathcal{M}, w \in \mathcal{W} \tag{15} \]

\[ f_{ip} \in \mathbb{N}_0 \quad \forall i \in \mathcal{I}, p \in \{1, 2\}, s \in \mathcal{S} \tag{16} \]

\[ f_f \in \mathbb{N}_0 \quad \forall f \in \mathcal{F}, s \in \mathcal{S} \tag{17} \]

\[ f_s \in \mathbb{N}_0 \quad \forall s \in \mathcal{S} \tag{18} \]

\[ r_s \in \mathbb{N}_0 \quad \forall s \in \mathcal{S} \tag{19} \]

Constraint (2) ensures that every part is assigned to a line feeding policy and a vehicle type. Constraint (3) guarantees that all members of the same part family are assigned to a single feeding policy and a single vehicle type. Furthermore, [34] showed that the benefits of relaxing this assumption are negligible. A similar conclusion is drawn by Baller et al. [3] who found for their case study that only 0.36% of the total feeding cost could be saved when allowing individual part decisions. The assignment of parts to boxed supply is restricted if parts do not fit into a box due to resource capacities such as volume (see Constraint (4)). This restriction was made to avoid confusion of workers. Constraint (5) determines for each station, whether stationary kits are used for an end product. For each station, the resource requirements such as volume and weight of all kit parts must not exceed the maximum resource capacities in a kit. Constraint (6) ensures that no more than one stationary kit per station is used. Constraint (7) determines whether travelling kits are used for an end product on the assembly line. Similar to stationary kits, the resource requirements such as volume and weight of all kit parts must not exceed the maximum resource capacities in a kit. Constraint (8) ensures that no more than one travelling kit is used. Constraints (9) to (19) define the decision variables’ domains.

3.1.3. Route determination

Fig. 2 schematically illustrates the movement of the different vehicle types between the warehouse and the assembly line for line stocked parts. Likewise, Fig. 3 illustrates this movement between the supermarket and the assembly line for parts assigned to other feeding policies. It is shown in Figs. 2 and 3 that a route can either cover all stations or a subset of stations at the BoL.

Just as in [2], we define a route as a path between the warehouse (\( w = 1 \)) or supermarket (\( w = 2 \)) and a sequence of stations at the assembly line linked to a route number \( r \). The number of stations in a route ranges from one to all stations of the assembly line. Moreover, only consecutive stations can be present in a route. On the one hand, this minimises transportation cost as the inclusion of non-consecutive stations forces vehicles serving such a route to pass by at least one dormant station; a station is dormant if the vehicle passes through the station without providing parts at the station’s BoL. On the other hand, building routes, only consisting of consecutive stations, avoids congestion at dormant stations. Thus, the corresponding routes for an assembly line with \( n = |\mathcal{S}| \) stations are calculated a priori and included in the model as follows: \( \{1\}, \{2\}, \{3\}, \ldots, \{n\}, \{1, 2\}, \{2, 3\}, \{3, 4\}, \ldots, \{n - 1, n\}, \{1, 2, 3\}, \{2, 3, 4\}, \{3, 4, 5\}, \ldots, \{n - 2, n - 1, n\}, \ldots, \{1, 2, 3, \ldots, n - 1\}, \{2, 3, 4, \ldots, n\}, \{1, 2, 3, \ldots, n\}. \) Hence, there are \( n + (n - 1) + (n - 2) + \ldots + 3 + 2 + 1 \) (or equivalently, \( \binom{n+1}{2} \)) possible routes in an assembly line with \( n \) stations. Notice that route \( r = 1 \) corresponds to a route that only visits the first station. Travelling kits are always transported on this route. A simplified illustration of such routes is depicted in Fig. 4.

In this study, we accurately determine each route’s length. That is, the length of route \( r \) depends on the distance between the warehouse or supermarket and the first assembly station of route \( r \), the distance between the first and last assembly station of route \( r \) and the distance between the last assembly station of route \( r \) back to the supermarket or warehouse (see Figs. 2 and 3). In addition, we determine each route’s travelling frequency. To determine the frequency, a unique \( t \), i.e., number of taktks from the set of taktks \( \mathcal{T} \) must be used. For instance, when \( t = 2 \) is chosen, the vehicle assigned traverses the route every two taktks. Combining the length and frequency of a route allows an accurate transportation cost calculation. In addition, routes must be selected such that a vehicle of any type can only traverse one out of the possible routes at any frequency passing through a station. An example of such a scenario is illustrated in Fig. 4: an AGV can either traverse Route 1 and Route 2 or Route 3 but it cannot traverse all three routes. This restriction was set to avoid inefficient and chaotic transportation. The model optimises the selection of routes and taktks for each vehicle.
hicle type and thereby minimises number of transports navigating between the warehouse or supermarket and assembly line. This is reflected in Constraint (20)

$$\sum_{t \in T} \sum_{R \in R_s} v_{pmrw} \leq 1 \quad \forall p \in P, m \in M', s \in S, w \in W_p$$  \hspace{1cm} (20)

To summarise, Constraint (20) allows part delivery to an assembly line from the warehouse or supermarket by vehicle type \( m \in M' \) through a chosen route from the possible routes \( r \in R_s \) passing through station \( s \) with a selected frequency. Constraint (21) ensures the selection of a suitable route and takt for each vehicle type if any part is assigned to that vehicle type.

$$\sum_{i \in I_s} x_{ipm} \leq \mathcal{M} \sum_{t \in T} \sum_{r \in R_s} v_{pmrw} \quad \forall p \in P, m \in M', s \in S, w \in W_p \hspace{1cm} (21)$$

### 3.1.4. Line-sided storage

The frequency of the routes also allows to efficiently manage the space needed to store parts at the BoL without going out of stock between consecutive arrivals. When the delivery frequency of a parts is lower, more space will be dedicated to store it at the BoL. On the contrary, less space is assigned to parts with more frequent deliveries.

$$\frac{\lambda_{fs}}{H \rho_p d_p} x_{ipm} - \mathcal{M} \left(1 - \sum_{r \in R_s} v_{pmrw}\right) \leq f_{ipm} \forall p \in \{1, 2\}, s \in S, i \in I_s, t \in T, m \in M', w \in W_p$$  \hspace{1cm} (22)

$$\sum_{s \in S} w_i \sum_{i \in I_s} f_{2s} \leq t_s \quad \forall s \in S$$  \hspace{1cm} (23)

$$\frac{\lambda_{x_{3m}}}{H \rho_f} x_{3m} - \mathcal{M} \left(1 - \sum_{r \in R_s} v_{pmrw}\right) \leq f_{fs} \quad \forall s \in S, f \in F_s, i \in I_f : f_{i-1} \neq f, t \in T, m \in M'$$  \hspace{1cm} (24)

$$\frac{\lambda_{k_{ms}}}{H \rho_m} x_{m} - \mathcal{M} \left(1 - \sum_{r \in R_s} v_{pmrw}\right) \leq f_s \quad \forall s \in S, t \in T, m \in M'$$  \hspace{1cm} (25)

$$\lambda_{q_p} \sum_{m \in M} X_{ipm} \leq f_{ips} \quad \forall p \in \{1, 2\}, s \in S, i \in I_s$$  \hspace{1cm} (26)

$$\lambda_{q_s} \sum_{m \in M} X_{3im} \leq f_{fs} \quad \forall s \in S, f \in F_s, i \in I_f : f_{i-1} \neq f$$  \hspace{1cm} (27)

$$\lambda_{q_a} \sum_{m \in M} k_{ms} \leq f_s \quad \forall s \in S$$  \hspace{1cm} (28)

$$\sum_{i \in I_s} L_1 f_{1s} + L_2 r_s + \sum_{f \in F_s} L_3 f_{fs} + L_4 f_{is} \leq L_s \quad \forall s \in S$$  \hspace{1cm} (29)

Constraints (22) to (29) refer to the space constraints at the BoL. First, Constraint (22) ensures that containers used for line stocked and boxed supply parts, delivered by vehicle type \( m \in M' \) to each station, between consecutive deliveries, do not exceed the number of facings reserved for them. A facing describes some space along the BoL whose size is just enough to store a single container of a corresponding load carrier, e.g., the size of a pallet in line stocking or a box in boxed supply. Constraint (23) determines the number of racks used for boxed supply. Additionally, Constraints (24) and (25) ensure the number of containers used for sequencing and stationary kitting delivered by vehicle type \( m \in M' \) to each station, between consecutive deliveries do not exceed their respective facings. Furthermore, Constraints (22) to (25) ensure allocation of less space to frequently delivered parts, whereas parts with lower delivery frequency receive more space by linking the number of facings to the transportation variables \( v_{pmrw} \) if parts are delivered by AGV or tow train. Consequently, these constraints allow efficient space management at the BoL. Constraints (26) to (28) provide a lower bound on the number of facings available for every part delivered to the BoL by a vehicle of type \( m \in M' \). Constraint (29) ensures that the summed length of all facings do not violate the space available at any station.

### 3.1.5. Fleet size determination

Besides modelling the operation cost of the travelling process, we also consider investment costs for the vehicle fleet which can be determined precisely due to the fine-grained modelling approach.

$$\frac{t}{H} \sum_{s \in S} \sum_{i \in I_s} \sum_{r \in R_s} \frac{\lambda_{ipm}}{H \rho_p d_p} \left( \frac{m_r}{n_{pmrw} \mu_m v_m} + l_{pm} \right) x_{ipm} - \mathcal{M} (1 - v_{pmrw}) \leq \beta t u_{pmrw} \forall p \in \{1, 2\}, m \in M', s \in S, w \in W_p$$  \hspace{1cm} (30)

$$\frac{t}{H} \sum_{s \in S} \sum_{i \in I_s} \sum_{r \in R_s} \frac{\lambda_{ipm}}{H \rho_p d_p} \left( \frac{m_{r2}}{n_{pmrw} \mu_m v_m} + l_{pm} \right) x_{ipm} - \mathcal{M} (1 - v_{pmrw}) \leq \beta t u_{pmrw} \forall m \in M', t \in T, r \in R$$  \hspace{1cm} (31)

$$\frac{t}{H} \sum_{s \in S} \sum_{i \in I_s} \sum_{r \in R_s} \frac{\lambda_{ipm}}{H \rho_p d_p} \left( \frac{m_{r3}}{n_{pmrw} \mu_m v_m} + l_{pm} \right) x_{ipm} - \mathcal{M} (1 - v_{pmrw}) \leq \beta t u_{pmrw} \forall m \in M', t \in T$$  \hspace{1cm} (32)

$$\frac{t}{H} \sum_{s \in S} \sum_{i \in I_s} \sum_{r \in R_s} \frac{\lambda_{ipm}}{H \rho_p d_p} \left( \frac{m_{r3}}{n_{pmrw} \mu_m v_m} + l_{pm} \right) x_{ipm} - \mathcal{M} (1 - v_{pmrw}) \leq \beta t u_{pmrw} \forall m \in M', t \in T$$  \hspace{1cm} (33)

$$\sum_{p \in P} \sum_{w \in W} u_{pmrw} \leq u_{m} \quad \forall m \in M', w \in W$$  \hspace{1cm} (34)

$$\frac{1}{H} \sum_{i \in I} \sum_{p \in P} \sum_{m \in M} \sum_{j \in J} \frac{\lambda_{ipm}}{n_{j}} \left( \frac{d_{w}}{n_{j} f_{j} l_{j}} + l_{f} \right) x_{ipm} \leq u_{w}$$  \hspace{1cm} (35)

$$\frac{1}{H} \sum_{i \in I} \sum_{p \in P} \sum_{m \in M} \sum_{j \in J} \frac{\lambda_{ipm}}{n_{j}} \left( \frac{d_{w}}{n_{j} f_{j} l_{j}} + l_{f} \right) x_{ipm} \leq u_{w}$$  \hspace{1cm} (36)

$$\frac{1}{H} \sum_{i \in I} \sum_{p \in P} \sum_{m \in M} \sum_{j \in J} \frac{\lambda_{ipm}}{n_{j}} \left( \frac{d_{w}}{n_{j} f_{j} l_{j}} + l_{f} \right) x_{ipm} \leq u_{w}$$  \hspace{1cm} (37)

Constraints (30) to (33) provide the number of vehicles of type \( m \in M' \) used for every possible route and takt during the planning horizon, by considering the ratio of the taks' number between
two consecutive deliveries and the total number of taks in the planning horizon. Furthermore, it is ensured for every vehicle of type \( m \in M \) that delivery times do not exceed the available time, defined by the assigned task. Constraint (34) guarantees that the number of available vehicles of type \( m \in M \) is sufficiently large to carry out all routes at any takt. Constraints (35) to (37) provide the fractional number of forklifts needed to carry out all deliveries from the warehouse to the supermarket, the warehouse to the line, and the supermarket to the line, respectively. Moreover, Constraint (35) ensures the usage of forklifts for the replenishment process.

### 3.2. Cost parameter calculation

In this section, we provide a breakdown of the cost associated with the model. Cost calculations are built up similar to [34]. Within this framework, four different sources of costs are taken into consideration: (i) cost incurred through part replenishment at the supermarket, denoted by \( c_{\text{pm}} \); (ii) cost incurred through part preparation for all feeding policies that require preprocessing, denoted by \( c_{\text{ppm}} \); (iii) cost incurred through transportation of parts to the BoL. Cost accrued through part transportation using forklifts is denoted by \( c_{\text{pfp}} \) for line stocked and boxed supply parts, \( c_{\text{pf}} \) for sequenced parts, \( c_{\text{pf}} \) and \( c_{\text{p}} \) for kits stocked (stationary and travelling kits, respectively). By contrast, costs incurred for each part transported by AGVs or tow trains is denoted by \( c_{\text{pmtrw}} \). (iii) cost incurred through part usage at the assembly line, denoted by \( c_{\text{ip}} \). Those cost sources are mapped to the cost parameters used in the optimisation model above in Equations (38) to (42).

\[
c_{\text{pm}} = c_{\text{pfp}} + c_{\text{ppm}} + c_{\text{pf}} + c_{\text{p}} \quad \forall i \in I, p \in P, m \in M \quad (38)
\]

\[
c_{\text{pmtrw}} = c_{\text{pmtrw}} \quad \forall p \in P, m \in M, t \in T, r \in R, w \in W \quad (39)
\]

\[
c_{\text{pf}} = c_{\text{pf}} \quad \forall f \in F, m \in M \quad (40)
\]

\[
c_{\text{p}} = c_{\text{p}} \quad \forall m \in M, s \in S \quad (41)
\]

\[
c_{\text{ip}} = c_{\text{ip}} \quad \forall m \in M \quad (42)
\]

Subsequently, all cost elements will be defined and their effects on decision making will be discussed. The notation and parameter values used in the model can be found in Table 3. The values are based on discussions with practitioners and data from case studies described in literature.

#### 3.2.1. Replenishment costs

As mentioned above, replenishment of parts deals with refilling the supermarket with pallets. The supermarket is located between the warehouse and the assembly line and serves as a preparation area to preprocess parts before delivering them to the BoL. Clearly, line stocked parts do not require any preprocessing.

\[
c_{\text{pfp}} = \left( \frac{\lambda_i}{n_{i2}} \right) \left( \frac{d_{\text{ms}}}{n_{i2} \mu_2 v_F} \right) + l_{iF} \quad \forall i \in I, p \in P_2 \quad (43)
\]

Replenishment times depend on a part’s demand \( \lambda_i \) divided by the number of parts that fit into a pallet \( n_{i1} \), which results in the total number of pallets. This result is divided by the number of pallets a forklift can contain at once \( n_{i2} \), and the forklifts’ utilisation rate \( \mu_2 \), which is further multiplied by the time taken by a forklift to go back and forth from the warehouse to the supermarket; this time is calculated by dividing the distance covered by forklift to go back and forth from the warehouse to the supermarket \( d_{\text{ms}} \) by the forklifts’ velocity \( v_F \). Moreover, the total number of pallets is multiplied by the corresponding loading and unloading time \( l_{iF} \). Finally, the outcome is multiplied with the cost of using a forklift for one time unit \( c_{\text{ip}} \) (see Equation (43)).

#### 3.2.2. Preparation costs

Preparation describes the process of repacking parts into different (generally smaller) load carriers conducted in the preparation area. The preparation cost varies across all line feeding policies and depends on the specific tasks that define a policy’s preparation.

\[
c_{\text{ppm}}^p = c^p \left[ \frac{\lambda_i}{n_{i3}} \left( \frac{A L_2}{n \tau_2 o v} \right) + (h t_{12} + s_{12}) \lambda_i \right] \quad \forall i \in I, m \in M \quad (44)
\]

\[
c_{\text{pmtrw}}^p = c^p \left[ \frac{\lambda_i}{n_{i3}} \left( \frac{A L_2 + m v}{o v} \right) + (h t_{13} + s_{13}) \lambda_i \right] \quad \forall i \in I, m \in M \quad (45)
\]

\[
c_{\text{p}}^p = c^p \left[ \left( \frac{A L_4 d}{o v m} \right) + (h t_{4} + s_{4}) \lambda_i \right] \quad \forall i \in I, m \in M \quad (46)
\]

\[
c_{\text{pmtrw}}^p = c^p \left[ \left( \frac{A L_4 d}{o v m} \right) + (h t_{5} + s_{5}) \lambda_i \right] \quad \forall i \in I, m \in M \quad (47)
\]

To prepare parts for boxed supply, a pick list containing parts to be prepared for a particular batch must be followed. Preparation times depend on a part’s demand \( \lambda_i \), the number of parts that fit into a bin \( n_{i2} \), and the batch size \( n \tau_2 \). The batch size is the number of bins a trolley, used to support the picker, can bear. This is multiplied by the time taken to walk through the preparation area for one batch, i.e., the length of an aisle \( A L_2 \) (it is assumed that the operator walks on average half of the length of an aisle in both direction) divided by the operator’s velocity \( o v \). In addition to this, the search \( s_{12} \) and handling times \( h t_{12} \) must be considered. All these are multiplied by the logistics operator’s cost \( c^p \) to obtain the corresponding preparation cost (see Equation (44)). In sequencing, preparation of parts is similar to boxed supply (see Eq. 45). However, dealing with different variants of the same part family leads to an additional term, \( m v \), for walking through the area where these variants are located in the preparation area. Furthermore, the preparation times do not depend on the batch size since only one sequencing container fits in a trolley. In stationary kitting, preparation costs are derived from dividing the products’ demand \( d \) by the number of kits that fit into a container \( n_{i3} \). This is multiplied by the incremental time required to walk through the preparation area for kitted parts, i.e., \( A L_4 \) divided by the operator’s velocity \( o v \). The searching time \( s_4 \) and handling time \( h t_4 \) are similar to the feeding policies described above (see Eq. 46). The preparation costs for travelling kitting are derived identically to the ones in stationary kitting (see Eq. 47).

#### 3.2.3. Transportation costs

Part transportation is a key activity in assembly line feeding. It entails the transportation of parts from a warehouse or supermarket to the BoL, regardless of the chosen line feeding policy. Transportation times for forklift depend on a part’s demand \( \lambda_i \) divided by the number of parts that fit into a feeding policy’s load carrier \( n_{i p} \), which results in the total number of load carriers requiring transport for that feeding policy. This result is divided by the number of load carrier a forklift can contain at once \( n_{i p} \) and the forklifts’ utilisation rate \( \mu_m \). This is further multiplied by the time taken by a forklift to go back and forth from the warehouse to the BoL (line stocked parts) or the supermarket to the BoL (preprocessed parts); this time is calculated by dividing the distance covered by a forklift to go back and forth from the warehouse to the BoL \( d_{\text{ms}} \) (or from the supermarket to the BoL \( d_{\text{ms}}' \)) by the forklifts’
velocity $v_{mn}$. Moreover, the total number of load carriers requiring transport for feeding policy $p$ is multiplied by the loading and unloading time $l_{pm}$ required for vehicle type $m$. Finally, the outcome is multiplied with the cost of using vehicle type $m$ for one time unit $c_{om}$. The loading and unloading times of tow trains are calculated similarly to forklift’s. Transportation times for tow train or AGV-based transportation depend on the number of times a milk-run route is carried out during the planning horizon $H$, divided by the vehicle's or utilisation rate $\mu_{m}$. This is further multiplied by the time to perform a milk run either from a warehouse or a supermarket $T_{mm}$, $m_{rw}$ is the milk run’s length between the warehouse or the supermarket and the line. $v_{m}$ represents the average velocity of vehicle type $m$. Finally, transportation times are multiplied with the corresponding vehicle usage costs per time unit $c_{om}$.
(see Equation (53)).

\[
\begin{align*}
C_{12m}^i & = c_{n2m} \left[ \frac{d_2^i}{n_i} \right] + l_{1m} \quad \forall i \in \mathcal{I}, m \in \mathcal{M} \\
C_{22m}^i & = c_{n2m} \left[ \frac{d_2^i}{n_i} \right] + l_{2m} \quad \forall i \in \mathcal{I}, m \in \mathcal{M} \\
C_{2m}^f & = c_{n2m} \left[ \frac{d_2^f}{n_f} \right] + l_{2m} \quad \forall i \in \mathcal{I}, f \in \mathcal{F}, m \in \mathcal{M} \\
C_{4m}^i & = c_{n4m} \left[ \frac{d_4^i}{n_i} \right] + l_{4m} \quad \forall i \in \mathcal{I}, m \in \mathcal{M} \\
C_{m}^o & = c_{n5m} \left[ \frac{d_5^o}{n_o} \right] + l_{5m} \quad \forall i \in \mathcal{I}, m \in \mathcal{M} \\
C_{pm}^{mrw} & = c_{n6m} \left[ \frac{H_{mrw}}{\mu_m \lambda_m} \right] \quad \forall p \in \mathcal{P}, m \in \mathcal{M}, t \in \mathcal{T}, r \in \mathcal{R}, w \in \mathcal{W}_p
\end{align*}
\]

While Equation (48) represents transportation cost of line stocked parts transported from the warehouse directly to the BoL by forklifts, Equations (49) to (52) provide the transportation cost incurred by forklift transportation from the supermarket to the BoL for preprocessed parts. Furthermore, Equation (53) defines the transportation cost of every part delivered by an AGV or a tow train from the warehouse or supermarket to the BoL. Forklifts’ transportation times for kitted parts depend on product demand \( d \) divided by number of kits that fit into a kit container \( n_k^\lambda \) (or \( n_k^\mu \)), number of kit containers that can be transported at once \( n_k^m \) (or \( n_k^m \)), and utilisation rate \( \mu_m \) of vehicle type \( m \).

Before concluding this section, we will briefly discuss the loading and unloading processes for the vehicle types considered in this study. There are two kinds of AGVs, namely: large and small AGVs. Large AGVs are responsible for transportation of feeding policy containers, whereas, small AGVs conduct the loading and unloading of containers on/from large AGVs. Henceforth, large AGVs will be referred to as AGV and are used the same way as tow trains. Just as [9] pointed out, the tow train can be operated manually by a driver or automatically as an AGV. The loading and unloading processes for every feeding policy container over all vehicle types is described in Table 4.

Please note that loading and unloading costs only pertain to parts transported to the line by forklifts and tow trains, as loading and unloading operations on AGVs is automated and, therefore, incorporated in the acquisition cost for small AGVs.

### 3.2.4. Usage costs

The usage process, carried out at the assembly station, consist of multiple operations, namely: walking back and forth from the assembly line to the BoL, searching for parts at the BoL (depending on the line feeding policy), handling parts, and assembling parts. This process is solely performed by the assembly worker. In this research, we clearly state that the usage cost comprises cost related to walking between assembly station and BoL, and searching cost. This is because, other costs, such as handling and assembling cost, are common to all feeding policies and as such, have no effect on line feeding decisions. Hence, usage cost is defined as the cost of assembly operators multiplied by searching and walking time.

\[
\begin{align*}
C_{ip}^i & = c_2 \lambda_i \left( \frac{w_p}{v} + s_i \right) \quad \forall i \in \mathcal{I}, p \in \{1, 2\}, m \in \mathcal{M} \\
C_{ipm}^i & = c_3 \lambda_i \left( \frac{w_p}{v} \right) \quad \forall i \in \mathcal{I}, p \in \{3, 4\}, m \in \mathcal{M}
\end{align*}
\]

In Equations (54) and (55), we present the usage cost, depending on the line feeding policy, as the product of wage of an assembly operator \( c_2 \), a part’s demand, the search time \( s_i \), and the walking time, i.e., the distance to go back and forth from the assembly station to the BoL \( w_p \) divided by operator’s walking velocity \( v \). The search time must only be taken into account for line stocking or boxed supply. This is simply because the assembly operator has no options to choose from in sequencing and kitting. Since traveling kits are placed on the assembly line, no walking is required either.

### 4. Solving procedure

In order to lower computation times for the model, \( b_i = M \) values are defined individually for each constraint and valid inequalities are added to the model. Within the next two subsections, these adaptations to the model will be discussed, respectively.

#### 4.1. Big-M constraints

There are eight \( b_i = M \) constraints in the model and six \( b_i = M \) constraints in the Model extension (see Appendix A of the Appendices). It is, therefore, advisable to choose the \( b_i = M \) values carefully to define them as tight as possible while avoiding cutting off valid solutions. To this end, the \( b_i = M \) values in constraints (21), and (62) are chosen as the cardinality of the set that sums up the left hand side of each constraint. On the contrary, the \( b_i = M \) values in constraints (22), (24), (25), and (30) to (33) as well as (58), (59), (60), (63), and (64) can be described as the sum of the coefficients of variables on the left-hand side of each constraint.

#### 4.2. Valid inequalities

To tighten the convex hull of the model, two valid inequalities are defined in Equations (56) and (57). They link \( x_{ipm} \) to the corresponding \( u_{n1} \) and \( u_{n2} \) variables.

\[
\begin{align*}
x_{ipm} & \leq u_{n1} \quad \forall i \in \mathcal{I}, m \in \mathcal{M} \\
x_{ipm} & \leq u_{n2} \quad \forall i \in \mathcal{I}, p \in \mathcal{P}, m \in \mathcal{M}
\end{align*}
\]

This means that once a part is assigned to a feeding policy and vehicle type, a corresponding vehicle of that type must be used for such assignment. This leads to Proposition 8.01. This Proposition as well as its proof can be found in Appendix B of the Appendices.

### 5. Computational experiments

We solve the model proposed in Section 3 using Gurobi 9 on a High Performance Computing (HPC) infrastructure. For this, 16 GB of RAM and 6 out of the available 24 cores of an Intel Xeon Gold 6130 processor with 2.10 GHz were allotted. A maximum runtime of 3600 seconds was set. The model was solved for 60 problem instances, that have been artificially created on the basis of Limère et al.’s [26,27] case study and Schmid et al.’s [34] problem instance generation algorithm. Each problem instance contains 15 stations, the number of parts ranges from 162 to 490, and the number of part families ranges from 74 to 171. The set of frequencies that dictates the movement of every vehicle type \( m \) is given as \( \lambda = \{1, 2, 3, \ldots, 10, 20, 30, 40, 50\} \). The planning horizon is the maximum number in set \( \lambda \), which is 50 in this case. The set \( \lambda \) was chosen such that it offers a range of possible frequencies over the planning horizon, while striking a balance with computation time. Moreover, the valid inequalities discussed in Section 4 were
added during the solving process, as a Cut and Branch (C&B) procedure. Table 5 depicts the results obtained by the two solving approaches. The number of problem instances (# inst.), number of instances that attained optimality within the time limit (# opt), the average solving time in seconds (time(s)), and average optimality gap (gap%) were recorded. The solving process was carried out for both versions of the model namely: single feeding policies in a route and mixed feeding policies in a route. In both versions, the C&B approach outperforms Gurobi, as can be seen by the average computation times and gaps. We remark that the model described in Subsections 3.1 and 3.2 can be referred to as "single feeding policy in a route" since any vehicle of type \( m \in M \) performing a milk run on any of the routes can only transport parts assigned to a single feeding policy. The second version of the model which is known as "mixed feeding policies in a route" can be found in Appendix A of the Appendices.

In the following subsections, we will discuss decision making insights that may serve as rules of thumb for production managers. Furthermore, in Section 5.5, we will show the positive effects of optimised vehicle type selection in terms of cost reductions. Note, results are dependent on the parameter settings and drastic changes in these may influence the results.

### 5.1. Line feeding decisions

Fig. 5 depicts the share of parts assigned to each feeding policies for all problem instances. Clearly, all feeding policies are used, confirming the results of Schmid et al. [34] as well as Baller et al. [3], and confirming the necessity to consider all feeding policies. Fig. 5 illustrates the extent to which each of the feeding policies is used by indicating the median (orange line), the first to third quartile (box), the minimal and maximal values (bar) as well as the outliers (red square) in a classic box plot representation. Line stocking, boxed supply, and sequencing are applied less than kitting. The relatively low usage of line stocking, which is the only line feeding policy that does not require preparatory material handling, can be largely attributed to proper space management at the BoL, where a large variety of parts has to be placed. Boxed supply and sequencing are the least preferential feeding policy with an average of less than 1%. The low usage of boxed supply may be explained by the number of parts in the problem instances and the capacity of travelling kits. That is, many parts fit into a travelling kit and thus there is no need to set up flow racks at the BoL. Similarly, the low usage of sequencing can be attributed to the size of the problem instances and kits’ capacity, i.e., a sequenced container can only hold a limited number of parts. When more stations and parts are considered, the amount of boxed supply and sequenced parts may very well increase. In addition, sequencing will benefit more from parts with large volumes and variety. By contrast, stationary kitting and travelling kitting are well used, and travelling kitting is by far the most preferred feeding policy in this research. This result can be largely attributed to space management as travelling kits do not consume space at the BoL.
5.2. Vehicle type selection

In this section, the use of vehicle types is analysed. As represented in Fig. 6, all vehicle types are used which proves the importance of selecting between different vehicle types. More specifically, Fig. 6 depicts the percentage of parts using each vehicle type in a box plot. Forklift is the least employed vehicle type, with an average of less than 1%. This result can be attributed to a high acquisition cost as well as operating costs as drivers need additional qualification; moreover, forklifts cannot serve more than one station at a time. On the other hand, AGVs and tow trains are used more, which is a consequence of their ability to serve multiple stations at once in a milk run, and thereby, minimising the share of empty trips. More precisely, tow train is the most employed vehicle type. The reason behind this result is not far-fetched; the acquisition cost incurred by a tow train is generally low when compared to other vehicle types. In general, the results indicate that an optimal solution consists of a variety of vehicle types.

5.3. Routing decisions

Figs. 7 and 8 show the number of tours for every possible length of a route for both versions of the model i.e., single and mixed feeding policies in a route, respectively. The results are obtained over all problem instances and combined for AGVs and tow trains. The length of a route is characterised by the number of stations that belong to that route. For both versions of the model, we observe that routes with multiple stations are generally preferred to routes with a single station, which may be a result of lower transportation cost attached to multiple-station routes. Figs. 7a and 8a illustrate the similarities in route trend for both versions of the model. This is because parts transported from the warehouse to the assembly station are only concerned with line stocking. On the other hand, there is a clear difference between Figs. 7b and 8b. In Fig. 7b, mixed feeding policies are not allowed in a route. Consequently, there is more likelihood of using multiple-station routes with shorter length. By contrast, mixing policies may force AGVs or tow trains to use multiple-station routes with longer length especially routes with 15 stations, as depicted in Fig. 8b.

5.4. Vehicle type selection by feeding policy

In this section, we show the distribution of vehicle types for each of the feeding policies over 60 problem instances. We sum over all parts assigned to any feeding policy and retrieve the percentage value of each vehicle type used in relation to that number. To this end, the percentage value is presented for routes with single policy and mixed policies in Table 6.
When policies are not mixed in a route, it can be inferred that line stocked, boxed supplied, sequenced and stationary kitted parts are mostly transported by AGVs, whereas, tow trains generally deliver travelling kitted parts. In general, parts are rarely delivered to the BoL by forklifts as forklifts are the least used vehicle type over all feeding policies. When mixing policies in a route, results change slightly. For example, line stocked, boxed supply, and sequenced parts are mostly transported by AGVs, whereas, stationary and travelling kitted parts are generally delivered by tow trains. Nevertheless, forklift is the least used vehicle type over all feeding policies in both versions of the model.

5.5. Multi-scenario analysis

In industry, vehicle types are usually preassigned to different material flows. With this analysis we want to examine the positive impact of optimising vehicle selection in comparison to an assignment a priori, as in industry. Fig. 9 illustrates the test setting.

On the one hand, the top left (VS) and bottom left (VM) quadrants delineate the two versions of the V model, where V stands for variable vehicle type per feeding policy. In both cases, the model optimises the assignments of parts to vehicle types and feeding policies, however, VS represents single feeding policy in a route, whereas VM represents mixed feeding policies in a route. On the other hand, the top right (FS) and bottom right (FM) quadrants depict a pre-assignment of vehicle types to policies, where analogously FS represents single feeding policy, and FM represents mixed feeding policies in a route. To summarise, the top and bottom left quadrants illustrate assignment of parts to a suitable feeding policy and vehicle type, whereas, the top and bottom right quadrants depicts assignment of parts to a suitable feeding policy while pre-assigning a vehicle type to the chosen feeding policy.

To fully test the impact of heuristic approaches for the pre-assignment of vehicles to feeding policies, different scenarios will be compared in this section, represented in Table 7. These scenarios were motivated by industry and by the results in Table 6 where it can be seen that most policies have a preference to be assigned to a particular vehicle type.

In Scenario 1, every part assigned to line stocking has been pre-assigned to forklift. Furthermore, parts assigned to box supply, sequencing, stationary, and travelling kitting can only be delivered to the BoL by a tow train. Scenario 1 is inspired by discussions with
practitioners and site visits as practitioners generally pre-assign these vehicle types to the corresponding feeding policies. Moreover, it can be clearly seen in Table 1 that Scenario 1 is widely used as transportation means for these feeding policies in the literature. Similarly, in Scenario 2 to Scenario 4, a vehicle type is pre-assigned to each feeding policy. However, these scenarios are derived from the results obtained in Table 6. As depicted in the table, parts assigned to line stocking, boxed supply, sequencing, and stationary kits prefer to be delivered to the BoL by AGV, whereas, travelling kits prefers tow train delivery; this formed Scenario 2. Scenario 3 and 4 were drawn from Scenario 2, however, the second best vehicle type delivering stationary and travelling kits to the BoL were considered while forming these scenarios.

The mean of the average total feeding cost per item over the 60 instances is presented for all the cases shown in Fig. 9. Average feeding cost per item is given as the total feeding costs divided by the sum of demand over all parts $\sum_{i \in T} a_i$. The results of the optimisation model (VS and VM) and the heuristic scenarios (FS and FM) are compared in Table 8.

We aim to compare the results from the optimisation model (VS and VM) to the heuristics scenarios (FS and FM) and it turned out, some problem instances are infeasible in the heuristic settings. To be fair with the cost comparison in Table 5, we created a separate column (i.e., feeding cost $V (\text{€ } / \text{unit})$) for the optimisation model, which corresponds to the solution of the feasible instances in each scenario. We repeated the same process for the lower bound (see lower bound $V (\text{€ } / \text{unit})$). A problem instance is infeasible for a heuristic scenario when the available space at the border of line is insufficient or when transportation demands exceed capacity. The latter is linked to Constraint (20) which allows only a single vehicle of any type over all routes for a station and all taks.

We provide information about the following in Table 8: the number of feasible instances (# feas); the mean for the average feeding cost per item over the feasible instances (feeding cost $\text{€ } / \text{unit}$) and the corresponding lower bound (lower bound $\text{€ } / \text{unit}$); the mean for the average feeding cost per item for the optimisation model (i.e., VS and VM) over each scenario’s feasible instances (feeding cost $V (\text{€ } / \text{unit})$) and the corresponding lower bound (lower bound $V (\text{€ } / \text{unit})$); the percentage change in cost between the optimisation model and the heuristic scenarios ($\Delta$ feeding cost (%) and the percentage change in the lower bounds ($\Delta$ lower bound (%)). The results show that the average feeding cost per item in V models is at least 7.54% cheaper than in the first scenario. Furthermore, the model proves to be better than the other three scenarios by at least 0.83%. Notice that these three scenarios are among the best possible scenarios based on the result in Subsection 5.4. In addition, when comparing the VS result to the VM result, we found out that up to 0.05% of the feeding cost would
be saved when allowing mixed feeding policies in a route. This implies that the average feeding cost per item of VM is cheaper than that of VS by 0.05%.

To highlight the effect of forkift inclusion in the vehicle types considered, in an additional experiment, we prevented the optimisation model (VS and VM) from allowing forkift deliveries. The results indicate that, just as in scenarios 2, 3, and 4, almost half of the problem instances are infeasible. This further underlines the importance of forkift usage for in-house transportation. For the feasible problem instances, there is a negligible (< 0.01%) change in costs between the optimal solution and the solution without forkift deliveries for both versions of the model. Similarly, we prevented the optimisation model (VS and VM) from allowing AGV deliveries, in order to quantify the importance of AGV inclusion in the vehicle types considered. In this case, all problem instances remain feasible. However, average feeding costs per item increase by 0.93% and 0.90% for the VS and VM settings, respectively.

Table 9 shows the computational aspects of solving the different scenarios. The number of feasible problem instances (# feas), number of instances that attained optimality within the time limit (# opt), the average solving time (time(s)) and average gap (gap(%)) over all feasible problem instances were recorded for all cases discussed in Table 8. It can be seen that the average solving time for each of the scenarios is lower than the average solving time of the model. This is expected due to the fact that the model is simplified in those scenarios due to the pre-assignment of vehicle types. Furthermore, the actual gain of the model may be slightly higher as gaps are smaller for the heuristics approaches.

In addition, the flexibility and adaptability of the model can be further assessed with respect to the number of feasible problem instances. While the model could solve all instances successfully, the first scenario could only solve 54 problem instances and in scenarios 2, 3, and 4 only around half of the problem instances were feasible.

### 6. Conclusion and future research

In this paper, we proposed a mixed integer programming model for the optimisation of in-house transportation and assembly line feeding. This research includes a number of novelties: (i) vehicle type selection decisions for all parts are introduced for the first time. In combination with assembly line feeding decisions per part, this precisely determines logistics flows for all vehicle types selected; (ii) we accurately build routes for AGVs and tow trains that are used for periodic milk-runs. These routes involve paths between the warehouse or the supermarket and a sequence of consecutive assembly stations; (iii) the model includes the possibility of transporting parts assigned to either a single feeding policy or a mixture of feeding policies on the same route and vehicle; (iv) for every vehicle type, the model calculates the fleet size.

To reduce solving times for the model, we proposed a cut and branch (C&B) approach with valid inequalities. Moreover, the results for 60 instances were compared to heuristic scenarios that may be applied in practice and it was shown that the model outperforms these approaches by up to 7.56%. In addition, the model demonstrated a high level of flexibility by solving all problem instances considered, whereas, some of the heuristic scenarios encountered infeasibilities in up to half of the problem instances considered.

Section 3 lists some assumptions that are made in this study. As a consequence, some directions for future research will be discussed briefly when these assumptions are considered relaxed. Firstly, while replenishment of parts is carried out by forklifts, it could be interesting to assign preprocessed parts to vehicle types in order to further reduce replenishment costs and ultimately, the total feeding cost. Secondly, in order to avoid computational issues, Constraints (6) and (8) were included to allow only one type of stationary kit per station and one travelling kit respectively. This assumption can however be relaxed by modelling the problem differently. Furthermore, space at the BoL was considered to be the same across all stations. Therefore, BoL space was provided as an input into the model. Relaxing this assumption could further increase the computation time as the model would have to decide the size of every station. Finally, while part demand is assumed to be deterministic, i.e., part demand is known with certainty, it could be interesting to explore stochastic part demand. Even though further studies might lead to exciting results, it is expedient to state that the assumptions considered in this study are realistic to a certain extent as some of the assumptions are used in practice. e.g., forklifts are generally used for part replenishment, parts are initially placed in pallets at the warehouse. Moreover, assumptions such as deterministic part demand, fixed space at the BoL as well as one type of stationary and travelling kit are often used in the literature. This research shows how vehicle type selection can be incorporated into the ALFP. However, other decisions related to factory design such as supermarket planning are included as an input. This can be compared to the optimisation of a brown-field plant. Thus, future research may optimise a green-field plant by additionally incorporating supermarket planning decisions into the ALFP. This would entail the location of supermarkets on the shop floor and the location of parts within the different supermarkets. Moreover, the computational speed of the model may be further improved upon. This will enable the model to solve larger instances such that the impact of real industry instances can be analysed. Furthermore, we have included three vehicle types in this research; the inclusion of additional vehicle types may reduce the transportation cost and ultimately, the total feeding costs even further.

### Appendix A. Model extension

As mentioned in Section 5, the model described in Subsections 3.1 and 3.2 can be referred to as "single feeding policy in a route" since any vehicle of type $m \in M'$ performing a milk run on any of the routes can only transport parts assigned to a single feeding policy. To allow mixed feeding policies in a route, the model is modified. Namely, Constraints (22), (24) and (25) are replaced by Constraints (58), (59) and (60), respectively.

\[
\frac{\lambda_{st}}{H_{ps}} d_{ps} x_{pms} - M \left(1 - \sum_{t \in T} v_{mrw} \right) \leq f_{ps} \\
\forall p \in \{1, 2\}, s \in S, i \in I, m \in M', t \in T, w \in W_p
\]

\[
\frac{\lambda_{st}}{H_{fs}} x_{fms} - M \left(1 - \sum_{t \in T} v_{mrw} \right) \leq f_{fs}
\]
∀s ∈ S, f ∈ F, i ∈ I_f : f_{i+1} ≠ f_i, t ∈ T, m ∈ M′  
\sum_{t \in T} k_{ms} - M(1 - \sum_{r \in R_s} v_{mtr2}) \leq f_i, \quad ∀s ∈ S, t ∈ T, m ∈ M′ \tag{59}

Constraints (20) and (21) are replaced by Constraints (61) and (62).

\sum_{t \in T} v_{mtrw} \leq 1 \quad ∀m ∈ M′, s ∈ S, w ∈ W \tag{61}

\sum_{i \in I_s, p \in P_s} x_{pm} \leq M \sum_{t \in T} \sum_{r \in R_s} v_{mtrw} \quad ∀m ∈ M′, s ∈ S, w ∈ W \tag{62}

Furthermore, Constraints (63) and (64) will replace Constraints (30) to (33).

\sum_{s \in S, i \in I_s} \frac{\lambda_i}{n_1} (\frac{f_{mtr1}}{n_1 m v_m} + I_{m1}) x_{1m} - M(1 - v_{mtr1}) \leq bt_{mtr1} \quad ∀m ∈ M′, t ∈ T, r ∈ R \tag{63}

\sum_{s \in S, i \in I_s} \frac{\lambda_i}{n_2} \left( \frac{f_{mtr2}}{n_2 m v_m} + I_{m2} \right) x_{2m} + \sum_{j \in J_s, j \neq i} \frac{\lambda_j}{n_1} \left( \frac{f_{mtr1}}{n_1 m v_m} + I_{m1} \right) x_{1m} + \sum_{s \in S, i \in I_s, j \neq i, j \in I_s} \frac{\lambda_i}{n_1} \left( \frac{f_{mtr1}}{n_1 m v_m} + I_{m1} \right) x_{1m} + \sum_{s \in S} \frac{\lambda_i}{n_4} \left( \frac{f_{mtr2}}{n_4 m v_m} + I_{m4} \right) k_{ms} + \sum_{s \in S, i \in I_s} \frac{\lambda_i}{n_5} \left( \frac{f_{mtr2}}{n_5 m v_m} + I_{m5} \right) - \lambda(1 - v_{mtr2}) \leq bt_{mtr2} \quad ∀m ∈ M′, t ∈ T, r ∈ R \tag{64}

Finally, Constraint (34) will be replaced by Constraint (65).

\sum_{r \in R} u_{mwr} \leq u_{mwr} \quad ∀m ∈ M′, w ∈ W \tag{65}

The modification of the model allows a new optimisation approach. The base model considers (i) single feeding policies in a route, whereas the modification considers (ii) mixed feeding policies in a route. The impact of this change can be found in the analysis in Section 5.

Appendix B. Proposition

Proposition B0.1. Assigning a part to a feeding policy and vehicle type requires that at least one vehicle of that type must be available to perform the task.

Proof. To verify this proposition, it will suffice to show the authenticity of Equations (56) and (57). Equation (56) was inspired through observation of Constraint (36) for parts assigned to forklifts as well as integrating Constraints (30) and (34) for parts assigned to AGVs and tow trains. Constraint (36) provides the number of forklifts used for line stocked parts over the planning horizon. To this end, each part assigned to line stocking and forklift must be delivered to the Bol by a forklift. This results in Equation (56). On the contrary, a segment of Constraint (30) provides the number of vehicles of type m ∈ M′ used in delivering line stocked parts to the Bol over all possible routes at different intervals. Furthermore, Constraint (34) ensures that these vehicles do not exceed vehicles of type m ∈ M′ used during the planning horizon. Consequently, every part assigned to line stocking and vehicle type m ∈ M′ must be delivered to the Bol by the same vehicle type. This also results in Equation (56). Equation (57) can be derived similarly by considering Constraint (37) for parts assigned to forklifts as well as integrating Constraints (34) and each of the Constraints (30) to (33) for parts assigned to AGVs and tow trains. □

References


[17] Caputo AC, Pelagagge PM, Salini P. Selection of assembly lines feeding policies based on parts features. IFAC-PapersOnline 2016;49(12):185–90.


[34] Schmid NA, Limère V, Raa B. Mixed model assembly line feeding with discrete location assignments and variable station space. Omega (Westport) 2020:102286.