Vibration suppression in multi-body systems by means of disturbance filter design methods

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Abstract

This paper addresses the problem of interaction in mechanical multi-body systems and shows that subsystem interaction can be considerably minimized while increasing performance if an efficient disturbance model is used. In order to illustrate the advantage of the proposed intelligent disturbance filter, two linear model based techniques are considered: IMC and the model based predictive (MPC) approach. As an illustrative example, multivariable mass-spring-damper and quarter car systems are presented. An adaptation mechanism is introduced to account for linear parameter varying LPV conditions. In this paper we show that, even if the IMC control strategy was not designed for MIMO systems, if a proper filter is used, IMC can successfully deal with disturbance rejection in a multivariable system, and the results obtained are comparable with those obtained by a MIMO predictive control approach. The results suggest that both methods perform equally well, with similar numerical complexity and implementation effort.

Keywords

Model based control, internal model control, predictive control, disturbance rejection, interaction, diophantine filter, dynamic compensation, vibration control, damping, multivariable system

1. Introduction

The importance of feedback compensators has been recognized as being key elements to guarantee the desired performance of a process. Two main objectives are required: the ability to track a desired set-point (the *servo* problem) and the property to efficiently reject disturbances (the *regulatory* problem) (Skogestad and Postlethwaite, 2005; Jiang, 2006). The challenge for the control engineer consists in developing a strategy to deal with these objectives, which assumes some sort of trade-off between fast closed loop dynamics and robustness.

To achieve higher performance and autonomy, both the system hardware and the control logic must advance. The system hardware, including sensors, actuators, computing platforms, and communication networks determines the ultimate autonomy levels and performance that can be achieved in an ideal world free of practical worries such as disturbance, sensor failures, and uncertain and varying operating conditions. The control algorithm, comprising feedback

control as well as feedforward and reference generation, determines how much of this performance and autonomy is actually achieved in the nonideal industrial reality.

Due to their simplicity, control schemes currently used in industry are incapable of fully exploiting the continually increasing potential of hardware. According to reports from the actual industrial landscape, 60% of controllers (almost entirely PID based control) are poorly or manually tuned. Moreover, the complex dynamics of industrial processes are clearly in

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constant change, leading to poor performance of these controllers after only six months (Bauer et al., 2016). It is mandatory thus that i) PID controllers are used for low level control and tuned based on specifications with CAD tools, and ii) advanced control is employed for plantwide optimization. The advanced control algorithms need to cope with complex systems dynamics as hardware evolution generally leads to systems with internal flexibilities, multiple sensors, and multiple actuators. Combining complex dynamics with the need for higher performance and robustness makes the control task very challenging.

In the literature, various methods have already been developed to deal with external Gaussian noise, but most of these are designed in state-space formulation (Li et al., 2016) and based on an H_∞ approach (Azman et al., 2015; Wang et al., 2015). However, to the knowledge of the authors, there are no or limited papers that discuss the rejection of periodical disturbances and/or the oscillation introduced by the dynamics of the system. Moreover, Samad (2017) has shown that the most relevant control algorithms for industry are PID and MPC followed later by robust control. Hence, it is our goal to provide pragmatic approaches relevant to industry, and therefore two model based advanced control methodologies are proposed in answer to these industrial challenges. A feedback based controller based on an internal model (IMC) is proposed with augmented disturbance filter design. Used in a single input single output context, periodic disturbances and repetitive tasks may be easily embedded in this disturbance filter, and consequently the overall performance can be significantly increased. Used in a multivariable setting, the interactions between subsystems are thus compensated, including possibly additional dynamic nodes. The second advanced method is a predictive control (MPC) algorithm with intrinsic feedforward compensation capabilities. This algorithm also contains an augmented disturbance filter and can serve similar purposes to the IMC algorithm.

In this paper, we investigate the potential of the IMC as an integrated solution to effectively reject disturbances entering at the input of the process. Compared to previous studies, the proposed methodology does not require any frequency weights to design the controller. The proposed design methodology has closed loop performance oriented tuning parameters, making it a versatile method. It uses concepts of generalized disturbance rejection by means of diophantine equations. Similar concepts are encountered in model based predictive algorithms (Meadows and Badgwell, 1998; Maciejowski, 2000; Qin and Badgwell, 2002; Camacho and Bordons, 2004), in which the authors are well-experienced through their development of the Extended Prediction Self-Adaptive

Controller (EPSAC) (De Keyser, 2003). In the EPSAC control formulation, if the main frequency of the disturbance is known, it is possible to model it and to provide this additional information to the controller, obtaining significant improvement in both closed loop performance and disturbance rejection capability (De Keyser and Ionescu, 2003).

The class of dynamic processes that can be addressed by the proposed methodology includes (but is not limited to) heavy duty industry utility machinery (weaving, agricultural combined harvesting, mining, and transportation machinery), all being composed of interacting subsystems and vibration propagation features. Theoretical assumptions are demonstrated on an illustrative example from mechanical systems, known to be relevant for vibration suppression, interaction mechanism, and stochastic disturbance profiles. The performance and efficiency of the proposed strategy is evaluated using a multivariable mass-spring-damper system with poor damping properties. As such, this specific fourth-order system represents a challenge regarding the design of any regulator (Alcantara et al., 2011b).

The paper is organized as follows: in Section 2, the IMC controller and the prediction-based controller is presented. Next, the mass-spring-damper system used as a testbench for the proposed methodology is described in Section 3. The outcome of the proposed methodology for general disturbance rejection is analyzed and the results of the designed controllers (IMC and EPSAC) along with some implementation aspects are presented in Section 4. Finally, a conclusion section summarizes the main outcomes of this work.

2. Proposed methodologies

2.1. IMC with augmented disturbance filter

Internal Model Control (IMC) (Garcia and Morari, 1982; Rivera et al., 1986; Morari and Zafiriou, 1989) is a control strategy based on the use of a plant model that captures the main process characteristics and allows a proper compensation of the process output, followed by an algebraic formulation that provides the optimal manipulated variables. Internal Model Control (IMC) techniques are a subset of model based control techniques, which are often used in chemical process control, but have recently been borrowed in other applications (e.g. mechatronics). These techniques have the potential to achieve good closed loop performance while taking into account the model structure of the process (e.g. varying time delays and periodic disturbances). Many successful implementations in real life processes have been reported (e.g. Rivera et al. (1986); Morari and Zafiriou (1989); Bequette (2003), to mention just a few). It is however observed

that, although basic IMC provides adequate suppression of output disturbances, it does a poor job of suppressing input disturbances when the process dynamics are significantly slower than the desired closed-loop dynamics (Chien and Fruehauf, 1990; Ho et al., 1994). Consequently, later studies have been focused on the search for new filters and/or alternative procedures to improve the closed-loop bandwidth and robustness, as presented in Campi et al. (1994). The conventional filter was modified to improve input disturbance attenuation on stable plants (Horn et al., 1996) and was later extended to unstable plants (Lee et al., 2000). Some of the ideas of robust control were introduced in Dehghani et al. (2006), where a numerical design based on H_∞ ideas was used. Following the same trend, Alcantara et al. (2011a) propose a simpler IMC-like H_∞ , which requires less assumptions, thus overcoming some basic limitations of similar approaches. An improvement to the previous method was obtained in Alcantara et al. (2011b), where an analytical solution based on H_2 was proposed to achieve a compensator that balances the input/output disturbances' rejection performance. One of the limitations of the previous strategy is the difficulty of finding the required weighting design parameters for the case of low-damping systems with more than one resonant frequency (i.e. higher than second order).

The Internal Model Control (IMC) philosophy relies on the Internal Model Principle, which states that control can be achieved only if the control system encapsulates, either implicitly or explicitly, some representation of the process to be controlled. In particular, if the control scheme has been developed based on an exact model of the process, then perfect control is theoretically possible (Bequette, 2003). Thus, assuming a process described by the transfer function $P(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})}$, the ideal will be to design a controller defined as $R(q^{-1}) = \frac{A(q^{-1})}{B(q^{-1})}$, which is the inverse of the transfer function of the process. In this way, by having complete knowledge about the process being controlled and by designing such a controller, we can achieve perfect control with the output of the system, even in open loop, always equal to the setpoint.

However, in practice, process-model mismatch is common; the process model may not be invertible and the system is often affected by unknown disturbances. Thus, it is not possible to design a “perfect” controller $R(q^{-1}) = \frac{A(q^{-1})}{B(q^{-1})}$, and the above open loop control arrangement will not be able to maintain output at setpoint. The Internal Model Control (IMC) strategy has the generic block structure depicted in Figure 1.

In this figure, d is an unknown disturbance affecting the system. The manipulated input u is introduced to both the process and its model. The process output, y ,

Figure 1. Schematic overview of the IMC structure.

is compared with the output of the model $\hat{x}$, resulting in a signal $\hat{d}$. If the process is well known, then a perfect estimation of the disturbances will be reached. It is important in IMC control to avoid an unstable or noncausal compensator transfer function by adding a filter $F(q^{-1})$ to make the compensator proper (Bequette, 2003), and to separate the model in “invertible” and “non invertible” parts in order to design the controller. Within the IMC context, the controller is tuned by using the inverse of the process to compensate the process dynamics. In order to apply this approach, the process must be split into an invertible (good) part and a noninvertible (bad) part. For instance, if the transfer function of the process has a time delay or presents nonminimum phase dynamics, these make up the bad part of the process denoted as $B_b(q^{-1})$. The remainder of the transfer function makes up the good part of the process and is denoted as $B_g(q^{-1})$. The inverse of the invertible part is a causal and stable transfer function with $B(q^{-1}) = B_g(q^{-1})B_b(q^{-1})$.

As mentioned earlier, a causal controller is designed by adding a filter $F(q^{-1})$. This filter is tuned with respect to the system disturbance and the user may choose between two approaches. For the first approach, hereafter called a *basic filter*, the filter is designed in order to reject a step disturbance d at the output of the process and its form is given by

$$F(q^{-1}) = \frac{(1+a)^n}{(1+a \cdot q^{-1})^n} \quad (1)$$

with steady state gain $F(1) = 1$ and a a design parameter defined as $a = -e^{-T_s/\lambda}$ where T_s is the sampling period. The (negative) values of this design parameter are within $0 \ll |a| < 1$, and they are related to the closed-loop speed as follows: if λ increases, the $|a|$ converges towards 1 and the settling time will be larger.

Note that the *basic filter* cannot reject a step disturbance at the input of the process or a ramp disturbance at the output of the process. The second approach overcomes this limitation, hereafter called the *extended*

filter, given by

$$F(q^{-1}) = \frac{(1+a)^n}{(1+a \cdot q^{-1})^n} \cdot (1-f+fq^{-1}) \quad (2)$$

with

$$f = \frac{na}{1+a} - (b_1 + 2b_2 + 3b_3 \dots + mb_m) \quad (3)$$

and the parameters b_i the coefficients of the bad part of the process

$$\begin{cases} B_b(q^{-1}) = 0 + b_1q^{-1} + b_2q^{-2} + \dots + b_mq^{-m} \\ \sum_{i=1}^m b_i = 1 = B_b(1) \end{cases} \quad (4)$$

However, for most industrial applications, the process disturbance has a different dynamics than a step or a ramp. Thus, the generic idea is to design a proper filter that can model any type of disturbance and provide this additional information to the controller. This approach, called a *diophantine filter*, was presented in detail in De Keyser et al. (2015) and has concepts borrowed from predictive control (De Keyser and Ionescu, 2003).

Given a repetitive (e.g. sinusoidal) disturbance acting on the system, this can be modelled and used as an augmented disturbance filter in the IMC algorithm. This can be best viewed in Figure 2, where the nominal closed loop can be defined as in the following equation, based on the IMC control structure (Figure 1) and on $B(q^{-1}) = B_g(q^{-1})B_b(q^{-1})$

$$y(t) = B_b(q^{-1})F(q^{-1})w(t) + [1 - B_b(q^{-1})F(q^{-1})] \frac{C(q^{-1})}{D(q^{-1})} h(t) \quad (5)$$

Figure 2. Conceptual block scheme of the augmented disturbance filter.

where $h(t)$ is a discrete-time unit impulse. The disturbance model can be defined as a function of the type and location of the disturbance

- step at process output: $\frac{C(q^{-1})}{D(q^{-1})} = \frac{\dots}{1-q^{-1}}$
- step at process input: $\frac{C(q^{-1})}{D(q^{-1})} = \frac{\dots}{(1-q^{-1})A(q^{-1})}$
- ramp at process output: $\frac{C(q^{-1})}{D(q^{-1})} = \frac{\dots}{(1-q^{-1})^2}$
- periodic/repetitive: $\frac{C(q^{-1})}{D(q^{-1})} = \frac{\dots}{(1-e^{j\omega T_s}q^{-1})(1-e^{-j\omega T_s}q^{-1})}$

A periodic disturbance is not the only dynamics that can be rejected. Other types of input/output disturbances, as well as specific process dynamics, can be filtered if the disturbance filter $F(q^{-1}) = \frac{F_N(q^{-1})}{F_D(q^{-1})}$ is adequately designed (De Keyser et al., 2015).

According to the principle of internal model control, the main idea is to include the disturbance signal, defined by $D(q^{-1})$, in the controller in order to have efficient disturbance rejection. By including the dynamics of the disturbance signal $D(q^{-1})$ into the filter design, the poles of the disturbance signal become zeros in the controller, and thus perfect compensation can be achieved. Following this reasoning, the numerator of $[1 - B_b(q^{-1})F(q^{-1})]$ has to explicitly include $D(q^{-1})$, which employs the following equivalence

$$F_D(q^{-1}) - B_b(q^{-1})F_N(q^{-1}) = D(q^{-1})Q(q^{-1}) \quad (6)$$

Given $D(q^{-1})$, $F_D(q^{-1})$, and $B_b(q^{-1})$, one needs to find the polynomials $F_N(q^{-1})$ and $Q(q^{-1})$ using concepts of generalized disturbance rejection via the diophantine equation. Solving the diophantine equation leads to the coefficient identification procedure, taking into account that the orders of F_N and Q have to be chosen such that the diophantine equation results in a unique solution. Based on the fact that the controller $R(q^{-1})$ for an IMC structure (Figure 1) is given by

$$R(q^{-1}) = \frac{A(q^{-1})F(q^{-1})}{B_g(q^{-1}) - F(q^{-1})B(q^{-1})} \quad (7)$$

it can be seen that the poles $D(q^{-1})$ of the disturbance filter are then indeed poles of the controller.

2.2. Prediction-based control MPC-EPSAC with augmented disturbance filter

Model based predictive control (MPC) is a control methodology that uses a process model on-line for calculating predictions of future plant output, and based on that optimizes future control actions.

Figure 3. The MPC principle.

The MPC principle is depicted in Figure 3. Referring to this figure, the following strategy is followed:

- At each “current” moment t , the process output $y(t+k)$ is predicted over a time horizon $k = N_1 \dots N_2$. The predicted values are indicated by $y(t+k|t)$, and the value of N_2 is called the prediction horizon. The prediction is done by means of a model of the process and depends on the past inputs and outputs, but also on the future control scenario $\{u(t+k|t), k = N_1 \dots N_2\}$.
- A reference trajectory $\{r(t+k|t), k = N_1 \dots N_2\}$, evolving towards the setpoint w , is defined over the prediction horizon, describing how we want to guide the process output from its current value $y(t)$ to its setpoint w .
- The control vector $\{u(t+k|t), k = N_1 \dots N_2\}$ is calculated in order to minimize a specified cost function. For illustration here, the simplest kind of cost function is shown, where the control effort is not taken into account ($\lambda = 0$)

$$\sum_{k=N_1}^{N_2} [r(t+k|t) - y(t+k|t)]^2 \xrightarrow{\text{Minimize}} u(t+k|t) \quad (8)$$

where N_1 and N_2 are called the cost horizons.

- The first element $u(t|t)$ of the optimal control vector is actually applied to the real process. All other elements of the calculated control vector can be forgotten. This approach is typical for the receding horizon principle. Hence, at the next sampling time, this optimization is repeated again, taking into account the new measurement information. This actually introduces the feedback component into the whole strategy, resulting in a closed-loop configuration.

MPC is not one particular control strategy but in fact stands for a large family of different control strategies. In this paper, the linear EPSAC-approach (Extended Prediction Self-Adaptive Control) as described in De Keyser (2003) is used.

Unlike other academic MPC strategies, the EPSAC approach is based on input-output filtering techniques, thus making it very attractive for industrial applications. In fact, it has been successfully embedded and applied in manifold industrial settings, ranging from very fast processes (e.g. electronics) (De Keyser et al., 2006; De Keyser and Donald, 2007; Castano et al., 2015; Dutta et al., 2016) to moderate and slow processes (e.g. thermal plants) (De Keyser and Hernandez, 2014; Dutta et al., 2014; Ionescu et al., 2016). The simplicity of the methodology allowed its implementation in FPGAs and PLC execution platforms (Folea et al., 2016).

In the EPSAC-approach to MPC, the future response $y(t+k|t)$ can be considered the sum of two parts

$$y(t+k|t) = y_{base}(t+k|t) + y_{opt}(t+k|t) \quad (9)$$

The first part is the base response $y_{base}(t+k|t)$, which is the effect of the past inputs $u(t-1), u(t-2) \dots$, a future base control sequence $u_{base}(t+k|t)$, and the predicted disturbance $n(t+k|t)$. The second part of the future response is $y_{opt}(t+k|t)$, which represents the effect of the optimizing control actions $\delta u(t|t), \dots, \delta u(t+N_u-1|t)$ with $\delta u(t+k|t) = u(t+k|t) - u_{base}(t+k|t)$, in a control horizon N_u .

The optimized output can be expressed as the discrete-time convolution of the unit impulse response coefficients $h_1, \dots, h_{N_2}$ and unit step response coefficients $g_1, \dots, g_{N_2}$ of the system as follows

$$y_{opt}(t+k|t) = h_k \delta u(t|t) + h_{k-1} \delta u(t+1|t) + \dots + g_{k-N_u+1} \delta u(t+N_u-1|t) \quad (10)$$

This can all be summarized in matrix notation as

$$\begin{bmatrix} y_{opt}(t+N_1|t) \\ y_{opt}(t+N_1+1|t) \\ \vdots \\ y_{opt}(t+N_2|t) \end{bmatrix} = G \begin{bmatrix} \delta u(t|t) \\ \delta u(t+1|t) \\ \vdots \\ \delta u(t+N_u-1|t) \end{bmatrix} \quad (11)$$

where G is given by

$$G = \begin{bmatrix} h_{N_1} & h_{N_1-1} & \dots & g_{N_1-N_u+1} \\ h_{N_1+1} & h_{N_1} & \dots & \dots \\ \dots & \dots & \dots & \dots \\ h_{N_2} & h_{N_2-1} & \dots & g_{N_2-N_u+1} \end{bmatrix} \quad (12)$$

This can be again summarized in the key EPSAC equation

$$Y = Y_{base} + Y_{opt} = \bar{Y} + GU \quad (13)$$

The cost function which needs to be optimized contains both the control error and the control effort

$$\sum_{k=N_1}^{N_2} [r(t+k|t) - y(t+k|t)]^2 + \rho \sum_{k=0}^{N_u-1} [\Delta u(t+k|t)]^2 \quad (14)$$

where ρ is a weighting parameter (another design parameter) and variable $\Delta u(\cdot)$ is the control increment ($\Delta u(t+k|t) = u(t+k|t) - u(t+k-1|t)$) (De Keyser, 2003).

Once the output is predicted, it is possible to optimize the control signal U by minimizing the cost function

$$V(\mathbf{U}) = \sum_{k=N_1}^{N_2} [r(t+k|t) - y(t+k|t)]^2 \quad (15)$$

Notice that the controller cost function (equation (15)) can be easily extended to many alternative cost functions (similar to the approach in optimal control theory) as described in De Keyser (2003).

Of particular interest when applying MPC-EPSAC in a multivariable setting is its capability to optimize either i) at plantwide level, or ii) at subsystem level, described briefly hereafter.

Plantwide optimization takes into account the interaction between subsystems and uses it to its advantage, temporarily allowing slightly reduced performance in one subsystem to help the interacting subsystem benefit from faster recovery. Seen as an integrated optimization of heterogeneous subsystems, this robust optimization seems to give the best results in terms of performance and energy efficiency.

Since the identified system is linear, a linear predictor can be used as in

$$\begin{aligned} y_1 &= G_{11} \cdot \Delta u_1 + G_{12} \cdot \Delta u_2 + f_1 \\ y_2 &= G_{21} \cdot \Delta u_1 + G_{22} \cdot \Delta u_2 + f_2 \end{aligned} \quad (16)$$

with $\Delta = 1 - q^{-1}$ the difference filter and y_k and u_k ($k = 1, 2$) the vectors of future outputs and inputs of the predictor

$$\begin{aligned} y_k &= [y_k(t+N_1) \dots y_k(t+N_2)]^T \\ u_k &= [u_k(t) \dots u_k(t+N_u-1)]^T \end{aligned} \quad (17)$$

with $k = 1, 2$, N_1 the minimum prediction horizon (i.e. the number of delays in samples plus one), N_2 the maximum prediction horizon (i.e. tuning of the closed loop response), N_u the control horizon (usually kept equal to one), and G_{jk} a matrix containing the coefficients of the step responses of the k -th input to the j -th output starting from N_1 to N_2 . The number of future control

samples that can be changed is equal to N_u . The terms f_k denote the free response of the predictor at the k -th output, i.e. the response y_k for the predictor when $\Delta u_j = 0$, with $j = 1, 2$.

If no constraints are active, the cost function adapted from equation (14) for the multivariable case is given by

$$J(\Delta u_1, \Delta u_2) = \sum_{k=1}^2 \|r_k - y_k\|_2^2 + \rho \|\Delta u_k\|_2^2 \quad (18)$$

with $\|\cdot\|_2$ as the quadratic norm of the vector, ρ a weighting factor and r_k the vector of the future set-points of the k -th output. Minimization with respect to Δu_k results in the optimal control action

$$\Delta u = (G^T G + \rho I)^{-1} G^T (r - f) \quad (19)$$

with G , Δu , and $r-f$ defined by

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}; \quad \Delta u = \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix}; \quad r - f = \begin{bmatrix} r_1 - f_1 \\ r_2 - f_2 \end{bmatrix}$$

and I a unity matrix. At time t , only the control action $u_k(t)$, ($k = 1, 2$) is applied to the system, and the whole procedure is repeated at the next time step.

For *subsystem optimization*, the tracking error and the control effort are optimized taking into account the interaction with other subsystems, but weighting only its own performance, at the cost of disturbing other, interconnected subsystems. The cost function can be written as

$$\begin{aligned} y_1 &= G_{11} \cdot \Delta u_1 + G_{12} \cdot \Delta u_2 + f_1 \\ y_2 &= G_{21} \cdot \Delta u_1 + G_{22} \cdot \Delta u_2 + f_2 \end{aligned} \quad (20)$$

which expresses the fact that during the prediction of the output y_1 , the input u_2 is treated as a known disturbance. Similarly for y_2 . The optimal cost is

$$J_k(\Delta u_k) = \|r_k - y_k\|_2^2 + \rho \|\Delta u_k\|_2^2 \quad (21)$$

which results in two equations linear in the unknown control vectors Δu_k , ($k = 1, 2$). Solving these equations gives the optimal control strategy per subsystem as

$$\Delta u = (G_{diag}^T G + \rho I)^{-1} G_{diag}^T (r - f) \quad (22)$$

with elements defined as

$$G_{diag} = \begin{bmatrix} G_{11} & 0 \\ 0 & G_{22} \end{bmatrix}$$

the decentralized matrix, whereas G contains all interaction elements as well. The robustness of the MPC control strategy has been discussed in Castano et al. (2015) and is beyond of the scope of this paper.

3. Illustrative example

As an illustrative example of a multi-body mechanical system, i.e. a quarter car module, a mass spring

Figure 4. A mass-spring damper system.

damper multivariable system is presented. For this process, the IMC controller was designed in order to minimize the interaction between subsystems. The proposed idea is to apply a decentralized approach and to use the augmented disturbance filter to compensate for the interaction between the subsystems (considered as interconnected disturbances). Note that low level controllers such as PID's cannot efficiently deal with such a complex structure of systems. In this paper, an IMC controller for MIMO systems was developed based on diophantine equations and the disturbance filter concept. The IMC filter is designed taking into account the dynamics of the subsystems, and thus the proposed controller not only rejects input disturbances but also minimizes the interaction between subsystems. The results obtained using the IMC approach are comparable with the results obtained by a multivariable predictive control approach.

The MIMO mass spring damper benchmark system is given by the following state space model

Figure 5. Bode plot of the 2DOF mass spring damper system.

(Shahrurz, 2009)

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{m_1} & \frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{b_1}{m_1} \\ \frac{k_1}{m_2} & -\frac{k_1 + k_2}{m_2} & \frac{b_1}{m_2} & -\frac{b_1 + b_2}{m_2} \end{bmatrix} \quad (23)$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \quad (24)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (25)$$

Figure 6. Time evolution of the output of the system for input step disturbance using IMC controller with: (a) extended filter; (b) diophantine equation; (c) diophantine equation with pre-filter for input tracking.

with $k_1 = 1$ N/m; $k_2 = 4$ N/m; $b_1 = 0.2$ Nm/s; $b_2 = 0.1$ Nm/s; $m_1 = 1$ kg; $m_2 = 2$ kg. This system can approximate well the dynamics of a simplified half-car suspension pendulum with two body mass, a simplified model of coupled combine harvester (1st mass) and header (2nd mass) dynamics. This system has been used to report iterative learning control algorithms (Doh and Ryoo, 2008), disturbance estimators (Tang et al., 2014), and distributed control (Yook et al., 2001), to mention a few.

The Bode plot of the system is given in Figure 5 below. The peak frequencies are in the vicinity of 1 rad/s frequency.

This section briefly summarizes the control strategies used in this study. The equivalent transfer function matrix resulting from equations (23), (24), and (25) is used for both prediction and emulation of the real process. All simulations used the same sampling period of 0.1 seconds and the same prediction horizon of

50 samples. The control horizon and delay have been set to 1 sample.

In order to implement the transfer function of the process in real time, we need to find its equivalent in discrete time. A suitable sampling period T_s of 10 ms has been chosen. The discrete time equivalent transfer function expressed in q^{-1} is given by

$$P_{11}(q^{-1}) = \frac{\left\{ 0.004963q^{-1} - 0.004799q^{-2} - 0.004775q^{-3} \right\}}{1 - 3.931q^{-1} + 5.828q^{-2} - 3.863q^{-3} + 0.9656q^{-4}} \quad (26)$$

$$P_{12}(q^{-1}) = P_{21}(q^{-1}) = \frac{\left\{ 1.856e^{-5}q^{-1} + 7.132e^{-5}q^{-2} \right\}}{\left\{ 1 - 3.931q^{-1} + 5.828q^{-2} - 3.863q^{-3} \right\} + 0.9656q^{-4}} \quad (27)$$

Figure 7. Output of the system considering input step disturbance with MIMO-EPSC approach using: (a) default filter; (b) intelligent filter.

Figure 8. Default filter—Comparison between: (a) IMC controller and (b) EPSAC controller.

$$P_{22}(q^{-1}) = \frac{\begin{Bmatrix} 0.002482q^{-1} - 0.002421q^{-2} - 0.002433q^{-3} \\ +0.002421q^{-4} \end{Bmatrix}}{\begin{Bmatrix} 1 - 3.931q^{-1} + 5.828q^{-2} - 3.863q^{-3} \\ +0.9656q^{-4} \end{Bmatrix}} \quad (28)$$

For implementing MPC-EPSAC, this process model is used directly for prediction. For the IMC approach, it is necessary to split $B(q^{-1})$ into a good part and a bad part as follows

$$\begin{aligned} B_g(q^{-1}) &= 0.005 - 0.0098q^{-1} + 0.0049q^{-2} \\ B_b(q^{-1}) &= 0.5013q^{-1} + 0.4987q^{-2} \end{aligned} \quad (29)$$

It is well known that the dynamics of the system play an important role in disturbance rejection. Thus, in order to efficiently compensate for the dynamics of the system, the disturbance model should include the denominator of the process transfer function such that $\frac{C(q^{-1})}{D(q^{-1})} = \frac{1}{(1-q^{-1})A(q^{-1})}$. For the case study presented here, the

denominator of the disturbance model is defined as

$$D(q^{-1}) = 1 - 4.9310q^{-1} + 9.7591q^{-2} - 9.6908q^{-3} + 4.8282q^{-4} - 0.9656q^{-5} \quad (30)$$

As previously discussed, to obtain a proper/semi-proper transfer function of the controller, a filter needs to be designed. For the IMC controller, an *extended filter* and a filter based on a diophantine equation (De Keyser et al., 2015) was considered. Note that both filters were designing within the same specifications, and the parameter a was set to -0.8 . Since the *extended filter* is well known in the literature, numerical example is shown only for the *diophantine filter* as follows

$$F(q^{-1}) = \frac{\begin{Bmatrix} 0.7044 - 2.5714q^{-1} + 3.5425q^{-2} - 2.1829q^{-3} \\ +0.5076q^{-4} \end{Bmatrix}}{\begin{Bmatrix} 1.0000 - 4.5000q^{-1} + 8.4375q^{-2} - 8.4375q^{-3} \\ +4.7461q^{-4} - 1.4238q^{-5} + 0.1780q^{-6} \end{Bmatrix}} \quad (31)$$

Figure 9. Comparison between (a) IMC controller and (b) EPSAC controller when intelligent filter is employed.

To reject the disturbance and to compensate for the dynamics of the subsystem interactions, the poles introduced by the disturbance model should be canceled by the zeros of $(1 - B_b(q^{-1})F(q^{-1}))$. Next, the poles of the disturbance model D are given as

$$\begin{cases} 0.9739 + 0.1629i \\ 0.9739 - 0.1629i \\ 1.0000 \\ 0.9916 + 0.0845i \\ 0.9916 - 0.0845i \end{cases} \quad (32)$$

while the zeros of $(1 - B_b(q^{-1})F(q^{-1}))$ (where F is computed using the diophantine equation) are

$$\begin{cases} 0.9739 + 0.1629i \\ 0.9739 - 0.1629i \\ 1.0000 \\ 0.9916 + 0.0845i \\ 0.9916 - 0.0845i \\ -0.0779 \end{cases} \quad (33)$$

4. Results and discussion

The designed control architecture for the mass spring damper benchmark system was implemented and tested in Matlab. The simulation examples are derived from a real life experimental case study and have been not included here for confidentiality reasons. Currently, these methods are employed on a real life system, but we do not have clearance for using real life data. However a similar approach for a single-input-single-output system was designed and successfully implemented on a real life process consisting of a mass-spring-damper system with two masses (De Keyser et al., 2015). Consequently, the designed controllers were validated using a hardware-in-the-loop configuration. Here, a Speedgoat real-time target machine SN3192 with an Intel Core i3 2.4 GHz CPU and 2048 MB RAM was used. Multiple experiments with different input disturbances were performed in order to show the efficiency of the proposed algorithm. For the first experiment, a step input disturbance was employed.

The results for disturbance rejection are summarized in Table I and illustrated in Figures 6(a) and 6(b) for the IMC controller. One can see that the extended IMC controller cannot efficiently reject the disturbance.

Table 1. Performance index for output error in the different control strategies.

Controller	IAE _{step dist}	IAE _{sinus dist}	Control effort _{step dist}	Control effort _{sinus dist}
IMC ext	12.5510	30.8887	0.0932	0.0026
IMC dio	5.6030	7.2511	0.0725	0.0030
EPSAC def	14.3494	98.7865	0.0595	0.0025
EPSAC int	1.8905	3.8886	0.0602	0.0023

However, if the IMC is designed based on the diophantine equation, the disturbance is successfully rejected, and the interaction between subsystems is minimized. Thus, the proposed IMC approach with the diophantine equation can also be used to compensate for special dynamics in the system.

Both these filters have zeros, hence they will introduce significant overshoot dynamics for step setpoint changes. A counter-acting solution is to use a pre-filter on the setpoint signal and cancel the zeros of the filter. If a pre-filter is used, it will have no effect on the disturbance rejection but will greatly influence the setpoint tracking and implicit control effort—see Figure 6(c).

Next to the IMC controller, a predictive controller (i.e. via the EPSAC approach) was designed and implemented. Similar to the IMC, a *default filter* (which does not take the system dynamics into account) and an *intelligent filter* (which includes the system dynamics) were used for disturbance rejection. The results obtained are illustrated in Figures 7(a) and 7(b). The second experiment investigated here consider a sinusoidal input disturbance, and the results for the IMC and EPSAC controller are given in Figures 8(a)–9(b). We may conclude that the disturbance is efficiently rejected and the dynamics of the system are well compensated if an adequate filter is used. Note that the oscillations of the control action are induced by the input disturbance signal, the effort to compensate for the dynamic of the system, and also by the quantization error effect. The results indicate that both controllers (IMC and EPSAC) have similar performance.

To further prove the effectiveness of the proposed methodology, the performance of the controllers is evaluated using the well known performance index *integral of the absolute magnitude of the error* (IAE) with mathematical definition given by the equation

$$IAE = \int |error| dt \quad (34)$$

Since the $A(q^{-1})$ is present in both disturbance models, any change in process model will adapt the disturbance filter, thus the proposed methodologies can deal with varying dynamics from poorly damped to critical damped systems.

5. Conclusions

Two model based control strategies (IMC and MPC-EPSAC) with augmented disturbance filter design have been proposed and evaluated in this paper. The illustrative example was carefully chosen to clearly allow maximal exploitation of controller properties to the benefit of the output performance in the presence of strong subsystem interaction. The proposed IMC strategy makes use of the diophantine equation in order to design a filter which effectively minimizes subsystem interactions. As has been shown in the paper, both methodologies can be successfully used to compensate for the dynamics of the process, allowing fast disturbance rejection even for poorly damped processes. The results obtained reveal the benefit of the intelligent filter and indicate that both control strategies provide good performance in disturbance rejection and subsystem interaction in a MIMO context.

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