THE INFLUENCE OF LINE BALANCING ON LINE FEEDING FOR MIXED-MODEL ASSEMBLY LINES

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KEYWORDS
Decision Making, Optimization, System Management

ABSTRACT

Though, recent research on mixed-model Assembly Line Balancing Problems (MALBP) and Assembly Line Feeding Problems (ALFP) aims to incorporate real-world aspects, research on the integration of both areas is still limited. This paper helps closing this gap by studying the influence of different balancing objectives on line feeding decisions and costs. For line balancing, different objective functions were formulated and the results were used as input when solving the ALFP. Although, no large cost differences were found, we observed that decision making in line feeding does depend on the balance.

INTRODUCTION

Nowadays, a rising number of models is produced on mixed-model assembly lines (Schmid et al. 2018) while each product requires its specific parts and the organization of tasks differs. Consequently, a large variety of parts has to be supplied to the assembly line, resulting in high storage space requirements at the Border of Line (BoL) being the area where parts are stored before usage. This leads to an increasing relevance of the material supply configuration for the effectiveness of the production system.

In the Assembly Line Feeding Problem (ALFP), decisions on provision and storage of parts at the BoL are taken by assigning them to policies such as line stocking (providing parts on a pallet) or kitting (presorting multiple parts in a smaller container). Mostly, the ALFP is solved after tasks have been assigned to individual assembly stations by solving the Mixed-model Assembly Line Balancing Problem (MALBP) (Sternatz 2015). As the latter determines the amount of tasks and, therefore, parts at every station it affects decision making in line feeding. In a MALBP, several objectives may be used to optimize a line, thereby resulting in different line configurations. Most research deals with both problems separately and also in practice those two optimization problems are solved successively. But, as there are attempts to integrate decision making in line balancing and feeding (Battini et al. 2016, Sternatz 2015), it is of strong interest to investigate the effect of varying objective functions in line balancing on the selection of line feeding policies and the corresponding costs as this has not been done so far.

Our results indicate that, even balancing lines in a pretty different way, effects on line feeding costs are marginal. However, it is found that the actual assignment of parts to line feeding policies is varying with the chosen balancing objective function to a large extent. Furthermore, some line feeding policies are more robust against changing objective functions than others.

The remainder of this research is organized as follows. First, a literature review is provided. Next, MIP models for line balancing and feeding are formulated. Thereafter, preliminary results are described and discussed. In the last section, a conclusion is drawn.

LITERATURE REVIEW

Mixed-model assembly line balancing problems

In MALBP, tasks are distributed among working stations with respect to some cost and/or capacity objective. The best known problem formulations are MALBP-F, MALBP-1, MALBP-2, and MALBP-E (Becker and Scholl 2006). In the MALBP-F, both, cycle time and number of stations are given and a feasible line balance has to be found. In MALBP-1, cycle time is given and the number of stations is minimized whereas in the MALBP-2, the number of stations is given and the cycle time is minimized. In MALBP-E, a line’s efficiency is maximized by minimizing both, cycle time and number of stations. Finally, for smoothing the workload, one can smooth varying times for different models at ev-
ery station, i.e. horizontal balancing (Becker and Scholl 2006). For a classification of research on the ALBP the reader is referred to (Boysen et al. 2007).

Assembly line feeding problem

According to (Kilic and Durmusoglu 2015) part feeding systems have three main components: parts storage, parts transportation and feeding policy. However, preparation of parts might also be taken into account. In literature, five different line feeding policies are distinguished (Schmid et al. 2018). In line stocking, homogeneously packed parts, e.g. on a pallet, are directly transported from a warehouse to the BoL. In kanban, parts are repacked into smaller, homogeneously filled, bins. Sequencing describes that different variants of a part presorted in containers according to the sequence of consumption. Kitting, as an extension of sequencing combines various components and their variants in a container. Stationary kits only contain parts required at a single station, whereas traveling kits contain parts for multiple stations and travel together with the product along the assembly line.

(Limere et al. 2012) developed a decision model to determine the optimal line feeding considering two feeding policies, line stocking and stationary kitting. (Caputo et al. 2015) developed an IP model for finding an optimal feeding policy mix (line stocking, kitting or JIT, i.e. kanban) in a single-model assembly line. (Schmid et al. 2018) developed a MIP cost minimization model to determine the optimal feeding policy for every individual part in a mixed-model assembly line. This model considers the, above described, five policies. Traveling kits are placed on an assembly line’s start and removed at the end, thus allowing only one kit per product.

Integration of line balancing and line feeding

(Sternatz 2015) finds potential productivity gains by simultaneously solving line balancing and feeding problems considering direct (line stocking) and indirect supply (stationary kitting). He states that indirect supply can be avoided and hence logistical costs are reduced when decisions are integrated. (Battini et al. 2016) formulate a similar model, additionally including ergonomic aspects. They also distinguish direct part feeding and indirect part feeding and confirm productivity gains as described by (Sternatz 2015). Both models minimize the number of required assembly and supermarket operators, having a fixed cycle time. Therefore, it seems promising, to investigate different objective functions’ effect on line feeding.

MATHEMATICAL MODELS

We formulated four MIP balancing model with different objective functions and used the results to apply the ALFP model described by (Schmid et al. 2018) as well as an extension, allowing multiple traveling kits.

Mixed-model assembly line balancing problem

In this section, four balancing models are formulated, i.e. MALBP-1, MALBP-2, MALBP-E and MALBP-E with horizontal balancing. Each MALBP is reduced to a simple form by making the following assumptions.

1. Serial (straight) assembly line layout.
2. Paced assembly line.
3. Deterministic task times.
4. Only precedence constraints.
5. Demand for all products is equal.

Following notations are used.

\[
\begin{align*}
S & \quad \text{Set of stations, index } s \\
P & \quad \text{Set of products, index } p \\
J & \quad \text{Set of tasks, index } j \\
t_j & \quad \text{Task time for task } j \\
\lambda_{jp} & \quad 1 \text{ if task } j \text{ is needed for product } p, 0 \text{ otherwise} \\
c & \quad \text{Cycle time} \\
m & \quad \text{Total number of stations installed} \\
DS & \quad \text{Set of definite stations, } DS = \{1, \ldots, m\} \\
PS & \quad \text{Set of probable stations, } PS = \{m+1, \ldots, m+m\} \\
\end{align*}
\]

MALBP-1

The number of stations is minimized (equation (1)) while the cycle time is constant. Two binary decision variables are used: \( y_s \) equals 1 if station \( s \) is installed and \( x_{js} \) equals 1 if task \( j \) is assigned to station \( s \).

\[
\min \sum_{s \in S} y_s \tag{1}
\]

\[
s.t. \sum_{j \in J} t_j \cdot x_{js} \cdot \lambda_{jp} \leq c \cdot y_s \quad \forall s \in S, p \in P \tag{2}
\]

\[
\sum_{s \in S} x_{js} = 1 \quad \forall j \in J \tag{3}
\]

\[
\sum_{w \in S} \sum_{h \in \text{Pred}_j} x_{hw} \geq x_{js} \quad \forall s \in S, \forall j \in J \tag{4}
\]

\[
y_{s-1} \geq y_s \quad \forall s \in S \tag{5}
\]

Constraint (2) ensures that the station load for any product does not exceed the cycle time, as assumption 2 specifies a paced assembly line. Furthermore, constraint (2) ensures that tasks can only be assigned to installed stations. Constraint (3) guarantees that all tasks are assigned to exactly one station while constraint (4) enforces precedence relations. Finally, constraint (5) was added for adjacency of all installed stations.
**MALBP-2**

Opposite to MALBP-1, the cycle time is minimized (equation (6)) while the number of stations is constant. This approach maximizes productivity.

\[
\begin{align*}
\min c \\
\text{s.t. } & \sum_{j \in J} t_j \cdot x_{js} \cdot \lambda_{jp} \leq c & \forall s \in S, p \in P \\
& \sum_{p \in P} \delta_p \geq c & \forall s \in S
\end{align*}
\]

(3), (4)

Constraint (7) is similar to constraint (2) and makes sure that station loads do not exceed the cycle time. The remaining constraints are the same as before.

**MALBP-E**

The proposed model for the MALBP-E below is an extension of the formulation given by (Esmaeilbeigi et al. 2015). In SALBP-E, line efficiency is maximized by minimizing both cycle time and number of stations. Similarly, in MALBP-E, the weighted average line efficiency, based on the product demand, is maximized. (Esmaeilbeigi et al. 2015) prove that the line efficiency is maximized by minimizing line capacity \( T = c \cdot m \), alternatively defined as \( T = t_{\text{sum}} + \delta_{\text{total}} \), with \( t_{\text{sum}} \) denoting the sum of all task times and \( \delta_{\text{total}} \) denoting the total idle time over all stations. As we assume equal demand for all products (assumption 5), idle times of a station for a product, and the average station idle time, denoted as \( \delta_{sp} \) and \( \delta_s \) respectively, can be calculated as

\[
\begin{align*}
\delta_{sp} &= c - \sum_{j \in J} t_j \cdot \lambda_{jp} & \forall p \in P, \forall s \in S \\
\delta_s &= \frac{\sum_{p \in P} \delta_{sp}}{|P|} & \forall s \in S
\end{align*}
\]

The line efficiency can thus be calculated as

\[
T \cdot |P| = \sum_{p \in P} \left( \sum_{s \in S} \delta_{sp} + \sum_{j \in J} t_j \cdot \lambda_{jp} \right),
\]

which is equivalent to

\[
T = \sum_{s \in S} \delta_s + \frac{\sum_{p \in P} \sum_{j \in J} t_j \cdot \lambda_{jp}}{|P|}
\]

Since \( t_j, \lambda_{jp} \) and \(|P|\) are parameters, minimizing the line capacity \( T \) is equivalent to minimizing the sum of average idle times over all stations. Hence, a similar linearization method as proposed by (Esmaeilbeigi et al. 2015) can be used. Summarizing, the following decision variables are used: \( x_{js}, y_s, c \) and \( \delta_{sp} \).

\[
\begin{align*}
\sum_{j \in J} t_j \cdot \lambda_{jp} \cdot x_{js} + \delta_{sp} & \leq c & \forall s \in DS, \forall p \in P \\
\sum_{j \in J} t_j \cdot \lambda_{jp} \cdot x_{js} + \delta_{sp} & \geq c + \tau \cdot (y_s - 1) & \forall s \in PS, \forall p \in P \\
\sum_{j \in J} t_j \cdot \lambda_{jp} \cdot x_{js} + \delta_{sp} & \leq \tau \cdot x_{js} & \forall s \in PS, \forall p \in P \\
x_{is} \leq y_s & & \forall j \in J, \forall s \in S
\end{align*}
\]

(3), (4), (5)

The objective function (12) seeks to minimize the total idle time over all products and stations. Constraint (13), (14), (15) and (16) make sure that the station load for a product does not exceed the cycle time (assumption 2). For these constraints, a lower and upper bound on both the cycle time, \( c \) and \( \tau \), and the number of stations, \( p \) and \( m \) are specified. Next, constraint (17) stipulates that all stations in the set of definite workstations should be installed and constraint (18) makes sure that tasks can only be assigned to installed stations. Finally, constraint (19) restricts the value for the cycle time to lie between its bounds.

**MALBP-E with horizontal balancing**

In order to balance the varying workload of the stations, caused by different models, we added horizontal balancing to the MALBP-E. (Thomopoulos 1970) proposed an objective function that minimizes the sum of the absolute deviation of the station time of a product from the average station time, i.e. \( \sum_{s \in S} \sum_{p \in P} |t_{ps} - t_s| \). This objective function value was linearized using variables \( v_{sp} \) and \( w_{sp} \), and added to the objective function of the MALBP-E. The decision variables are: \( x_{js}, y_s, c, \delta_{sp}, v_{sp} \) and \( w_{sp} \).

\[
\begin{align*}
\min & \sum_{s \in S} \sum_{p \in P} \left( \delta_{sp} + l \cdot (v_{sp} + w_{sp}) \right) \\
\text{s.t.} & \ v_{sp} \geq 0, w_{sp} \geq 0 & \forall p \in P, \forall s \in S \\
& \ v_{sp} \geq t_{ps} - t_s & \forall p \in P, \forall s \in S \\
& \ w_{sp} \geq t_s - t_{ps} & \forall p \in P, \forall s \in S
\end{align*}
\]

(3), (4), (5), (13), (14), (15), (16), (17), (18), (19)

with \( t_{ps} = \sum_{j \in J} \lambda_{jp} \cdot x_{js} \cdot t_j \) \ \forall p \in P, \forall s \in S

\[
t_s = \frac{\sum_{p \in P} t_{ps}}{|P|} \forall s \in S
\]

In objective function (20), the total absolute deviation from the average station time over all models and stations is added to the MALBP-E objective function. A
The MIP formulation was extended by adding the following elements: transportation costs for every traveling kit, and usage costs for replacing an empty traveling kit with a full one.

The usage cost calculation in equation (29) is similar to the calculation of the usage cost for stationary kits, however the handling time $h_T$ is counted double as both the empty and new kit have to be handled. As traveling kits are replaced at the end of the station (see equation (34)), a walking distance only equaling two times the distance $d_e$ between station and BoL needs to be covered.

Furthermore, some additional constraints were added.

$$D_s \cdot l_D + \sum_{i \in I} x_{is} \cdot l_i^L + \sum_{f \in F} x_{fs} \cdot l_s^T + x_s^K \cdot l^K + y_s^T \cdot l_T \leq EP_s - SP_s \quad \forall s \in S$$

(30)

Firstly, by adapting constraint (30), storage space at the BoL for the replacement traveling kits is reserved.

$$\sum_{f \in BOM_p} y_{pfs} \cdot v_f \leq V_T \cdot y_{ps} \quad \forall p \in P, \forall s \in S$$

(31)

Secondly, equation (31) enforces that the volume of the parts in the traveling kit does not exceed the kit volume.

$$y_{pfs} = 0 \quad \forall s \in S, \forall f \in F, \forall u \in S : u < s$$

(32)

Next, by adding constraint (32), we assure that a traveling kit is not taken out of the line when it still holds parts needed at stations downstream.

$$y_f^T + y_{pfgu}^T - 1 \leq y_{pfu}^T \quad \forall p \in P, \forall s \in S, \forall f \in BOM_p \cap F_s, \forall q \in S : q \leq s, \forall g \in BOM_p \cap F_q, \forall u \in S : u \geq s$$

(33)

Constraint (33) ensures that only one traveling kit can travel along with the product at the same time.

$$y_f^T + y_{pfs}^T - 1 \leq y_{pfs}^T \quad \forall p \in P, \forall s \in S, \forall f \in BOM_p \cap F_s$$

(34)

Constraint (34) enforces that a newly inserted travelling kit does not contain any parts needed at the station of insertion. By adding this limitation, a new travelling kit is forced to be inserted near the end of the station.

Lastly, some additional auxiliary constraints were added.

$$y_{pfs} \cdot |BOM_p| \geq \sum_{f \in BOM_p} y_{pfs}^T \quad \forall p \in P, \forall s \in S$$

(37)
\[
y^T_p \cdot |S| \geq \sum_{s \in S} y^T_{ps} \quad \forall p \in P \quad (38)
\]
\[
y^T_s \cdot |P| \geq \sum_{p \in P} y^T_{ps} \quad \forall s \in S \quad (39)
\]

The model specified by (Schmid et al. 2018) is referred to as ALFP with single traveling kit and it’s extension as ALFP with multiple kits.

**PRELIMINARY RESULTS**

First, all balancing models are optimized to obtain a line configuration. Secondly, both line feeding models were solved for every obtained line balance. All mathematical models are implemented and solved using CPLEX (A time limit of 3600s yielded to average LP-gaps of 1.5% and maximal gaps of 15.1%).

For testing purposes, 2 sources of data were merged in order to create 16 instances. For balancing, datasets from (Scholl 1993) and instances generated by the NTI-GEN software (Serrano et al. 2014), were used. For the ALFP in a mixed-model environment, 16 instances from (Schmid et al. 2018), were used. Both data sources were merged by linking tasks with part families (describing all variants of a part). Since the balancing related instances from literature only provide precedence links for a single product assembly line, we merged precedence constraints of two products.

**Assembly line balancing**

The MALBP-E model with horizontal balancing was solved with different weights \(l\), i.e. 100% and 50% (see equation (20)). The former is denoted as 'MALBP-E+1HB' and the latter as 'MALBP-E+0.5HB'.

As can be seen in table (1), MALBP-2 balances logically comprise more stations, and consequently less tasks per station, than the other balances. Admittedly, it must be said that the resulting balance of MALBP-E depends on the chosen bounds for cycle time and number of stations.

<table>
<thead>
<tr>
<th>MALBP</th>
<th>1</th>
<th>2</th>
<th>E</th>
<th>E+1HB</th>
<th>E+0.5HB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stations</td>
<td>13.4</td>
<td>19.7</td>
<td>13.9</td>
<td>12.9</td>
<td>13.1</td>
</tr>
<tr>
<td>Tasks/station</td>
<td>37.2</td>
<td>21</td>
<td>32.7</td>
<td>35</td>
<td>33.3</td>
</tr>
</tbody>
</table>

**Assembly line feeding with single traveling kitting**

Table (2) reveals that the maximum difference in costs only amounts to a maximum of 6% on average comparing MALBP-2 and MALBP-E+0.5HB.

<table>
<thead>
<tr>
<th>MALBP</th>
<th>1</th>
<th>2</th>
<th>E</th>
<th>E+1HB</th>
<th>E+0.5HB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost/part</td>
<td>97.82</td>
<td>95.74</td>
<td>95.93</td>
<td>98.95</td>
<td>101.25</td>
</tr>
<tr>
<td>st.dev.</td>
<td>31.98</td>
<td>32.58</td>
<td>28.30</td>
<td>31.10</td>
<td>32.75</td>
</tr>
</tbody>
</table>

It can be seen that a similar line feeding policy mix is used for lines resulting from MALBP-1, MALBP-E and MALBP-E with horizontal balancing (see table (3)). Whereas in MALBP-2 more stations, and hence more storing space is available, allowing more space consuming policies such as line stocking and sequencing.

<table>
<thead>
<tr>
<th>MALBP</th>
<th>1</th>
<th>2</th>
<th>E</th>
<th>E+1HB</th>
<th>E+0.5HB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line stocking</td>
<td>19%</td>
<td>31%</td>
<td>10%</td>
<td>9%</td>
<td>31%</td>
</tr>
<tr>
<td>Kanban</td>
<td>26%</td>
<td>37%</td>
<td>15%</td>
<td>7%</td>
<td>15%</td>
</tr>
<tr>
<td>Sequencing</td>
<td>19%</td>
<td>26%</td>
<td>11%</td>
<td>16%</td>
<td>28%</td>
</tr>
<tr>
<td>St. kitting</td>
<td>17%</td>
<td>28%</td>
<td>10%</td>
<td>13%</td>
<td>32%</td>
</tr>
<tr>
<td>Tr. kitting</td>
<td>17%</td>
<td>30%</td>
<td>11%</td>
<td>13%</td>
<td>29%</td>
</tr>
</tbody>
</table>

Some parts are assigned to the same policy irrespective of the underlying balance (see table (4): L denotes line stocking, D kanban, S sequencing, K stationary kitting and T traveling kitting). Overall, 28.2% of all parts are always assigned to the same feeding policy. Furthermore, one can observe that parts, assigned to line stocking in all ALB solutions, have on average a considerable higher volume and demand than parts that are assigned to other policies, whereas it seems to be the opposite for sequenced parts. Parts in stationary kit switch policies in most cases (92.4%), indicating that it is the least favorable option.

<table>
<thead>
<tr>
<th>MALBP</th>
<th>L</th>
<th>D</th>
<th>S</th>
<th>K</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same policy [%]</td>
<td>35.8</td>
<td>23</td>
<td>29.7</td>
<td>7.6</td>
<td>36.8</td>
</tr>
<tr>
<td>Avg part volume [dm^3]</td>
<td>37.6</td>
<td>5.7</td>
<td>3.3</td>
<td>3.1</td>
<td>7.3</td>
</tr>
<tr>
<td>Avg part demand [%]</td>
<td>32.5</td>
<td>0.5</td>
<td>0.7</td>
<td>45</td>
<td>7</td>
</tr>
</tbody>
</table>

Table (5) shows the change of a part’s feeding policy from being supplied with one policy in a certain ALB solution to another policy in another ALB solution. For example, it can be seen that on average 14% of the parts that were line stocked in one ALB solution, are provided with kanban in another ALB solution.

**Assembly line feeding with multiple traveling kitting**

For all instances that could be solved close to optimality (12 out of 16), exactly the same results as when using
Table 5: Average changing behaviour of the part feeding policy assignments

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>D</th>
<th>S</th>
<th>K</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>L</strong></td>
<td>36%</td>
<td>14%</td>
<td>3%</td>
<td>7%</td>
<td>7%</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>21%</td>
<td>59%</td>
<td>12%</td>
<td>28%</td>
<td>13%</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>2%</td>
<td>4%</td>
<td>58%</td>
<td>12%</td>
<td>6%</td>
</tr>
<tr>
<td><strong>K</strong></td>
<td>4%</td>
<td>11%</td>
<td>12%</td>
<td>36%</td>
<td>7%</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>10%</td>
<td>11%</td>
<td>15%</td>
<td>17%</td>
<td>67%</td>
</tr>
</tbody>
</table>

a single traveling kit were found. However, a serious reduction of available space indicated some benefits when multiple kits were used.

**Discussion**

As we aimed to understand the effect of different balancing methods on line feeding, we found that balancing does influence decision making in line feeding, costs however, are not as strongly affected as expected (Sternatz 2015). This is probably due to the use of five line feeding policies providing flexibility in the assignments of parts to line feeding policies. Furthermore, space borrowing might also have affected this outcome. Therefore, not allowing space borrowing might show different results. As some models are not solved to optimality, numbers might change slightly, when solved to optimality.

So far, no benefits of using multiple traveling kits could be found in the conducted experiments. This is probably reasoned in the small datasets used as well as in large containers for traveling kits. Although it could not be proven, we expect that cost savings will outweigh fixed costs for traveling kits in larger instances and multiple traveling kits become an interesting option.

**CONCLUSION**

This research analyzed the influence of using different objective functions in the MALBP on the line feeding policy decisions, which varied for 72% of the parts, and the resulting line feeding costs, varying at most up to 6% . We also found that stationary kits seem to be the least favourable option and, therefore, the assignment of parts to stationary kits is less robust. As the scope of this research is limited to small assembly lines with two products, more research, in terms of computational experiments, is needed to gain a better insight in more advanced production lines. Additionally, revealing the influencing factors for decision making on line feeding policy assignment might be worthwhile. Furthermore, this research indicates that the possibility of using additional traveling kits is not beneficial in small assembly lines. However, more research is required to evaluate the usage of multiple traveling kits in larger production systems.

**REFERENCES**


