

## IMC based PID Control Applied to the Benchmark PID18

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**Abstract:** In this paper an Internal Model Control (IMC) based proportional-integral-derivative (PID) control is presented and evaluated on the benchmark system presented at the 3<sup>rd</sup> IFAC Conference on Advances in Proportional-Integral-Derivative Control (PID18). The controller is designed based on the model of the benchmark system. Its performance is compared with a computer-aided design tool based on frequency response (FRtool) and against the benchmark reference controller. The results show that the proposed method has a better performance due to the fact that IMC based PID parameters depend totally on the model.

*Keywords:* PID control, Multivariable system, IMC, FRtool, Refrigeration system.

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### 1. INTRODUCTION

The refrigeration systems are everywhere in our daily life, such as applications for private consumers, and also in industrial facilities (Bejarano et al., 2017). Most refrigeration systems work in a similar manner. The refrigerant works in inverse Rankine cycle between equipments including evaporator, compressor, condenser and expansion valve (Buzelin et al., 2005). However, due to strong nonlinearity, variables coupling and dead time, many refrigeration systems operate at conditions of low efficiency, inaccurate cooling temperatures and unpleasant conditioning speed. Hence, there is an urgent need for effective control strategies to overcome these problems.

In recent years, some advanced control strategies have been developed to improve the performance of refrigeration systems, including fuzzy control (Chia et al., 1997; Aprea et al., 2004; Chiou et al., 2009), backstepping method (Rasmussen et al., 2008), model predictive control (Larsen et al., 2005; Hovgaard et al., 2012; Ma et al., 2012), robust  $H_\infty$  control (Bejarano et al., 2015; Rahnama et al., 2017), and sliding mode control (Huang et al., 2017; Koo et al., 2012). In these literatures, satisfactory performance of refrigeration was obtained. However, due to the complexity existing in these advanced control strategies, it is difficult to be comprehended by engineers and to be applied in actual project.

PID controllers are still the most widely used controllers in industrial control systems due to their simplicity and low cost

(Åström and Hägglund, 2006). In practical industrial processes, the conventional PID control is most of the time designed for Single-Input and Single-Output (SISO) systems, which means the classical PID controllers do not consider the mutual interaction for Multi-input and Multi-output (MIMO) systems explicitly (Kawai et al., 2017). Therefore, much research is carried out to solve the application problem of PID controller in MIMO systems. Various evolution methods were applied to obtain optimal multivariable PI and PID controllers (Iruthayarajan and Baskar, 2009). Chaotic firefly algorithm approach based on Tinkerbell map was developed for multi-loop PID parameters tuning (Dos Santos Coelho and Leandro, 2012). A simple two-step procedure was proposed for deriving PID settings for typical process control applications, and better results are obtained compared with other PID tuning methods (Skogestad, 2003).

In this paper, IMC based PID controller for multivariable systems is proposed. Firstly, using the prediction error estimation algorithm, a linearized model of the refrigeration system is obtained around the normal operation point. Then, the interaction effects are neglected based on decentralised approach. Finally, IMC based PID controller is obtained for the MIMO system. PID controller based on FRtool is designed for benchmark system to validate the performance of the proposed method.

This paper is structured as follows. In section 2, the MIMO refrigeration control system is described, and the system identification is performed. The detailed theory of PID controller based on IMC and FRtool computer aided design toolbox is shown in section 3. Finally, the simulation results

and conclusions are given in section 4 and section 5 respectively.

## 2. DESCRIPTION AND MODELING OF THE PROCESS

### 2.1 Description of MIMO refrigeration system

The schematic of refrigeration system is shown in Fig.1. The inputs in this system are compressor speed  $N$  and expansion valve opening  $A_v$ , and the outputs are outlet temperature of the evaporator secondary flux  $T_{e,sec,out}$  and the degree of superheating  $T_{SH}$ . Operating points and operating ranges of other variables can be found in Table 1 (The detailed meaning of these variables are shown in Bejarano et al., 2017). This system works as follows. Firstly, the refrigerant enters the evaporator at low temperature and pressure and it evaporates while removing heat from the evaporator secondary flux. Then, the compressor increases the refrigerant pressure and temperature and it enters the condenser. Thirdly, the refrigerant condenses and may become subcooled liquid while transferring heat to the condenser secondary flux. Finally, the expansion valve closes the cycle by upholding the pressure from the condenser to the evaporator (Bejarano et al., 2017).

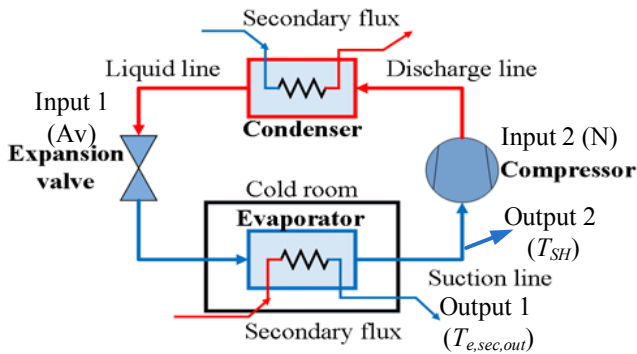


Fig. 1. Schematic picture of one-compression-stage, one-load-demand vapour-compression refrigeration cycle

Table 1. Variable ranges and operating point

Variables		Range	Operating point	Units
Input variables	$A_v$	[10-100]	$\approx 50$	%
	$N$	[30-50]	$\approx 40$	Hz
Disturbances	$T_{c,sec,in}$	[27-33]	30	$^{\circ}\text{C}$
	$\dot{m}_{c,sec}$	[125-175]	150	$\text{g s}^{-1}$
	$P_{c,sec,in}$	—	1	bar
	$T_{e,sec,in}$	[-22 - -18]	-20	$^{\circ}\text{C}$
	$\dot{m}_{e,sec}$	[55-75]	64.5	$\text{g s}^{-1}$
	$P_{e,sec,in}$	—	1	bar
	$T_{surr}$	[20-30]	25	$^{\circ}\text{C}$
Output variables	$T_{e,sec,out}$	—	$\approx 22.15$	$^{\circ}\text{C}$
	$T_{SH}$	—	$\approx 14.65$	$^{\circ}\text{C}$

### 2.2 Identification of MIMO refrigeration system

In order to design the PID controller based on IMC and FRtool, the model of the MIMO system is required. In this

process, the model is identified by applying Pseudo-Random Binary Signals (PRBS) to the inputs of the system. As it is a system with multiple inputs, identification is performed by applying a PRBS signal to one of the inputs while keeping the other inputs constant at the initial operating point. A generic structure for this type of systems of two inputs and two outputs is shown in Fig.2.

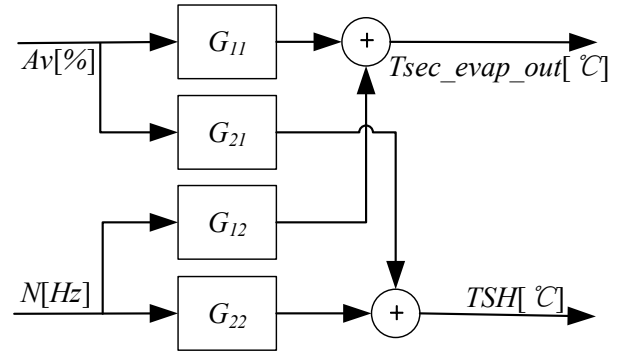


Fig. 2. Model Structure for Refrigeration Control System

By using the prediction error method (PEM), a linearized model of the refrigeration system is obtained around the normal operation point: expansion valve opening = 50% and compressor speed = 40Hz. The continuous model is shown as follow:

$$G(s) = \begin{bmatrix} \frac{-0.2219s - 0.004757}{s^2 + 5.834s + 0.2373} & \frac{-0.004638}{s^2 + 93.24s + 3.802} \\ \frac{-2.425}{s^2 + 2.099s + 6.634} & \frac{1.208s + 0.03219}{s^2 + 6.743s + 0.1946} \end{bmatrix} \quad (1)$$

with transmission zeros:

$$\begin{aligned} z_1 &= -93.198; & z_{2,3} &= -1.0484 \pm 2.3526i \\ z_4 &= -0.0408; & z_{5,6} &= -0.0254 \pm 0.0022i \end{aligned} \quad (2)$$

indicating that the process is minimum-phase. Secondly, based on decentralised approach, the Relative Gain Array (RGA) analysis of the multivariable process is performed.

$$\Lambda = \begin{bmatrix} 0.8815 & 0.1185 \\ 0.1185 & 0.8815 \end{bmatrix} \quad (3)$$

Since the main diagonal has positive values close to 1, the pairing 1-1/2-2 is suitable. Finally, the individual PID controllers are designed for each input-output ( $G_{11}$  and  $G_{22}$ ) pairing by neglecting the effect of the interaction loop ( $G_{12}$  and  $G_{21}$ ) based on decentralised approach.

## 3. CONTROL STRATEGIES DESIGN

### 3.1 IMC based PID control with Filter

The basic structure of the IMC is shown in Fig.3, where  $P(s)$  indicates the process,  $H_m(s)$  is the model of the process,  $H_{IMC}(s)$  is the IMC controller transfer function and  $H_c(s)$  is the equivalent controller for a traditional closed loop system (Bequette, 2003).

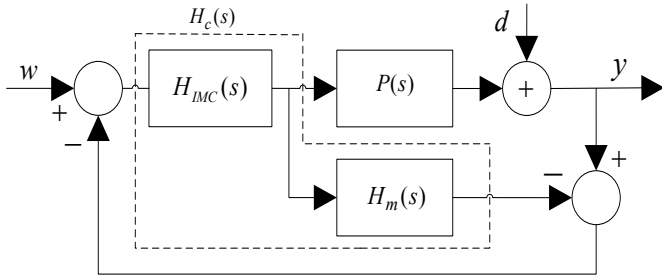


Fig. 3. Basic IMC structure

According to process model identified using PEM the transfer functions  $G_{11}$  and  $G_{22}$  have a structure of second order minimum-phase system without time delay.

$$H_m(s) = \frac{K(\beta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)}, \quad \tau_1, \tau_2, \beta > 0 \quad (4)$$

The equivalent controller for a traditional closed loop system  $H_c(s)$  can be computed as:

$$H_c(s) = \frac{H_{IMC}(s)}{1 - H_{IMC}(s)H_m(s)} \quad (5)$$

where,

$$H_{IMC}(s) = H_m^{-1}(s)F(s) \quad (6)$$

$$F(s) = \frac{1}{\lambda s + 1} \quad (7)$$

Replacing (4), (6) and (7) into (5) the equivalent PID controller with filter is obtained:

$$H_c(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{\beta s + 1} \quad (8)$$

$$K_p = \frac{\tau_1 + \tau_2}{\lambda K}; \quad T_i = \tau_1 + \tau_2; \quad T_d = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$$

where  $K$ ,  $\beta$ ,  $\tau_1$  and  $\tau_2$  are parameters of each input-output ( $G_{11}$  or  $G_{22}$ ). While  $\lambda$  is a tuning parameter for the speed of the closed loop. This controller differs from the traditional PID by the term  $1/(\beta s + 1)$  which is a low-pass filter (LPF) with cutoff frequency  $\omega_c = 1/\beta$  (rad/s). Because the process transfer functions ( $G_{11}$  or  $G_{22}$ ) are of minimum-phase.

### 3.2 Computer Aided PID Design: FRtool

In this section, the ‘in-house’ developed tool, namely the Frequency Response tool (FRtool) for Matlab is applied to the benchmark system as a reference performance controller as described in (De Keyser et al., 2006). The tuning of the PID controllers on the refrigeration process is realized with the following design specifications: overshoot  $\%OS < 5\%$ , robustness  $R_o > 0.5$  and settling time  $T_s < 100$  seconds for both outputs using decentralised approach for multivariable

processes. This design specifications are utilized according to the results obtained in (Bejarano et al., 2017).

## 4. SIMULATION RESULTS

In this section, the proposed IMC based PID controller with filter is compared to the PID controller tuned with FRtool and the benchmark reference controller ‘Ref.PID’. The model-based controllers via FRtool was designed according to the specifications described in section 3.2. Similarly, IMC based PID controllers are adjusted using the same design specifications. The parameters  $K = -0.02$ ,  $\tau_1 = 24.4$ ,  $\tau_2 = 0.17$ ,  $\beta = 46.64$  are obtained according to transfer function  $G_{11}$ . While,  $K = 0.1654$ ,  $\tau_1 = 34.5$ ,  $\tau_2 = 0.15$ ,  $\beta = 37.52$  are obtained according to  $G_{22}$ . Finally,  $\lambda_1 = 3$  and  $\lambda_2 = 1.5$  are chosen to obtain similar specifications that FRtool for outputs  $T_{e,sec,out}$  and  $T_{SH}$  respectively. Table 2 shows the PID parameters obtained with different tuning methods.

Table 2. PID Controller Parameters

Output	Tuning method	$K_p$	$T_i$	$T_d$	$\beta$
$T_{e,sec,out}$	FR tool	-47.23	3.98	0.53	-
	IMC-PID	-408.8	24.56	0.17	46.64
$T_{SH}$	FR tool	3.50	0.80	0.20	-
	IMC-PID	139.65	34.65	0.14	37.52

According to Table 2, it is important to note that the proportional-constant ( $K_p$ ) of both controllers is negative due to the gain of  $G_{11}$  is negative which corresponds to the transfer function for output  $T_{e,sec,out}$ . On the other hand, the reference signals and performance indexes used are all from the benchmark case, and the meaning of these indexes are shown in Table 3. Table 4 shows the performance indexes calculated for all the controllers.

Table 3. Meaning of performance indexes

Indexes	Meaning
$RIAE_1$ $RIAE_2$	Ratios of Integrated Absolute Error for two outputs
$RITAE_1$ $RITAE_2$	Ratios of Integrated Time multiplied Absolute Error for two outputs
$RIAVU_1$ $RIAVU_2$	Ratios of Integrated Absolute Variation of Control signal for two inputs
$J$	Mean value of the eight individual indices with weighting factor for each index

Table 4. Performance indexes for the different controllers  $C_1 = \text{Ref. PID}$ ,  $C_2 = \text{FRtool}$ ,  $C_3 = \text{IMC-PID}$

	$C_1$ vs $C_2$	$C_1$ vs $C_3$	$C_2$ vs $C_3$
$RIAE_1$	0.8863	0.8716	0.9822
$RIAE_2$	0.8198	0.8399	1.0245
$RITAE_1$	0.9371	0.2954	0.3153
$RITAE_2$	0.6441	0.6283	0.9755
$RITAE_2$	0.8803	0.7651	0.8692
$RITAE_2$	0.1522	0.3961	2.6025
$RIAVU_1$	3.5765	0.9024	0.2523
$RIAVU_2$	0.7376	0.7064	0.9576
$J$	0.8464	0.6332	0.9838

Since almost all indices are less than unit in the first comparison in Table 4, the PID controller based on FRtool has a better performance than benchmark reference controller. The second comparison between the proposed controller and benchmark reference controller has similar results with the first group, which indicates that the IMC based PID has a better performance than benchmark reference controller. In addition, the values obtained in this second comparison are much lower than those calculated in the first comparison. Which indicate that the proposed controller is better than the rest of the controllers. Similarly, the third group of Table 4 show the numerical values of the indices obtained from the comparison between the IMC based PID and PID controller tuned with FRtool. What corroborates the above indicated that the proposed controller achieves good load disturbance rejection, while maintaining a good reference tracking performance. The system outputs with different PID controllers are show in Fig. 4 and Fig.5. They also show that the proposed controller has the best performance.

proposed controller is due to the presence of the low-pass filter (LPF) in the structure of the IMC based PID controller, which allows to smooth the controller output signal. Consequently, the proposed controller has a better performance applied to the benchmark system.

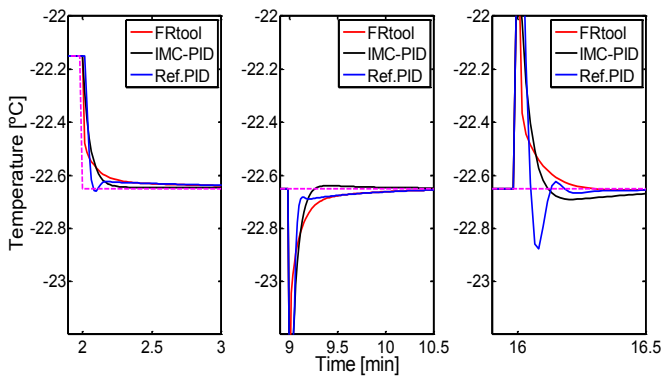


Fig. 4. Outlet temperature of the evaporator secondary flux ( $T_{e,sec,out}$ ) with different PID controllers.

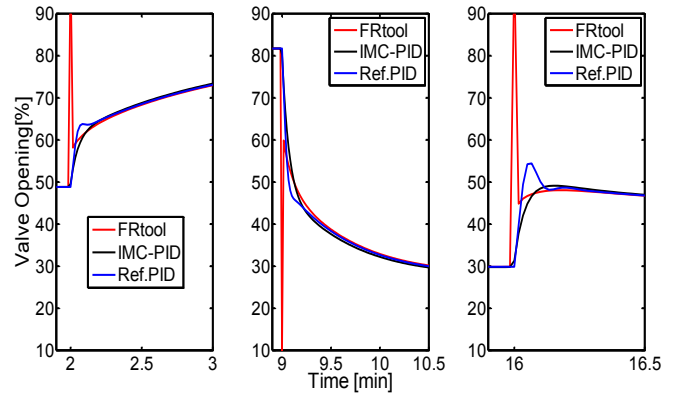


Fig. 6. Valve opening ( $A_v$ ) with different PID controllers.

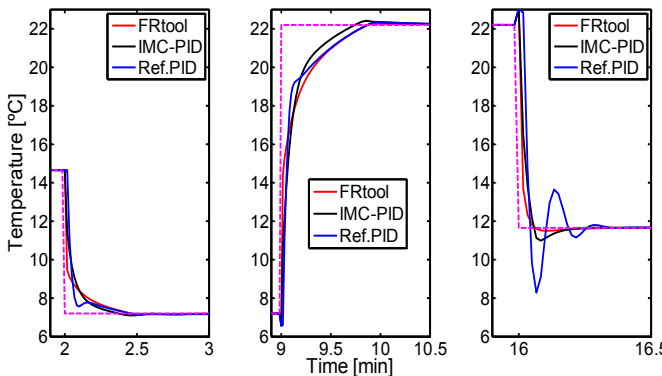


Fig. 5. Outlet temperature of the degree of superheating ( $T_{sh}$ ) with different PID controllers.

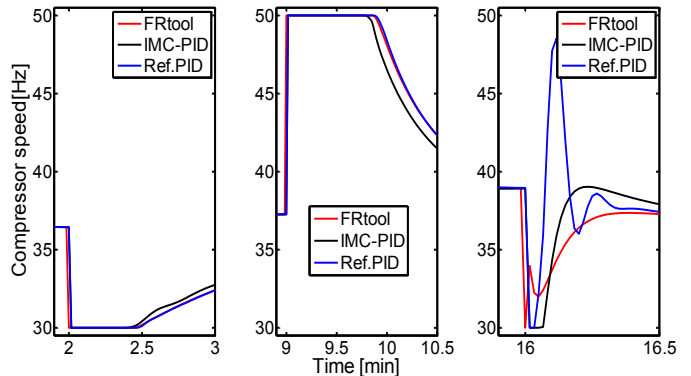


Fig. 7. Compressor speed ( $N$ ) with different PID controllers.

## 5. CONCLUSIONS

In this paper, an IMC based PID with filter is proposed. The proposed controller is applied to the benchmark system presented at the 3rd IFAC Conference on Advances in Proportional-Integral-Derivative Control (PID18). The performance of the proposed method is compared against the PID controller based on FRtool with full knowledge of the system, also against to the benchmark reference controller. The simulation results and numerical analysis show that the proposed controller has better performance in disturbance rejection, while maintaining a good reference tracking performance and low control effort compared with other controllers.

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On the other hand, the control effort signals of these controllers are shown in Fig. 6 and Fig.7 for valve opening ( $A_v$ ) and compressor speed ( $N$ ). It can be seen that the input of valve opening ( $A_v$ ) is higher in PID controller based on FRtool, therefore, the relative Index  $RIAVUI$  is greater than one. While, the proposed controller reflects in all the relative Indexes values which are less than one for all the comparisons shown in Table 4. This advantage of the

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