Initial probabilistic studies into a deflection-based design format for concrete floors exposed to fire

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Abstract

Performance Based Design is increasingly applied in structural fire engineering, sometimes entailing the use of deflection-based criteria to demonstrate adequate performance. Since no deflection-based design format currently exists which takes into account the many uncertainties associated with structural performance during fire, the attainment of an ‘adequate safety level’ is not necessarily ensured. Building on earlier studies, the feasibility of a deflection-based design format with a single global safety factor is explored for concrete floors exposed to fire. Considering the common assumption of lognormality for slab deflections, safety factors can be defined. The subsequent feasibility study however indicates that the assumption of a lognormal distribution is problematic in case of fire. A conceptual alternative to the global safety factor is explored, where the load on the slab is stepwise increased up to the point where a predefined deflection criterion is reached. This alternative approach seems promising, as it results in a known distribution type for the calculated maximum distributed load. Next steps for the development of this concept are identified.

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1. Introduction

Prescriptive structural fire safety requirements are increasingly avoided in favor of so-called performance based design (PBD) options [1]. Sometimes, this means that deflection-based criteria are used to ‘demonstrate’ adequate performance in fire (i.e. fire resistance). Whether these deflection-based designs result in adequate, quantifiable levels of safety, let alone optimum levels of investment in structural fire resistance, remains unclear.

On the other hand, it must be recognized that current calculation tools are, in general, not capable of modelling all possible modes of fire-induced structural failures for the full range of available structural materials and systems. More specifically, with current methods – notably including both implicit and explicit finite element methods (FEM) – the true load bearing capacity of many complex structural systems exposed to fire cannot generally be determined, leaving the structural fire safety profession with deflection limit criteria as one commonly chosen, calculable alternative.

This paper explores the feasibility of using a deflection-based design format for concrete floors exposed to fire. The paper builds on earlier studies in [2], where the possibility of demonstrating safety through deflection limits was demonstrated, and [3], where target reliabilities for structural fire design of concrete slabs were established. Those studies are summarily introduced below.

1.1. The possibility of demonstrating safety through deflection criteria [2]

The use of deflection criteria to demonstrate ‘adequate safety’ hinges on two important assumptions/requirements. First, the applied deflection criterion must be an unequivocally conservative approximation of the true strength-based (‘collapse’) failure criterion. For complex structural systems, this requirement cannot be verified with current calculation methods, and thus fundamentally hinges on expert-judgement. Second, the deflection criterion (or any other ‘unequivocally’ conservative approximation of the strength limit state) should only result in a minor additional investment when optimizing the design over the lifetime of the structure.

Both requirements have been explicitly demonstrated, considering deflection limits for the specific (calculable) example case of a simply supported fire-exposed concrete slab, in [2]. Parameters of the slab were as given in Table 3, and the acceptable deflection limits were assumed as 0.24/0.28/0.32 m. More restrictive deflection limits were found to be overly conservative for the cases considered. Taken together, the evaluations suggested the possibility of obtaining adequate structural fire safety (with respect to the collapse limit state) using deflection-based design.

1.2. Target reliabilities for design [3]

The work in [2] necessitated an evaluation of the uncertain future costs and benefits of the design, which is rather demanding for most practical applications. As in ambient structural design, a reliability-based design would be preferable as it allows design to be based on structural-engineering considerations only (i.e. it avoids the detailed valuation of future costs and benefits). However, a reliability-based design requires definition of target reliabilities. These target reliability indices, \( \beta_t \), correspond with maximum accepted failure probabilities, \( P_{f,t} \), through Eq. (1), with \( \Phi \) being the standard cumulative normal distribution function [4].

\[
P_f = \Phi(-\beta)
\]  

In [3], tentative target reliabilities for structural fire design were determined through Lifetime Cost Optimization (LCO) calculations for the same concrete slab as in [2]. The calculations were made as a function of the dimensionless ‘fire-damage parameter’ \( \eta_f \), defined by Eq. (2), with parameters as given in Table 1, making the results generally applicable to a wide range of applications. The proposed (tentative) target reliabilities specified in [3] are given in Table 2. For the specified concrete slab (span 4.8m, lever arm \( d = 180\text{mm} \)), these target reliabilities were found to be applicable for both the strength criterion (i.e. bending ultimate limit state) and when applying deflection limits of 0.24/0.28/0.32 m.
Table 1. Definition parameters Eq. (2)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_f$</td>
<td>Annual probability of a fully developed ('significant') fire [1/year]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Cost of fire-induced structural failure costs (incl. reconstruction) as a multiple of the initial construction cost [-]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Continuous discount rate [1/year]</td>
</tr>
</tbody>
</table>

Table 2. Tentative target reliabilities as a function of $\eta_f$ [3]. Global safety factor $\gamma_v$ in accordance with Eq. (9).

<table>
<thead>
<tr>
<th>$\eta_f$</th>
<th>Tentative classification</th>
<th>$\beta_{i,\gamma}$</th>
<th>$\gamma_v (V_v = 0.1)$</th>
<th>$\gamma_v (V_v = 0.2)$</th>
<th>$\gamma_v (V_v = 0.3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7-10</td>
<td>High relative failure consequences</td>
<td>3.9</td>
<td>1.48</td>
<td>2.18</td>
<td>3.22</td>
</tr>
<tr>
<td>0.08-0.7</td>
<td>Medium relative failure consequences</td>
<td>3.3</td>
<td>1.39</td>
<td>1.93</td>
<td>2.69</td>
</tr>
<tr>
<td>&lt;0.01-0.08</td>
<td>Low relative failure consequences</td>
<td>2.6</td>
<td>1.30</td>
<td>1.68</td>
<td>2.18</td>
</tr>
</tbody>
</table>

1.3. The next step: a reliability-based design format

The next step in the development of practical design guidance would be to define a reliability-based format for deflection-based structural fire design. Considering the introduction above, the format should be applicable to advanced nonlinear FEM.

Different possible safety formats for nonlinear FEM analysis have been listed by Cervenka [5], who concludes that a full probabilistic analysis is theoretically preferable but recommends a global (safety) factor from a practical perspective. Since this approach would be a natural choice for a design format, its feasibility for deflection-based structural fire design is investigated below.

As discussed further, application of the global safety factor format is not without difficulties. Therefore, an alternative concept is outlined in Section 3.

2. The feasibility of a global safety factor format for deflection-based design

2.1. Concept

The safety format recommended by Cervenka [5] can readily be derived if the distribution describing the stochastic model output is known (e.g. a lognormal distribution being a common assumption). Also, reference values for the output variability (i.e. the coefficient of variation, $V_v$) are needed.

Definition of a design format based on the application of a global safety factor corresponds with the acceptance criterion of Eq. (5), with $v_d$ being the design value of the mid-span deflection, $\mu_v$ being the mean mid-span deflection (can be approximated by the deflection calculated considering mean values for all stochastic variables), and $\gamma_v$ being the global safety factor. The acceptance criterion of Eq. (5) is intended to result in the same safety level as for a full-probabilistic evaluation, and thus in the limit adheres to Eq. (6), i.e. for the least restrictive acceptable $v_d$.

$$v_{\lim} \geq v_d = \gamma_v \mu_v$$  \hspace{1cm} (5)

$$P[v_{l/2} > v_d] = \Phi(-\beta_{i,\gamma})$$  \hspace{1cm} (6)

If $v_{l/2}$ follows a lognormal distribution with coefficient of variation $V_v$ and mean value $\mu_v$, then the left hand side of Eq. (6) corresponds to Eq. (7), where $\mu_{lnv}$ and $\sigma_{lnv}$ are the corresponding parameters of the lognormal distribution. Elaborating Eq. (7) results in Eq. (8), where the right-hand approximation results in a relative error of less than 3%
for $\beta_{t,fi} \leq 3.9$ and $V_e \leq 0.2$, and less than $7\%$ when $V_e \leq 0.3$. Combining Eq. (8) with Eq. (5), results in Eq. (9) for the global safety factor $\gamma_v$. Corresponding values for $\gamma_v$ have been listed in Table 2 for $V_e = 0.2$ and 0.3.

$$P\left[v_d > v_d\right] = \Phi\left(-\ln\left(v_d\right) + \mu_{lev}\right) \Rightarrow -\ln\left(v_d\right) + \mu_{lev} = -\beta_{t,fi}$$

(7)

$$v_d = \mu_v \exp\left(\beta_{t,fi} \sqrt{\ln\left(1 + V_e^2\right)} - \ln\left(1 + V_e^2\right)\right) \approx \mu_v \exp\left(\beta_{t,fi} V_e\right)$$

(8)

$$\gamma_v \approx \exp\left(\beta_{t,fi} V_e\right)$$

(9)

2.2. Study of applicability

The safety format above would be the format of choice for nonlinear structural design [5]. The derivation of Eq. (9) and the corresponding values in Table 2 are, however, conditional on the (common) assumption of a lognormal distribution. This assumption of lognormality is assessed in Fig. 1, where the observed density function and complementary cumulative density function obtained through 10000 Monte Carlo Simulations (MCS) are compared against lognormal approximations with mean value and coefficient of variation estimated through the MCS. The results relate to the simply supported slab configuration of Table 3 (as in [2,3]), considering different ISO 834 [6] standard fire durations $t_E$.

As expected considering the Eurocode design philosophy of EN 1990 [4], the slab has a non-zero probability of the load effect exceeding the resistance effect. In those situations, the slab deflection increases asymptotically. These very large (incalculable) deflections are not considered in the parameter estimation for the lognormal distribution.

For small fire durations (e.g. for $t_E = 0/15$ min), the probability of failure is small, and the lognormal approximation is (visually) appropriate. For larger fire durations however (e.g. $t_E = 30$ min), the lognormal approximation is less appropriate, with an increasing tail of the distribution giving a deviation from the lognormal approximation. For $t_E = 45/60$ min a distinct non-zero probability of failure (corresponding with very large deflections) makes the classic lognormal distribution infeasible. This is clearly visible in Fig 1b for $t_E = 60$ min, where $1.7\%$ of the slabs have insufficient capacity to carry the distributed loads (resulting in a horizontal asymptote for the complementary CDF at 0.017).

![Fig. 1. (a) Observed PDF and lognormal approximation; (b) Observed complementary CDF (1-CDF) and lognormal approximation.](image)
Table 3. Probabilistic models for basic variables concrete slab (unit width), based on [8,9]; *see [7] for details

<table>
<thead>
<tr>
<th>Property</th>
<th>Distribution</th>
<th>Mean μ</th>
<th>CoV V</th>
</tr>
</thead>
<tbody>
<tr>
<td>20°C concrete compressive strength, $f_{c,20}$ [MPa]; ($f_{c}=30$ MPa)</td>
<td>LN</td>
<td>42.9</td>
<td>0.15</td>
</tr>
<tr>
<td>20°C reinforcement yield stress, $f_y,20$ [MPa]; ($f_y=500$ MPa)</td>
<td>LN</td>
<td>581.4</td>
<td>0.07</td>
</tr>
<tr>
<td>$T^°C$ concrete compressive strength reduction factor, $k_{c}(T)$ [-]</td>
<td>Beta[$\mu$±3$\sigma$]</td>
<td>conform EN 1992-1-2</td>
<td>T-dependent*</td>
</tr>
<tr>
<td>$T^°C$ reinforcement yield stress reduction factor, $k_{y}(T)$ [-]</td>
<td>Beta[$\mu$±3$\sigma$]</td>
<td>conform EN 1992-1-2</td>
<td>T-dependent*</td>
</tr>
<tr>
<td>Concrete cover, $c$ [mm]</td>
<td>Beta[$\mu$±3$\sigma$]</td>
<td>15</td>
<td>0.33 ($\sigma_c=5$ mm)</td>
</tr>
<tr>
<td>Slab thickness, $h$ [mm]</td>
<td>DET</td>
<td>200</td>
<td>-</td>
</tr>
<tr>
<td>Slab free span, $l$ [m]</td>
<td>DET</td>
<td>4.8</td>
<td>-</td>
</tr>
<tr>
<td>Reinforcement axis spacing, $s$ [mm]</td>
<td>DET</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>Uniformly distributed permanent load, $g$ [kN/m²]</td>
<td>Normal</td>
<td>$g_s$</td>
<td>0.1</td>
</tr>
<tr>
<td>Uniformly distributed imposed load, $q$ [kN/m²]; (5-year reference)</td>
<td>Gumbel</td>
<td>$0.2q_i$</td>
<td>1.1</td>
</tr>
<tr>
<td>Characteristic value of the permanent load, $g_i$ [kN/m²]</td>
<td>DET</td>
<td>6.2</td>
<td>-</td>
</tr>
<tr>
<td>Characteristic value of the imposed load, $q_i$ [kN/m²];</td>
<td>DET</td>
<td>7.5</td>
<td>-</td>
</tr>
</tbody>
</table>

2.3. Section conclusion

Figure 1 suggests that the lognormal distribution is indeed an appropriate choice in ambient conditions, but also clearly shows that this approximation cannot be readily accepted for structural fire design. For the slab configuration of Table 3, the lognormal approximation is acceptable up to 30 min of ISO 834 in case of $\beta_{t,fi} = 2.6/3.3$, and up to 15 min for of $\beta_{t,fi} = 3.9$. The slab configuration of Table 3 however has a tabulated fire resistance of R60 in accordance with EN 1992-1-2 [11], and thus any performance-based design applications would reasonably be associated with longer fire durations.

The data in Fig. 1 relate only to a single example case, but clearly demonstrate the difficulty of transposing an ambient design format to structural fire safety applications. More generally, Fig. 1b highlights a fundamental issue with a purely deflection-based design, as the horizontal asymptotes in the complementary CDF make it difficult to propose an approximate distribution type for the derivation of a global safety factor (i.e. for application of Eq. (5)).

In the absence of a workable proposal addressing the above issues, an alternative to a purely deflection-based design format is sought.

3. Towards an alternative deflection-based design format, considering a load increase to cause ‘failure’

3.1. Problem reformulation

The problem identified in Fig. 1b (i.e. the horizontal asymptotes for the complementary CDF of the mid-span deflection) relates to situations where the load effect exceeds the resistance effect. It seems therefore reasonable to suggest avoiding design formats where the underlying decision variable (e.g. the deflection) is indeterminate (i.e. goes to infinity) when the design approaches structural failure.

Considering the statement in Section 1.1 that the deflection limit state is used as an approximation for the strength (ultimate) limit state, the strength limit state is written in Eq. (10), with $R$ being the resistance effect and $E$ being the load effect, in an effort to reformulate the problem.

For the slab configuration of Table 3 subjected to pure bending, the strength limit state is calculable and becomes Eq. (11), with $M_{R,fi,E}$ being the bending moment capacity during fire, $M_E$ being the bending moment induced by the load effect, $w$ being the total distributed load, and $l$ being the slab free span. For complex structural systems with multiple failure modes the strength limit state cannot be evaluated with current methods.
The bending moment capacity $M_{R,fi,E}$ can be associated with a maximum bearable total distributed load, $w_{\text{max,STR}}$, through Eq. (12).

$$Z_{\text{STR}} = R - E$$

$$Z_{\text{STR}} = M_{R,fi,E} - M_E = M_{R,fi,E} - w^2/8$$

$$w_{\text{max,STR}} = \frac{8M_{R,fi,E}}{l^2}$$

For the slab of Table 3, $M_{R,fi,E}$ (and thus $w_{\text{max,STR}}$) can be calculated using a numerical model [7], as in [2]. Alternatively, the maximum total distributed load can be approximated via deflection calculations, by step-wise increasing the total distributed load up to the point where the deflection limit $v_{\text{lim}}$ is reached. The corresponding maximum distributed load is denoted as $w_{\text{max,DEF}}$.

Monte Carlo simulations for $w_{\text{max,STR}}$ and $w_{\text{max,DEF}}$ were performed for different ISO 834 standard fire durations. Results are shown in Fig. 2, indicating how the strength-based limit value is closely approximated by the deflection-based limit ($v_{\text{lim}} = 0.32m$ for the considered example case). Since $w_{\text{max,STR}}$ directly relates to $M_{R,fi,E}$ through Eq. (12), corresponding alternative axes are given at the top and right-hand sides of Fig. 2.

![Fig. 2. Observed PDF for $w_{\text{max}}$ considering the strength limit state, and displacement-based approximation](image)

The observed PDFs in Fig. 2 can be described by a mixed-lognormal distribution. This has been shown in detail for $M_{R,fi,E}$ in [7,12], where it was observed that the uncertainty with respect to the concrete cover distorts the PDF of $M_{R,fi,E}$. For a fixed concrete cover, the slab bending moment capacity during fire was shown to correspond with a lognormal distribution, resulting in Eq. (13) for a complete description of the distribution of $M_{R,fi,E}$. In Eq. (13), $p_{ci}$ is the probability associated with a given concrete cover $c_i$, and $M_{R,fi,E,ci}$ is the associated lognormal distribution of the bending moment capacity, considering a fixed concrete cover equal to $c_i$; the summation runs over all possible (discretized) concrete covers $c_i$.
Considering the agreement between $w_{\text{max},\text{STR}}$ and $w_{\text{max},\text{DEF}}$, also the latter can reasonably be associated with a mixed-lognormal distribution. Omitting the subscript DEF and STR for brevity, the above implies that $w_{\text{max}}$ can be described by Eq. (14), with $w_{\text{max},ci}$ the (lognormal) maximum total distributed load for a fixed concrete cover $c_i$.

Consequently, the safety requirement for the slab is related to the target reliability index $\beta_{t,fi}$ through Eq. (15), with $g$ and $q$ the uniformly distributed permanent and imposed loads as defined in Table 3. Note that the formulation in Eq. (15) relates to distributions, i.e. the summation relates to the weighted sum of distributions in the mixed-lognormal distribution, and not to an actual weighted summation of distributed load values.

\[
M_{R,fi,cl} = \sum_{c_i} p_{c_i} M_{R,fi,cl,ci} \tag{13}
\]
\[
w_{\text{max}} = \sum_{c_i} p_{c_i} w_{\text{max},ci} \tag{14}
\]
\[
P_{f} = P \left[ \sum_{c_i} p_{c_i} w_{\text{max},ci} - (g + q) < 0 \right] \leq P_{f,\beta} = \Phi(-\beta_{t,fi}) \tag{15}
\]

3.2. Tentative outline of an alternative deflection-based design format

Eq. (15) has the same format as a traditional strength-based limit state (i.e. $Z = R - E = R - (G + Q)$), where the resistance effect $R$ is, however, described by a mixed-lognormal distribution. Furthermore, $w_{\text{max}}$ can be approximated through deflection-based simulations.

The formulation of Eq. (15) can be evaluated using a full probabilistic analysis. As noted by Cervenka [5], this is theoretically preferable for nonlinear FEM analyses. When applying a full probabilistic analysis, the reliability targets of Table 2 can be readily assessed. A possible methodology for a full probabilistic analysis requiring a (relatively) limited number of model evaluations has been presented in [13]. The results in [13] are particularly relevant because the ability of the method to (reasonably) capture a mixed-lognormal distribution has been explicitly illustrated.

A full-probabilistic analysis does not, however, comply with the practical preference for a ‘deterministic’ (or rather: semi-probabilistic) design format where safety factors are applied to the result of a single (or limited number of) model evaluations. In principle, Eq. (15) lends itself to the derivation of partial factors or a global resistance factor. Resulting (tentatively), in an acceptance criterion in the format of Eq. (16), where $w_{\text{max},k}$ is the maximum total load obtained by one or more deflection-based evaluation(s) considering ‘characteristic’ input values.

\[
\frac{w_{\text{max},k}}{\gamma_R} - \gamma_G s_k + \gamma_Q q_k \geq 0 \tag{16}
\]

Considering the concrete cover-dependency of the mixed lognormal distribution. A practical consideration would be to consider one model evaluation (or a limited number) where the concrete cover $c$ is set to a low characteristic value. The feasibility of this approach will be investigated in future work. More generally, future analyses are required to confirm the generality of the mixed-lognormal description of $w_{\text{max},\text{DEF}}$ as a function of the concrete cover. A general design format for situations with a resistance effect described by a mixed-lognormal distribution should also be developed. This definition of a design format for a mixed-lognormal resistance is currently considered to be the bottleneck of the approach. Once a general approach is established, safety factors, as in Eq. (16), can be calculated. Considerable further evaluations will still be required to generalize the results before the approach can reasonably be applied in practice.
4. Conclusions

The feasibility of a deflection-based design format for concrete floors exposed to fire has been investigated. Although a global safety factor for deflection-based design could be derived, the underlying assumption of a lognormal distribution for the slab mid-span deflection was shown to be inappropriate for structural fire design. This is because considerable probabilities of failure during fire correspond with non-negligible probabilities of extreme slab deflections (ultimately tending to infinity), resulting in an asymptote for the complementary cumulative density function. Consequently, correcting the safety factor through an improved choice of probability distribution is (apparently) not feasible.

Reformulating the problem, an approach is suggested where the total distributed load on the slab is step-wise increased up to the point where the predefined deflection limit is exceeded. The corresponding maximum total load is found to closely approximate the maximum load derived from a strength-based (bending moment capacity) evaluation, and can (by analogy) be described by a mixed-lognormal distribution. The reformulation of the design problem suggests that a design format based on the determination of the load at which a deflection limit is reached may be feasible. The further elaboration however requires the derivation of a design format for situations where the resistance is described by a lognormal distribution. The derivation of such a design format is considered as the current bottleneck for the (preliminary) proposal of a deflection based design format.

References