Target safety levels for structural fire resistance based on Lifetime Cost Optimization

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Abstract: Deflection criteria are sometimes used to assess the fire resistance of large structural floorplates for which the accurate modelling and prediction of all possible modes of fire-induced failure is not yet possible. The safety level obtained by applying deflection criteria during design and analysis is, however, unclear. To ensure adequate safety and allow for the derivation of a simplified deflection-based structural fire design format, target safety levels (i.e. target reliability indices) are derived herein using the concepts of Lifetime Cost Optimization. The example case of a simply supported concrete slab is evaluated in detail, since for this case both deflection limit states and strength (bending) limit states can be calculated relatively easily, and the obtained optimum design solutions and target safety levels compared. By introducing a ‘fire-damage parameter’ (which incorporates the fire damage cost, fire occurrence rate, and discount rate), a differentiation in target safety levels is obtained. Although exploratory, the results in the current paper suggest the feasibility of deriving target safety levels on this basis to be used in conjunction with deflection criteria, to ensure adequate structural fire resistance.

1 Introduction

1.1 Motivation

Advances in numerical methods and reductions in the costs of computational power have made it possible for engineering offices to apply complex numerical (typically finite element) models for evaluating the structural response and performance of large building floorplates in case of fire. As current numerical methods are not yet capable of accurately modelling all relevant modes of fire-induced failure for the range of available flooring systems, these numerical models are sometimes applied in conjunction with deflection response monitoring, with acceptance criteria based on specific limits on rate or magnitude of vertical deflection. For a given deflection based acceptance criterion, the numerical models allow designers to optimize the fire protection to the structural elements by placing any fire protection materials only where they are deemed to be necessary (in terms of the responses observed), and even to increase fire protection to structural elements which are
considered crucial for limiting deflections (or other responses). However, the safety level obtained by a thus optimized structural fire protection scheme is unclear, and in many cases not explicitly quantified. Furthermore, the applied deflection limits themselves are typically somewhat arbitrary and therefore open for debate.

To ensure consistency of safety between competing candidate designs, and to develop rational guidance for designers, a simple deflection-based design format is desirable. Any such design format should allow for a straightforward (deterministic) evaluation of ‘fire resistance’, while at the same time resulting in an adequate, quantified level of safety. What level of safety can be considered ‘adequate’ is not well defined (or widely discussed) within the structural fire design community. As a first step in deriving a simple deflection-based design format, target safety levels for structural fire resistance (defined herein using deflection criteria) need to be derived.

1.2 Concept background

Target safety levels for structural safety can be defined through the concepts of Lifetime Cost Optimization (LCO) [11]. When applying LCO, the optimum level of safety investment is determined which minimizes the total cost over the lifetime of the structure, taking into account the uncertain future occurrence of adverse events. This optimum level of safety investment thus corresponds with an ‘optimum’ safety level (probability of failure) associated with the specific parameters applied in the cost-optimization. By calculating the optimum safety level for a number of cases, it can be observed that the optimum safety level is disproportionately sensitive to some of these parameters, while being essentially insensitive to others. The non-influential parameters can therefore be neglected when generalizing the observed optimum safety levels to target safety levels for the design of new structures. In extreme cases, where none of the cost-optimization parameters significantly influence the optimum safety level (or where all influential parameters have well-defined values), the observed optimum safety levels can be generalized to a single target safety level. Whenever this is not the case, a differentiation in target safety level may be appropriate, resulting from a subdivision of the influential cost-optimization parameters into groups or ‘classes’ for which the optimum safety levels are approximately constant (or for which a single target safety level can be justified on other grounds, see Section 4).

Considering the motivation above, this paper derives target safety levels for application in conjunction with deflection calculations in case of fire. A first obstacle in the application of LCO with this intent is the fact that damage costs which can be associated with the exceedance of specific deflection limits are not known. This issue has been tentatively overcome in a recent paper [13] by introducing the following substitution hypothesis for LCO.

*The optimum level of investment in structural fire resistance obtained when substituting the true failure criterion for an approximate but unequivocally conservative failure criterion will correspond to a small additional investment in safety beyond the optimum, but these costs are considered negligible in practice.*

Applying the substitution hypothesis to the situation at hand, the ‘true’ failure criterion refers to the strength limit state, while the ‘unequivocally conservative failure criterion’
refers to a specific deflection limit state. The above hypothesis postulates that substituting deflection-based failure probabilities in a strength-based LCO (for which failure costs can be more readily estimated) will result in a reasonable but conservative approximation of the optimum design solution (i.e. small additional safety investment beyond the optimum). In [13], the validity of this hypothesis has been validated for the example of a simply supported concrete slab, under the additional constraint that the substituted limit state should be an unequivocally conservative approximation of the true failure phenomenon, while at the same time resulting in as close as possible an approximation. This additional constraint has been dubbed the ‘SAFE-requirement’.

The goal is to apply LCO and derive deflection-based target safety levels for large floorplates for which the strength limit state cannot currently be evaluated. As the true failure phenomena are currently incalculable, no direct validation will be possible. Therefore, the simple case of a simply supported concrete slab applied in [13] will be considered further for deriving target safety levels, allowing for a direct comparison of the results obtained through application of the strength criterion (STR) and different deflection criteria (DEF), and a tentative validation of the proposed approach.

The layout of the paper is as follows. First, the applied LCO concepts are summarily introduced. Subsequently, optimum safety levels (reliability indices) are evaluated for different cost parameters. Finally, target safety levels are proposed (for the example case) by grouping design situations in a limited set of ‘fire safety classes’. The obtained target safety levels should be generalized by considering more design cases (e.g. different slab span and thickness) before practical application of the approach is seriously considered.

2 Lifetime Cost Optimization for structural fire resistance

2.1 Total lifetime cost and optimum design criterion

Concepts of Lifetime Cost Optimization (LCO) for structural fire resistance have been described in [13]. Considering only fire-induced failure and neglecting the damage costs associated with partial failure for simplicity, the total lifetime cost, $K$, is given by equation (1), with $\theta$ the design parameter considered for optimization, $C_0$ the base construction cost independent of $\theta$, $\varepsilon$ the ratio of the additional safety investment to the base construction cost, $\xi$ the ratio of the failure costs to $C_0$, $\lambda_{fi}$ the fully developed fire rate, $\gamma$ the discount rate, and $P_{f,fi}$ the conditional probability of fire-induced failure given the occurrence of a fully developed fire. Introducing the fire damage parameter $\eta = \lambda_{fi}\xi / \gamma$, the optimum design criterion (minimum total cost) is given by Equation (2).

$$K(\theta) = C_0 (1 + \varepsilon(\theta)) + C_0 \frac{\xi \lambda_{fi}}{\gamma} P_{f,fi}$$  \hspace{1cm} (1)

$$\frac{dK(\theta)}{d\theta} = 0 \Rightarrow \frac{d\varepsilon(\theta)}{d\theta} + \eta \frac{dP_{f,fi}(\theta)}{d\theta} = 0$$  \hspace{1cm} (2)
2.2 Strength and deflection limit states

The conditional probability of failure \( P_{f,\beta} \) in equations (1) and (2) is calculated for a given limit state \( Z \) by application of the general definition of Equation (3). Considering a slab subjected to pure bending, the strength limit state, \( Z_{\text{STR}} \), is given by Equation (4), with \( M_{R,\beta,E} \) being the bending moment capacity during fire, \( K_R \) being the model uncertainty for the resistance effect, \( K_E \) the model uncertainty for the load effect, and \( M_E \) the bending moment induced by the loads. Considering a load situation with only a single imposed load \( Q \), \( M_E = M_G + M_Q \), with \( M_G \) the bending moment induced by the permanent load effect and \( M_Q \) the bending moment induced by the imposed load effect. The deflection limit state, \( Z_{\text{DEF}} \), is defined by Equation (5), where \( v_{l/2} \) is the mid-span deflection of the slab, \( v_{\text{lim}} \) is a set deflection limit and \( K_v \) is the model uncertainty for the deflection calculation. As in\ VAN COILE AND BISBY [13], the model uncertainties are in the evaluations further set equal to unity so as not to complicate the first level evaluations in this feasibility study. Although not directly applicable, the test standard EN 1365-2 [6] is sometimes cited for justifying the deflection limit, \( v_{\text{lim}} \), of Equation (6). Application to the slab configuration described further results in \( v_{\text{lim}} = 0.32 \text{ m} \). In [13], it has been observed that deflection limits of 0.24 m, 0.28 m and 0.32 m can be considered to fulfill the SAFE-requirement mentioned in Section 1.2. More stringent deflection criteria (for example 0.16 m) resulted in an overly conservative assessment of the optimum safety investment.

\[
P_f = P[Z < 0] \\
Z_{\text{STR}} = K_R M_{R,\beta,E} - K_E M_E = K_R M_{R,\beta,E} - K_E (M_G + M_Q) \\
Z_{\text{DEF}} = K_v v_{l/2} - v_{\text{lim}} \\
v_{\text{lim}} = \frac{l^2}{400d} = \frac{l^2}{400 \left( h - c - \frac{\varnothing}{2} \right)} \\
\]

2.3 Example slab and LCO application

Characteristics of the example slab configuration are given in Table 1, together with the probabilistic models considered in accordance with JCSS [9] and HOLICKY AND SYKORA [8]. The one-way spanning simply supported slab is designed in accordance with the Eurocodes [5], considering a characteristic value of the permanent load, \( g_k \), of 6.2 kN/m\(^2\) (self-weight + 1.5 kN/m\(^2\) finishes) and a characteristic value of the imposed load, \( q_k \), of 7.5 kN/m\(^2\) (warehouse: category E1 in EN 1991-1-1 [3]). The deflection of the slab in ambient conditions is acceptable in accordance with the simplified span to depth deflection assessment of EN 1992-1-1 [5]. The high imposed load has been chosen to result in the same slab configuration studied in [12], without resulting in a clear overdesign with respect to the ambient deflection limit. The Gumbel distribution describing the imposed load, \( q \), in Table 1 refers to a 5-year reference period so as to evaluate the probability of fire-induced failure taking into account the imposed load coinciding with a fire event. The fire is considered to be described by the ISO 834 standard fire as in EN 1991-1-2 [4], with \( t_E \) the specified number of minutes of standard fire exposure. Here, a given ISO 834 fire duration \( t_E \) is pragmatically interpreted as a proxy for the severity of the fully developed fire, in accordance with the historical basis of the ‘fire resistance’ concept. Additional information on the background of different parameters is given in [12] and [13].
Table 1: Probabilistic models for basic variables of the reference concrete slab

<table>
<thead>
<tr>
<th>Property</th>
<th>Distribution</th>
<th>Dimension</th>
<th>Mean μ</th>
<th>CoV V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design value for the bending moment capacity, ( M_{\text{Res}} )</td>
<td>DET</td>
<td>kNm</td>
<td>57.6</td>
<td>-</td>
</tr>
<tr>
<td>20°C concrete compressive strength, ( f_{c,20°C} ) (( f_{ck} = 30 ) MPa)</td>
<td>LN</td>
<td>MPa</td>
<td>42.9</td>
<td>0.15</td>
</tr>
<tr>
<td>20°C concrete yield stress, ( f_{y,20°C} ) (( f_{yk} = 500 ) MPa)</td>
<td>LN</td>
<td>MPa</td>
<td>581.4</td>
<td>0.07</td>
</tr>
<tr>
<td>25°C concrete compressive strength reduction factor, ( k_{f}(T) )</td>
<td>Beta[( \mu \pm 3\sigma )]</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25°C reinforcement yield stress reduction factor, ( k_{f}(T) )</td>
<td>Beta[( \mu \pm 3\sigma )]</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete cover, ( c )</td>
<td>Beta[( \mu \pm 3\sigma )]</td>
<td>mm</td>
<td>15</td>
<td>0.33</td>
</tr>
<tr>
<td>Slab thickness, ( h )</td>
<td>DET</td>
<td>mm</td>
<td>200</td>
<td>-</td>
</tr>
<tr>
<td>Slab width, ( b ) (unit width)</td>
<td>DET</td>
<td>mm</td>
<td>1000</td>
<td>-</td>
</tr>
<tr>
<td>Slab free span, ( l )</td>
<td>DET</td>
<td>m</td>
<td>4.8</td>
<td>-</td>
</tr>
<tr>
<td>Reinforcement axis spacing, ( s )</td>
<td>DET</td>
<td>mm</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>Reinforcement area, ( A )</td>
<td>N</td>
<td>mm²</td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>Uniformly distributed permanent load, ( g )</td>
<td>Normal</td>
<td>kN/m²</td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>Uniformly distributed imposed load, ( q ) (5-year reference)</td>
<td>Gumbel</td>
<td>kN/m²</td>
<td>0.2( q_k )</td>
<td>1.1</td>
</tr>
</tbody>
</table>

As fire resistance specifications for reinforced concrete are often stated in terms of the reinforcement axis distance \( a \) to the exposed surface, the LCO is performed with respect to \( a \). The slab thickness is considered as fixed at 200 mm. Consequently, a larger concrete cover is obtained by positioning the tensile reinforcement closer to the compression zone. To compensate for the associated reduction in lever arm and to maintain the design value of the ambient bending moment capacity, \( M_{\text{Res}} \), the reinforcement area per unit width is increased by reducing the horizontal axis spacing \( s \) between the reinforcement bars. Thus, the ratio \( e \) of the additional safety investment to the base construction cost is given by Equation (7), where \( \tau^* \) is the cost per unit length of a single reinforcing bar, and \( \kappa \) is the relative cost of a unit length of the reinforcing bar to the base construction cost. In [13], \( \kappa \) has been evaluated as approximately \( 5.2 \times 10^{-3} \) considering Western European construction costs [12]. For the remainder of this paper this value for \( \kappa \) will be maintained.

\[
\varepsilon(a) = \frac{\tau^* b}{C_0 \left( \frac{1}{s} - \frac{1}{s_{\text{ref}}} \right)} = 1000 \kappa \left( \frac{1}{s} - \frac{1}{s_{\text{ref}}} \right)
\]

Applying the total cost evaluation of Equation (1) with \( P_{f,fi} \) evaluated respectively through the STR limit state of Equation (5) and different deflection limit states of Equation (6) the results in Figure 1 are obtained for a fire-damage parameter, \( \eta \), of 0.5 and considering 120 minutes of ISO 834 fire exposure. Figure 1 has previously been presented in [13] and illustrates how substitution of deflection limit states in the STR-based LCO results in a small overestimation of the optimum level of safety investment when the substituted limit state
fulfils the SAFE-requirement (here for deflection limits of 0.24, 0.28, and 0.32 m). Furthermore, it has been shown in [13] that the increase in expected lifetime cost associated with the substitution of the deflection limit state is small (for this specific case, no more than 0.025% when considering $v_{\text{lim}} = 0.32$ m).

Figure 1: Relative total lifetime cost $K/C_0$ and optimum designs considering $t_E = 120$ min, $\kappa = 5.2 \cdot 10^{-3}$, $\eta = 0.5$, substituting different limit states for evaluating $P_{f,i}$.  

3 Optimum safety levels for structural fire resistance

The optimum design solutions are associated with a conditional probability of failure $P_{f,i}$ and thus with observed optimum values for the reliability index $\beta_{f,i,\text{opt}}$ through the definition of the reliability index, Equation (8). Observed optimum values for $t_E = 120$ minutes are visualized in Figure 2a for the different considered limit states, as a function of the fire-damage parameter $\eta$. The respective curves differ only slightly, and the same proximity of $\beta_{f,i,\text{opt}}$ for different limit states is observed when considering $t_E = 60/90/150/180$ minutes. The curves for $\beta_{f,i,\text{opt}}$ for the respective fire severities are shown in Figure 2b, where only the curve for the STR criterion is shown for clarity.

$$\beta = -\Phi^{-1}(P_f)$$

(8)

Only a small effect of $t_E$ is observed in Figure 2b, indicating that for the considered example case, the optimum reliability index can be considered essentially independent of the fire severity $t_E$. The curve for $t_E = 120$ min is considered further as a reference since this curve is the most central of the considered $t_E$ values in Figure 2b. On the other hand, both Figure 2a and 2b suggest a strong $\eta$-dependency for $\beta_{f,i,\text{opt}}$.

4 Target safety levels for different ‘fire safety classes’

As $\beta_{f,i,\text{opt}}$ appears to be largely independent of $t_E$, and since the curves in Figure 2a for the different limit states are similar, for this specific example case a single curve specifying $\beta_{f,i,\text{opt}}$ as a function of $\eta$ is considered to define a $\eta$-dependent target reliability index $\beta_{f,i,t}$.  

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[$\Phi$ is the cumulative distribution function of a standard normal distribution]
For practical applications, a limited set of target reliability indices may be preferred, as in EN 1990 [2] where the target reliability index for ambient design is differentiated with respect to the structure’s consequence class. Also, an evaluation of other design parameters (e.g. spans, support conditions, element types) may result in different curves for $\beta_{\text{fi, opt}}$. This generalization to other design problems is not considered in the current paper, but it strengthens the argument in favour of a limited set of ‘fire safety classes’, each associated with a single (constant value) target reliability index $\beta_{\text{fi, t}}$. A final proposal for these fire safety classes necessary follows a generalization of the results to other design problems, and are the subject of follow-on research. In the following, possible concepts for the definition of the fire safety classes are presented.

Figure 2: Observed optimum reliability index, $\beta_{\text{fi, opt}}$, for $t_E = 120$ min.

4.1 Concept

The concept presented here is inspired by the decision support tool introduced in [12] and derives an ‘acceptable range’ for the target reliability index $\beta_{\text{fi, t}}$ for given (nominal) fire-damage parameter $\eta_{\text{nom}}$. Where the acceptable ranges associated with different $\eta_{\text{nom}}$ overlap, a joint acceptable range for $\beta_{\text{fi, t}}$ is defined. Extending the range of $\eta_{\text{nom}}$-values in the selection, the joint acceptable range reduces up to the point where a single $\beta_{\text{fi, t}}$ is defined and no further extension of the range of $\eta_{\text{nom}}$-values is possible for which a joint acceptable range exists. In this manner a fire safety class is organically defined together with its associated target reliability index $\beta_{\text{fi, t}}$. The thus defined fire safety class is not, however, unique (since other groupings of $\eta$-values are acceptable as well), and therefore the final definition of fire safety classes in, for instance, a standardization committee can incorporate subjective considerations as well without negatively influencing the acceptability of the obtained target safety levels.

Focusing the discussion for clarity on the example slab described above, specifying a target reliability index $\beta_{\text{fi, t}}$ directly corresponds with the specification of a required reinforcement axis distance $a_{\text{req}}$ for a given fire severity $t_E$. For clarity, the direct relationship between $\beta_{\text{fi}}$ and $a$ is illustrated in Figure 3 for $t_E = 120$ min.

The required reinforcement axis distance, $a_{\text{req}}$, will however in general not correspond with the optimum axis distance $a_{\text{opt}}$ (which for a given $t_E$ is function of $\eta$), except when $\beta_{\text{fi, t}} = \beta_{\text{fi, opt}}(\eta)$. Consequently, adoption of a target reliability index $\beta_{\text{fi, t}}$ results in an expected total
lifetime cost exceeding the optimum (minimum) value, and thus in an expected lifetime ‘overspending ratio’ \( o \), defined by Equation (9). The different formulations in Equation (9) are all equivalent and are listed to indicate the \( \eta \)- and \( \beta_{fi,t} \)-dependency of \( o \). When defining fire safety classes associated with a single \( \beta_{fi,t} \), it is a logical consideration to want to limit the overspending ratio \( o \) to a maximum overspending fraction \( \zeta_{lim} \) deemed acceptable. Thus, unacceptable overspending is defined by Equation (10).

\[
o(\beta_{fi,t}, \eta) = \frac{K(\beta_{fi,t}, t_E)}{K(\beta_{fi,opt}, t_E)} - 1 = \frac{K(\beta_{fi,t}, t_E)}{K(\eta, t_E)} - 1 = \frac{K(a_{req}, t_E)}{K(a_{opt}, t_E)} - 1 \quad (9)
\]

\[
o(\beta_{fi,t}, \eta) > \zeta_{lim} \quad (10)
\]

\[
P[o(\beta_{fi,t}, \eta) > \zeta_{lim}] \leq P_{lim} \quad (11)
\]

Figure 3: \( \beta_{fi} \) as a function of \( a \), considering different limit states \( t_E = 120 \text{ min} \).

For a given type of structure (e.g. 500 m\(^2\) office floorplate with sprinkler protection) a nominal fire-damage parameter, \( \eta_{nom} \), can reasonably easy be assessed from literature data (e.g. evaluating the damage cost \( \zeta \) based on a study by KANDA AND SHAH [10], the fully developed fire rate as in ALBRECHT AND HOSER [1], and setting the discount rate equal to the long-term growth rate), but when evaluating the details of a specific design the fire-damage parameter will generally differ. Consequently, from a code-making perspective the fire-damage parameter \( \eta \) should be considered as a stochastic variable, and thus Equation (10) can only be evaluated as a probability of exceedance. This probability of exceedance should naturally be limited to a maximum probability \( P_{lim} \). The above acceptance criterion is written mathematically by Equation (11), with \( P[.\] the probability operator. For a given probabilistic description of \( \eta \) and function of \( \eta_{nom} \), the acceptance criterion of Equation (11) defines the acceptability for each possible value of \( \beta_{fi,t} \), thus resulting in an acceptable range for the target reliability index. By repeating this evaluation for different \( \eta_{nom} \) and determining the joint acceptable range, fire safety classes are defined as discussed at the start of this section. Application of these concepts is illustrated in the next section.
4.2 Lifetime overspend for a given $\beta_{fi,t}$ and $\eta$

Considering the definition of the overspending ratio, $o$, of Equation (9), results are visualized in Figure 4a for different $\beta_{fi,t}$ as a function of $\eta$, and in Figure 4b for different $\eta$ (deterministic value) as a function of $\beta_{fi,t}$. Both plots have been evaluated for $t_E = 120$ min, but $t_E$ has only a minor influence, as already noted in Section 3. At some points the curves diverge (for example the curves for $\eta = 1$ close to $\beta_{fi,t} = 3.1$ in Figure 4b) due to the numerical nature of the LCO evaluation, as described in [13]. The curves illustrate how large overspending ratios are associated with small $\beta_{fi,t}$ in case of large $\eta$, whereas the overspending ratio in case of large $\beta_{fi,t}$ for small $\eta$ remains limited to less than 1% overspending for the cases evaluated here.

![Figure 4: Overspending ratio $o$, for, $t_E = 120$ min.](image)

(a) as a function of $\eta$, for different $\beta_{fi,t}$

(b) as a function of $\beta_{fi,t}$, for different $\eta$

4.3 Acceptable range for $\beta_{fi,t}$

For illustrative purposes, $\zeta_{lim}$ is set to $10^{-3}$, $P_{lim} = 0.10$ and the fire-damage parameter $\eta$ is described by a Gumbel distribution with mean equal to $\eta_{nom}$ and a coefficient of variation of 0.3. The parameters $\zeta_{lim}$ and $P_{lim}$ are fundamentally subjective and should be set, for example, by consensus in a standardization committee. The distribution of the fire-damage parameter is not known, but assuming its variability is governed by the variability in the fire damage costs, a distribution with a heavy tail is considered most appropriate (based on analysis by FISCHER [7]); a Gumbel distribution is therefore adopted here. Setting the coefficient of variation to 0.3 implies that there is approximately a $10^{-2}$ probability that the $\eta$ exceeds twice its nominal value, and a $10^{-4}$ probability that $\eta$ exceeds three times its nominal value. Applying the above for the considered limit states and $t_E = 120$ min, Figure 5 is obtained, illustrating the definition of the acceptable range for a single $\eta_{nom}$.

Repeating the same calculation procedure for different $\eta_{nom}$, the acceptable ranges visualized in Figure 6 are obtained. These acceptable ranges have been calculated considering the DEF 0.32 m limit state, but differ imperceptibly from the acceptable ranges obtained through the STR limit state (i.e. $\Delta \beta_{fi,t} < 0.017$). Joint acceptable ranges are defined that incorporate as large a set of $\eta_{nom}$-values as possible, starting from $\eta_{nom} = 10$ and working towards lower fire-damage parameters. As indicated in Figure 6, this organically results in the creation of three fire safety classes as a function of $\eta_{nom}$, each with a clearly defined target reliability index $\beta_{fi,t}$. For the lowest fire safety class (lowest $\eta_{nom}$-values and lowest
the joint acceptable range encompasses multiple possible $\beta_{fi,t}$ (using a subjective criterion that a reasonable $\beta_{fi,t}$ should be defined with no more than a single decimal value). If this joint acceptable range is expanded to lower $\eta_{nom}$ (outside the current calculation scope), than a single $\beta_{fi,t} = 2.6$ would eventually be obtained.

Figure 5: Probability of exceeding the maximum acceptable overspending limit as a function of the applied $\beta_{fi,t}$, for $t_E = 120$ min and $\eta_{nom} = 0.1$. Indication of the acceptable range considering $P_{lim} = 0.1$.

Figure 6: Acceptable range for different $\eta_{nom}$, considering $t_E = 120$ min and 0.32 m DEF limit state. Acceptable set of joint acceptable ranges, defining fire safety classes.

5 Discussion

The mathematics of LCO are now well established, but the translation of the obtained optimum safety levels to target values for incorporation in standards and guidance documents remains unclear. The concept of the ‘acceptable range’ presented above partly overcomes
this difficulty by clarifying the range of target values that result in a safety investment close to the optimum value, explicitly taking into account the uncertainty associated with the parameters in the LCO evaluations. Where acceptable ranges for different nominal LCO-parameters overlap, the intersection of the acceptable ranges defines a joint acceptable range with target safety levels that are acceptable to all of the considered nominal LCO-parameters. Step-wise adding more nominal LCO-parameters to the selection (for which the acceptable range intersects with the current joint acceptable range), the size of the joint acceptable range decreases up to the point where the target safety level is clearly specified and no further overlap with the acceptable range of other nominal LCO-parameters exists. Doing so, safety categories are organically defined together with their associated target safety levels. It is noteworthy that in general a joint acceptable range applicable across all nominal LCO-parameters does not exist. For the evaluations in Section 4, no single joint acceptable range can be defined across all $\eta_{nom}$, see Figure 6.

The fire safety classes (safety categories) determined in Figure 6 are based on the 0.32 m DEF limit state, but again the difference with the STR limit state is very small (acceptable range boundaries for $\beta_{fi,t}$ differ with no more than 0.017). This close similarity is in principle not a requirement for the substitution hypothesis formulated in [13], but does strengthen the argument in favour of defining safety targets based on deflection criteria for some structural typologies. For large floorplates for example, the STR limit state is not clearly defined and therefore, an extension of the concepts presented in this paper to large floorplate analyses will necessarily build on the substitution hypothesis.

In the derivations presented in this paper a number of simplifications have been made to allow for an initial assessment of the feasibility of the proposed approach (for example: partial failure has been neglected and the model uncertainties have been set to unity). More detailed evaluations are being performed in follow-on research.

### 6 Conclusions

Optimum reliability indices for structural fire safety have been obtained through Lifetime Cost Optimization, for the simple example case of a simply supported solid concrete slab. The optimum reliability indices correspond to the optimum safety levels given exposure to a fully developed fire, taking into account the costs associated with fire-induced structural failure, the fully developed fire occurrence rate, the discount rate, and the costs associated with increasing structural fire resistance (here: costs associated with increasing the reinforcement axis distance to the exposed surface whilst maintaining an adequate ambient temperature design for a slab of constant total thickness). Both strength and deflection limit states have been applied, but the deflection-based optimum reliability indices are found to match closely with the strength-based results (for the considered deflection limit states, which all adhere to the SAFE-requirement presented by Van Coile and Bisby [13]). Furthermore, also the fire severity (using a duration of exposure to the ISO 834 standard fire as a proxy) is found to only slightly affect the optimum reliability index given the occurrence of a fully developed fire. The optimum safety level is, however, strongly dependent on the fire-damage parameter (which incorporates the failure costs, discount rate, and fire occurrence rate). By specifying the maximum acceptable lifetime overspending and ac-
ceptable probabilities of exceedance, and considering the uncertainty with respect to the fire damage parameter, acceptable ranges for the target reliability index are derived from the calculated optimum safety levels. The intersection of these acceptable ranges defines fire safety classes as functions of the fire damage parameter. For each fire safety class a single target reliability index applies. This tentative definition of a single target reliability index allows for a generalization of reliability-based design concepts to structural fire safety design.

7 References


