Numerical simulation of long-term creep tests on prestressed beams

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Abstract

The deformations of concrete elements can increase significantly over time as a result of creep and shrinkage. Different material models, which have been calibrated on large datasets, are available in literature in order to predict this time-dependent behaviour. A cross-sectional calculation tool which employs the age-adjusted effective modulus has been developed to verify the accuracy of six models with respect to creep data available for 24 prestressed beams. These prestressed beams with a span of 8 m were loaded up until 4.5 years in a four point bending configuration. This paper reports on the comparison between the measured and calculated compression strains and deflections. It was observed that the mid-span deflection of the prestressed beams at the end of loading is best prescribed by the model B3 and the Gardner Lockmann 2000 model.

Keywords: prestressed beams; time-dependent deformations; creep models; large-scale test

1 Introduction

Due to the aging linear viscoelastic behaviour of concrete its deformations change over time. Upon loading, concrete has an elastic deformation and, if the loading is sustained, the deformation changes as a result of shrinkage and creep of the concrete. In structural analysis, it can be difficult to predict the creep and shrinkage response since both phenomena are the result of the interaction of several physical mechanisms. In order to allow a more simplified analysis, creep and shrinkage are most often assumed to be independent and additive [1]. Under these assumptions, the total time-dependent deformation is the sum of the shrinkage deformation (determined as the deformation of an unstressed specimen) and the creep deformation (determined as the deformation as a result of the stress on the specimen). For loaded and stressed specimens the creep deformations are dominant at later ages. Furthermore, in the case of prestressed concrete the relaxation of the prestressing steel can also significantly contribute to the time-dependent behaviour.

The creep behaviour of concrete elements is well-known and well-documented in literature. Yet, up until now no universally accepted creep theory has been postulated. However, recent investigation proposes that creep of concrete is caused by a dissolution-precipitation mechanism [2]. An accurate prediction of the time-dependent behaviour is of paramount importance, as is demonstrated by the collapse of the Koror Babeldaob bridge in Palau. The failure of this bridge was partly attributed to an underestimation of the creep behaviour in the design phase [3].

Several material models have been proposed in literature to determine the time-dependent behaviour of concrete as a result of creep and shrinkage. All of these models have been calibrated on large datasets mainly consisting of data obtained from small plain concrete specimens.
Thus, the question rises if the contemporary material models are indeed capable of predicting the time-dependent behaviour of reinforced and prestressed concrete beams.

A large-scale testing programme was set up in Belgium to study the time-dependent behaviour of concrete beams. Six Belgian universities participated in this programme: Ghent University, Vrije Universiteit Brussel, Université libre de Bruxelles, University of Leuven, Université Catholique de Louvain, and Université de Liège. In the first stage, from 1967 until 1972, reinforced concrete beams were studied. In the second stage, from 1975 until 1980, prestressed concrete beams were studied, and in the final stage, from 1981 until 1985, also the time-dependent behaviour of partially prestressed concrete beams was studied.

The analysis and numerical simulation of the concrete beams of the first stage of the research programme has already been performed [4, 5]. This paper focusses on the analysis of the time-dependent behaviour of the prestressed beams. Six commonly used creep prediction models are considered, these are: CEB-FIP Model Code 1990-1999, [6], designated as MC90-99; fib Model Code 2010 [7], designated as MC2010; the model of EN1992-1-1 [8], designated as EC2; model B3 [9], designated as B3; the Gardner Lockmann 2000 model [10], designated as GL 2000; and ACI 209 [11], designated as ACI. For each model the deflection, as well as the strain near the top fibre, at mid-span in function of time are determined and compared with the measured results.

2 Test procedure

The prestressing steel, the passive reinforcement, the cement, the sand, and the coarse aggregates which were used for making the concrete for the prestressed beams were all ordered at the same time and were then distributed over the laboratories. The target mean compressive strength at 28 days on cubes with a side length of 200 mm was 50 MPa.

Three different cross-section shapes were tested: rectangular, T-shaped, and I-shaped. All three cross-section shapes were post-tensioned using several wires with a diameter of 7 mm. The I-shaped cross-section was also pre-tensioned using six prestressing strands of 0.5 inch. A detailed reinforcement scheme of each considered cross-section is shown in Figure 1.

The beams were prestressed at an age of 7, 14 or 56 days. Immediately after prestressing the beams were simply supported. After 28 or 56 days the beams were loaded by means of two hydraulic jacks in a four-point bending configuration. In order to maintain the applied force throughout the entire testing period an accumulation vessel (one for each beam) was added to the hydraulic system. The span of the beams was 8.00 m. The hydraulic jacks were placed at one fourth and three fourth of the span respectively. The force was equal to the calculated service load or half of the calculated service load and was kept constant during a period of 4.5 years. Additionally several beams were not loaded and thus only prestressed. The value of the prestressing force and the service load for all four cross-sections is given in Table 1. For each cross-section six beams were tested resulting in a total of 24 prestressed beams subjected to long-term loading. Throughout the entire testing period the beams were kept in an air-conditioned room with a temperature of 20°C ± 0.5°C and a relative humidity of 60% ± 3%.

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Prestressing force [kN]</th>
<th>Service load [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-tensioned rectangular</td>
<td>1120</td>
<td>63.75</td>
</tr>
<tr>
<td>Post-tensioned T-shaped</td>
<td>684</td>
<td>51.00</td>
</tr>
<tr>
<td>Post-tensioned I-shaped</td>
<td>750</td>
<td>62.80</td>
</tr>
<tr>
<td>Pre-tensioned I-shaped</td>
<td>757</td>
<td>54.90</td>
</tr>
</tbody>
</table>
3 Cross-sectional calculation method

The calculated deflections and strains, taking into account the time-dependent behaviour as a result of creep and shrinkage, were computed using a cross-sectional method as described in Ghali et al. [12]. This cross-sectional method is similar as the one which was used for the analysis of the reinforced beams of the testing programme [5].

The instantaneous strain $\varepsilon_0$ and curvature $\psi$ at a reference point O can be calculated by:

$$\begin{bmatrix} \varepsilon_0 \\ \psi \end{bmatrix} = \frac{1}{E_{ref}(A \cdot I - S^2)} \begin{bmatrix} I & -S \\ -S & A \end{bmatrix} \begin{bmatrix} N_{eq} \\ M_{eq} \end{bmatrix}$$ (1)

with $E_{ref}$ a reference modulus of elasticity which is taken equal to the modulus of elasticity of concrete measured at 28 days, $N_{eq}$ and $M_{eq}$ the equivalent normal force and moment on the cross-section, and with $A$, $S$, and $I$ respectively the transformed area of the cross-section, the transformed static moment about an axis through the reference point O, and the transformed moment of inertia about an axis through the reference point O. Note that equation (1) is only valid under the assumption of a linear elastic stress-strain relationship.

The strain and curvature change over time as a result of the stress redistribution due to creep and shrinkage of the concrete. The change in the strain and curvature are denoted as $\Delta \varepsilon_0$ and $\Delta \psi$ respectively. In order to calculate these changes it can be assumed that they are restrained by an artificial axial force $\Delta N$ and an artificial moment $\Delta M$ applied in the reference point O. They are calculated based on the free shrinkage and creep of the concrete section only. Using equation (1) but by substituting $N_{eq}$ and $M_{eq}$ by $\Delta N$ and $\Delta M$ the changes in the strain and curvature can be calculated. Since the creep and shrinkage of concrete develop over time, the restraining forces are not applied immediately at full strength but also develop over time. Therefore, $E_{ref}$ is
substituted by the age-adjusted effective modulus $E_c(t, t_0)$ for calculating the changes in the strain and curvature. Likewise, $A$, $S$, and $I$ need to be substituted by their age-adjusted equivalent.

The proposed method by Bažant [13] was applied to calculate the age-adjusted effective modulus:

$$E_c(t, t_0) = \frac{E_c(t_0)}{1 + \chi(t, t_0) \cdot \rho(t, t_0)}$$

where $E_c(t_0)$ is the instantaneous modulus of elasticity at time of loading $t_0$, $\rho(t, t_0)$ is the creep coefficient according to one of the contemporary material models and $\chi(t, t_0)$ is an aging coefficient. The aging coefficient was assumed to be constant and equal to 0.8. This assumption was verified by a computed aging coefficient and it was found that the difference was negligible.

Based on the measurements, it was assumed in the calculations that the prestressed beams were uncracked. This was later endorsed by the calculated stresses which stayed indeed above the mean tensile stress throughout the entire testing period.

With the strain and curvature known in the reference point, the strain in the rest of the cross-section can be determined under the assumption of a linear stress-strain relationship. Using the principle of elastic weights, the deflection of the prestressed beams can be calculated by integrating the curvatures over the length of the beam.

The relaxation of the prestressing steel was experimentally determined from relaxation tests. It was observed that the relaxation in the prestressing steel remained lower than 2%. This relaxation will be induced over time and will be smaller than the theoretical value determined from relaxation tests due to the effects of creep and shrinkage. Therefore, it was decided to not take into account the relaxation of the prestressing steel.

Apart from the prestressing steel also passive reinforcement was placed in the beams, as can be seen from Figure 1. The stress redistribution to the passive reinforcement was taken into account in all calculations.

4 Results

4.1 Deflection

By using the cross-sectional calculation tool, which was described above, the time-dependent deflection at mid-span of all the prestressed beams was calculated. These calculated deflections were compared against the measured deflections from the testing programme.

Figure 2 shows the deflections at mid-span for the I-shaped beams which were pre-tensioned at 14 days and loaded at 28 days. The deflections for three beams are shown: a beam subjected to the calculated service load, a beam subjected to half of the calculated service load, and a beam which was only prestressed but not loaded. In the figures a negative value represents an upward deflection and a positive value represents a downward deflection.

Before the load was applied, the beams underwent a small downward deflection as a result of shrinkage of the concrete. At an age of 14 days an instantaneous upward deflection is observed as the result of the transfer of prestress from the prestressing wires to the concrete. The considered material models predict the instantaneous deflection with good agreement. The calculated deflections according to the six material models are somewhat divergent for the unloaded beam, apart from the deflections according to MC2010, EC2, and MC90-99 which are similar. The B3 model predicts a larger deflection than the other models and provides the most accurate prediction for the unloaded beam and the beam subjected to half of the service load.

For the loaded beams the difference between the deflections according to the different models is slightly smaller than for the unloaded beam. Note that for the beam subjected to half of the service load the time-dependent deformation is very limited.

4.2 Strain

In Figure 3, a comparison is shown between the calculated and measured strains 10 mm below the top fibre of the I-shaped beams, which are pre-tensioned at 7 days and loaded at 28 days. The
strains for three beams are shown: a beam subjected to the calculated service load, a beam subjected to half of the calculated service load, and a beam which was only prestressed but not loaded. A negative strain is an indication of compression.

Before prestressing the observed strain is due to shrinkage. At the moment of prestressing there is a slight increase in the strain and a more pronounced increase in the strain after application of the load. The limited increase of strain at the moment of prestressing is the result of the limited stress which is induced near the top fibre due to the application of prestress near the bottom fibre.

For the unloaded beam the strains calculated according to the different models are very similar. For the loaded beams there is some more divergence. As the load increases, the difference between the strains calculated according to the different material models increases.

Figure 2. Calculated (continuous lines) and measured (dots) deflections at mid-span for three I-shaped beams pre-tensioned at 14 days. One beam was subjected to the service load at 28 days (I-LD-BS-P14-Q28-100), one beam was subjected to half of the service load at 28 days (I-LD-BS-P14-Q28-50), and one beam remained unloaded (I-LD-BS-P14-0).

Figure 3. Calculated (continuous lines) and measured (dots) compression strains (as a multiple of $10^{-3}$ at 10 mm below the top fibre) at mid-span for three I-shaped beams pre-tensioned at 7 days. One beam was subjected to the service load at 28 days (I-LD-BS-P7-Q28-100), one beam was subjected to half of the service load at 28 days (I-LD-BS-P7-Q28-50), and one beam remained unloaded (I-LD-BS-P7-0).
5 Discussion

5.1 Deflection

The difference between the deflections calculated according to the different material models is mainly the result of the divergent predictions of the instantaneous deflection at prestressing and the early-age creep just after prestressing. The higher deflections of the B3 model just after loading could be explained by both a relatively low modulus of elasticity at an early age and a high creep coefficient at an early age.

The predicted results regarding the deflection at mid-span of the loaded beams show a better agreement. This can be explained by the fact that two time-dependent deformations are superimposed: the upward deflection due to prestressing and the downward deflection due to loading.

The stress distribution over the height of the beams, loaded at half of the service load, is approximately uniform. Therefore, the creep results in an axial shortening while the curvature remains quite constant. As a result the deflection, which is calculated on the basis of the curvature, stays approximately constant.

The models of MC2010, EC2, and MC90-99 have a similar background. This is also visible when analysing the results in terms of the deflections, which are quite similar as can be seen from Figure 2.

Despite the somewhat divergent deflections calculated according to the different material models, the models describe the time-dependent deflection of the beams with reasonably good agreement. A significant contribution for the uncertainty is the calculation of the modulus of elasticity. At relative young ages there is a significant difference between the moduli of elasticity calculated according to the different material models. This results in a discrepancy of the calculated elastic deflections and early-age creep, due to prestressing or the application of loads, compared to the measured elastic deflections and early-age creep.

Overall, considering all the prestressed beams, the most accurate predictions of the deflection at the end of loading were given by B3 and MC2010. These two models have also been calibrated on more recent datasets containing a larger amount of creep tests with a longer duration.

5.2 Strain

The difference between the measured strain of the beam subjected to half of the service load and the unloaded beam is approximately the same as the difference between the beam subjected to the total service load and the beam subjected to half of the service load. This indicates that the linear relationship which was assumed between stresses and strains is a reasonable assumption.

The rate in which the time-dependent strain increases after loading is proportional to the loading level. This is as expected since the creep deformations are proportional to the load level if the stress level does not exceed 40%-45% of the mean concrete compressive strength.

The difference between the strains calculated according to the different material models increases as the load increases. However, it was noted that the models give a quite similar prediction of the shrinkage behaviour. The difference is thus explained by the discrepancies in the creep behaviour according to the different models, which also explains why the divergence is higher for the higher loaded beam.

The high calculated strains according to GL2000 are the result of a mediocre shrinkage prediction, as well as a mediocre creep coefficient for loading at 28 days and a low modulus of elasticity throughout the entire time domain compared to the other models.

6 Final comments

A simplified cross-sectional method is applied to determine the long-term behaviour of prestressed concrete beams. It was observed that current material models to assess the time-dependent behaviour give good agreement with the measurements.
7 Conclusions

A cross-sectional calculation tool incorporating the age-adjusted effective modulus and contemporary material models is able to describe the time-dependent behaviour of prestressed beams with sufficient accuracy.

While the proposed method shows good agreement for the long-term behaviour, the assessment of the deflection at a young age shows much more variation due to the rather different behaviour of the material models for young concrete ages.

Using the cross-sectional calculation tool described here, the most accurate deflections at the end of loading are calculated by B3 and MC2010.

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9 References