Development of a full probabilistic QRA Method for Quantifying the Life Safety Risk in Complex Building Designs

Bart Van Weyenberge1,2, Xavier Deckers1,2, Robby Caspeele3 and Bart Merci1
(1) Ghent University – UGent, Dept. Flow, Heat and Combustion Mechanics, Belgium
(2) Fire Engineered Solutions Ghent, Belgium
(3) Ghent University – UGent, Dept. Structural Engineering, Belgium
E-mail: bart.vanweyenberge@ugent.be

1 ABSTRACT
The present paper describes part of a framework for the development of a risk assessment methodology to quantify the life safety risk for people present in buildings in the context of the creation of a fire safety design. Complex building designs and regulations drive engineers towards quantitative risk analysis (QRA). One of the key aspects in quantitative risk analysis is finding the proper balance between simplification of the scope by using different models and taking important case specific features into account. In other words, a proper balance needs to be found between modelling cost versus accuracy. Increasing the accuracy by means of sophisticated models will increase the computational cost or might force to reduce the amount of representative scenarios to be analysed. In this paper, a method is discussed which quantifies the safety level with regard to building configuration detailing. Attention is given to the accuracy of the deterministic models, and at the same time approaches are analysed which reduce the computational cost. The output for the quantification of the life safety level of the building is determined by means of a procedure in which several steps are taken to obtain the risk outcome. The final risk value is calculated with regard to a failure probability in analogy with structural engineering.

KEYWORDS: Quantitative risk assessment, risk analysis, response surface modelling, performance based design, Polynomial Chaos Expansion, limit state design

2 INTRODUCTION
In prescriptive legislation regarding fire safety it is often implicitly assumed that if all the rules of the regulation are applied, the fire safety level is acceptable [1] [2] [3]. For a significant part of the buildings these prescriptive requirements are appropriate. However, architectural demands have become increasingly complex during the past decades as advances in structural engineering as well as material sciences have made it possible to realize buildings with complex configurations. These advances have outrun the prescriptive requirements in which complex geometries cannot be realized in accordance with the codes. Therefore, globally, more and more countries change their perspective codes and proceed to the design in function of objectives, such as an objective-based [2], a performance-based [1] [4] or a risk-informed [5] [6] format where the implicit acceptable safety level assumption in prescriptive rules now becomes explicit by showing the verified safety level. Although the aforementioned approaches still show some shortcomings [6], there is a consensus that a holistic approach is necessary in which the building configuration, user, content, safety systems and procedures are analysed together.

Risk-based methods provide a way to evolve towards such a holistic approach. More specifically, quantitative risk assessment techniques provide an opportunity to determine the safety level in a representative measure. The advantage is that both the magnitude and likelihood of hazards versus safeguards can be determined [7]. In [8] a procedure is developed to determine the risk value by means of a full-probabilistic risk assessment. The risk indication is represented by means of a failure probability in which the failure state can be predefined depending on the boundary conditions set up by the stakeholders.
A procedure for quantifying the life risk is developed. A flowchart and guidance is proposed in which successive steps are described for analysis [8]. Four of the major steps in the flow chart are elaborated in this paper. The first step discusses the preceding sensitivity analysis to reduce the number of input variables. The remaining variables are translated in discrete and continuous variables for the next part. The second step discusses the bow-tie structuring and important improvements incorporated in the bow-tie model in terms of reliability and efficacy. The third step deals with response surface modelling and reducing computational time for analysing complex multimodel tools. The results from the surrogate modelling are used for defining the failure limit stated design. This fourth step deals with the calculation of the failure probability in accordance to the limit state. In [8] more steps between the different models are discussed such as fault tree design, design of experiments, approximation of the failure domain, fire safety design, multimodel surrogate design, etc.).

3.1 Preceding sensitivity analysis

Generally, sensitivity analyses (SA) are performed to determine how uncertainty in the model output can be attributed to different sources of uncertainty in the model input [9]. The main objective of the preceding sensitivity analysis in fire safety risk assessment [10] & [11] is to reduce the dimensionality of the problem formulation. The larger the number of random variables, the higher the number of necessary solver evaluations. Considering the computational effort of several submodels, such as CFD and evacuation models, the dimensionality of several submodels needs to be reduced.

Before discussing different techniques in sensitivity analysis it is good to distinguish between two types of SA [9], [12]:

- **Local sensitivity analysis** is carried out to determine the local impact of parameters in the model. In this method are all derivatives taken at a single point. Usually, a local SA is only of practical importance when the variation around representative values of the input factors is small [13]. The main drawback of this approach is that interactions among factors cannot be detected, since they only become evident when the inputs are changed simultaneously.

- **Global sensitivity analysis**, the emphasis is on linking the output uncertainty to the uncertainty of the input factors. Global measures offer a comprehensive approach to model interaction analysis, since they evaluate the effect of a factor while all others are varying as well, exploring efficiently all dimensions of the input space. Therefore, this type of SA is persued.

![Figure 1: (Left) Responses for a 1D case with local and global effects and (Right) Representation of the balance between sensitivity analysis and computational efficiency. Taken from [14].](image)

The choice of an appropriate sensitivity analysis technique depends on several factors [15] such
as the computational cost of running the model. Another important factor is the number of input variables. In Figure 1 (right), a graphical representation is given for choosing an appropriate type of sensitivity method, without going into depth about all the different types of sensitivity analysis. For a particular situation, one starts somewhere in the neighbourhood between screening design and automated differentiation. Automated differentiation is not suitable for numerical models such as CFD, because of the underlying complexity due to time and geometrical dependence. Fractional factorial design as used in [16] is a possible alternative. However, a high number of solver evaluations would be necessary to have an estimation of the interaction effects. Therefore, a screening designs is suggested. More specific, a radial design sampling technique as developed in [12] is suggested. The advantage of this technique is that the amount of necessary evaluations doesn’t grow exponentially with the number of input variables to be analysed.

The radial design technique as investigated in [17] is a sampling technique in which the sensitivity of the different parameters is analysed similar to Morris sampling [18]. The main difference of the technique is that the Morris techniques uses random trajectories and radial sampling uses random star structures. A two dimensional case is shown in Figure 2.

The aim of the radial sampling is to calculate both the elementary effect and interaction effects between variables. In radial design each effect is computed over a different step size, equal to the distance between e.g. $x^{(u)}_i x^{(v)}$ which is equal to the difference between $x^{(u)}$ and $x^{(v)}$, where $u$ and $v$ denote two rows of the sampling matrix. Considering this notation, the absolute value of the elementary effect $EE_i$ can be calculated as [17]:

$$EE_i = \left| \frac{y(x^{(u)}_i x^{(u)}_{-i}) - y(x^{(v)}_i x^{(v)}_{-i})}{x^{(u)}_i - x^{(v)}_i} \right|$$

Consequently, the measure $\mu*$ can be calculated as the average over $r$ of these effects:

$$\mu_i^* = \frac{1}{r} \sum_{j=1}^{r} |EE_i^j|$$

The interaction effects can be calculated by means of the standard deviation:

$$\sigma_i^2 = \frac{1}{r-1} \sum_{j=1}^{r} (EE_i^j - \mu)^2$$

In which $\mu$ is the mean of the results.
3.2 Bow-tie methodology

A bow-tie method is suggested for representing and analysing discrete parameters as pathway factors [20]. The extent of the bow-tie model can be reduced by limiting the pathway factors and only taking the discrete parameters into account and to take the influence of continuous variables into account in the response surface model (step 3). Continuous parameters are taken into account in the response surface mode.

The bow-tie model is a combination of the fault and event tree analysis (FTA & ETA) with a critical event in the middle (Figure 3). FTA is a top down, deductive failure analysis in which an undesired state of a system is analysed using Boolean logic to combine a series of lower-level events. ETA is a forward, bottom up, logical modelling technique for both success and failure that explores responses through a single initiating event and lays a path for assessing probabilities of the outcomes and overall system analysis [21]. The bow-tie technique requires formation of fault structures at the left side and branch scenarios at the right side of the critical event. In this regard, risk is analysed from an engineering point of view by multiplying frequency and consequences. Each branch scenario has its own frequency and consequences in terms of fatalities per year. By providing preventive safety measures in the FTA and mitigation safety measures in the ETA part, the negative effects from fire situations can be reduced.

![Figure 3: Making best use of model. Taken from [21].](image)

It is important to include the effectiveness of safety systems. In engineering terms effectiveness can be defined as the product of reliability and efficacy [22]. Instead of only taking into account the probability of failure, the efficacy of the system should be integrated in the risk analysis. For example, a sprinkler system has a probability of activation and once activated it has a probability of containing or extinguishing the fire.

3.3 Response surface modelling

For every scenario branch to be analysed in the bow-tie structure, several non-fixed parameters are still variable and have to be taken into account. A method in order to deal with this aspect is developed by defining a response surface for every scenario and analysing the failure state (see part 3.4).

The purpose of the response surface model is twofold. At the one hand, it will take into account the complexity and combine different submodels [23] such as fire spread, smoke spread, evacuation, etc. At the other hand, it will reduce the computational effort necessary to analyse the scenarios. The purpose of the surrogate model is to create a response surface with only a few solver evaluations. The creation of the response surface makes it possible to generate a complex interpolation function by which the output can be generated for a new combination of input data without evaluating a new sample [24]. This means a high number of input
combinations can be analysed without additional computational effort, which is of great importance when performing limit state analysis in order to evaluate probabilities and ultimately risks.

The main objective of a response surface model is to approach the responses in the global domain for a certain model without understanding the physics of the system or when the modelling of the response becomes too complex. The response model can be formulated as:

\[ y = f(X) \]

in which \( y \) is the response and \( X \) is the vector of input variables (Figure 3).

\[ \text{Figure 4: Examples of a response surface model. Taken from [25] [14].} \]

A response surface model (RSM) can be used for limit state design [26] when the problem statement is not explicitly formulated [14] and thus not differentiable which is the case for CFD-models. It can be used when the limit state function is implicitly formulated, which is the case for numerical models [24]. It is the goal of a RSM to replace the output information of the complex model \( f(x) \) by an equivalent function \( \bar{f}(x) \) by which the computational procedures can be simplified. An example of such function can be a polynomial function of the following type:

\[ \bar{f}(x) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \]

(1)

in which \( x_i, (i = 1,..., n) \) are the basic variables and the parameters \( a, b_i, c_i, (i = 1,..., n) \) have to be determined.

The method for determining the model used for the current research is polynomial chaos expansion. Polynomial chaos expansions are a powerful surrogate modelling technique that aims at providing a functional approximation of a computational model through its spectral representation on a suitably built basis of polynomial functions. Without going into the fundamental mathematics, the polynomial chaos expansion of the model \( \mathcal{M}(X) \) is defined as [27]:

\[ \bar{f}(x) = \mathcal{M}(X) = \sum_{\alpha \in \mathbb{N}^m} y_\alpha \Psi_\alpha (X) \]

(2)

where the \( \Psi_\alpha (X) \) are multivariate polynomials orthonormal with respect to \( f_X, \alpha \in \mathbb{N}^m \) is a multi-index that identifies the components of the multivariate polynomials \( \Psi_\alpha \) and the \( y_\alpha \in \mathbb{R} \) are the corresponding coefficients.
3.4 Limit state design

In reliability based design one has to calculate the probability for the design to reach an undesired or unsafe state. Limit state design in fire safety engineering has been inspired by structural safety engineering because of the extensive experience with reliability analysis in this field. The general case of a limit state of a cross-section or construction element can be formulated in a limit state equation [28]:

$$ g(\mathbf{X}) = \mathbf{Z} = \mathbf{R} - \mathbf{E} = 0 $$

Where the vector $\mathbf{Z}$ consist of $n$ basic variables. For all the different variable an appropriate probabilistic model has to be chosen. In case a basic variable has a negligible variation or uncertainty, the variable can be considered as deterministic. The function is defined so that $g(\mathbf{X}) > 0$ corresponds to a safe condition, while $g(\mathbf{X}) < 0$ corresponds to failure. For life safety analysis this unsafe state would be reached when untenable conditions occur in the considered compartment or building before egress has been completed. The definition of untenable conditions can be translated towards Fractional Effective Dose (FED) criteria [29]. In other words, the probability of reaching a specific FED value for one person or a group of persons can be analysed.

An example of the failure region is shown in Figure 4 for a one and two dimensional case. The main purpose of the current step is to analyse the failure probability, defined above, by means of a level 2 or level 3 method [28]. In case the response surface can be formulated in closed form a level 2 method can be applied by means of the first order reliability method. In case no closed form can be derived, a level 3 method can be applied by means of methods such as Monte Carlo or importance sampling.

4 CASE STUDY

4.1 Problem description

The main purpose of the case study is to investigate the feasibility, efficiency and validity of the discussed methodology. In the current contribution, an analysis is performed in order to validate the response surface method (part 3.3) in which different slice files of toxicity components are estimated based on the surrogate model. The case study consists of a multi-purpose community assembly compartment. The fire scenario analysed is a fire on the stage area (red).
In the case study, the focus is put on the variability in the fire parameters. This means only one submodel, smoke and fire spread, is taken into account. This by means of a Computational Fluid Dynamics (CFD) model, more specifically the Fire Dynamics Simulator (FDS 6) tool.

From a preceding sensitivity analysis, it is decided to take three important parameters into account: the fire growth rate, the Heat Release Rate per Unit Area (HRRPUA) and the maximal area of the fire. In the table below the distributions for these parameters are described.

<table>
<thead>
<tr>
<th>Type</th>
<th>μ_log</th>
<th>σ_log</th>
<th>Analysed range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>α-growth</td>
<td>Var 1</td>
<td>Lognormal</td>
<td>-6.6</td>
<td>0.01 – 0.14</td>
</tr>
<tr>
<td>HRRPUA</td>
<td>Var 2</td>
<td>Lognormal</td>
<td>5.97</td>
<td>250 - 800</td>
</tr>
<tr>
<td>Max area</td>
<td>Var 3</td>
<td>Lognormal</td>
<td>0.78</td>
<td>2.0 – 71.0</td>
</tr>
</tbody>
</table>

Table 1: Input parameters case study.

Several methods have been investigated for the choice of the samples based on design of experiments. In this case study, the input samples are chosen based on Sobol [31] indices and the input range is chosen based on the domain of expected failure state of the probability density distributions. For academical purpose, 64 simulations are performed over a wide spectrum of the input range (see Table 1).
Apart from the 64 support points, 20 validation samples are evaluated. The validation samples are chosen between the ranges shown in Table 1, this in order to have an interpolation approach.

4.2 Results

In the figure below, the results are presented for a validation case different time steps for $\alpha = 0.0122$, HRRPUA $= 296\ kW/m^2$ and $A_{\text{max}} = 5.17\ m^2$ and compared with the results of the RSM. The figures show a good agreement between the estimated values based on the developed response surface and the validation set.

<table>
<thead>
<tr>
<th>Estimated set based on RSM</th>
<th>Validation set</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 120 s, equipotential [CO] = 120 ppm</td>
<td></td>
</tr>
<tr>
<td>t = 240 s, equipotential [CO] = 690 ppm</td>
<td></td>
</tr>
<tr>
<td>t = 300 s, equipotential [CO] = 1190 ppm</td>
<td></td>
</tr>
</tbody>
</table>
The average error in % is calculated between the estimated set and validation set over the entire xy-plane. This for every time step separately by means of:

\[
\text{Av}_\text{error} = \frac{1}{\#n} \sum_{i=1}^{x} \sum_{j=1}^{y} \frac{[CO]_{\text{estimated}} - [CO]_{\text{validation}}}{[CO]_{\text{validation}}}
\]

In the figure below, the average error in % is shown for the discussed simulation with 60 time steps (480 s). The results show that the average error is between -2.5 % and 2 % which is assumed to be a good agreement considering the highly turbulent flow in the compartment. It is expected to have better results in more stable conditions. The zero values for the initial values are because of two reasons: no concentrations are measured in the beginning, and errors on concentrations below 100 ppm were not taken into account because of insignificance compared to the major part of the other results.
In total 20 validation simulations were performed for the validation of the model. All of these simulations showed good agreement and small error compared to the validation set.

5 CONCLUSION

In this paper a procedure is developed for performing a full-probabilistic risk analysis to determine life safety risk in buildings in case of fire. The focus is put on the probabilistic part of the methodology. Several important steps are explained to determine the final risk value. The integrated approach involves several submodels for dealing with complex designs. The models can take into account all types of geometry and materials, human behaviour and different susceptibilities of people for smoke.

In the case study, the applicability of the PCE response surface model is investigated by performing a validation. The results show that approximation of the effluents of the fire is possible by means of proper surrogate modelling. Hence, the proposed approach indeed provides a possible alternative to reduce to reduce the computational efforts when analysing the life safety risk in fire safety calculations.

6 FUTURE PERSPECTIVE

In the ongoing research, the method will be tested with multiple interconnected submodels such as evacuation, fire spread, consequence models, etc. Secondly, the dynamic bow-tie structure will be developed.

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