Boundary Integral Equation Study of the Influence of Finite Conductivity on Antenna Radiation Using a 3-D Differential Surface Admittance Operator

Martijn Huynen, Daniël De Zutter, Dries Vande Ginste
 Department of Information Technology
 Ghent University, Belgium
 Email: martijn.huynen@UGent.be

Abstract—A novel method to simulate scattering and radiation from arbitrary materials, including good conductors, in a boundary integral equation (BIE) framework, is described in this contribution. The utilized technique relies on the differential surface admittance operator, which introduces an equivalent surface current density, effectively changing the antenna's material to that of the background medium. Constructed out of the volume's eigenmodes, the operator is employed to assess the radiation properties of a lossy Yagi-Uda antenna.

I. INTRODUCTION

The impact of the finite conductivity on scattering and antennas has been a research topic for over half a century. Initially, approximate current distributions were utilized to include the imperfect conductors' effect for thin-wire antennas [1]. This method has since been replaced by employing a surface impedance boundary condition, such as the Leontovich boundary condition, to couple the exterior electric and magnetic field without solving the interior problem [2]. This technique is, however, not applicable in all situations due to fundamental limitations and assumptions [3]. The differential surface admittance operator can circumvent these shortcomings [4]. This operator defines a fictitious surface current density that replaces the material by the background medium. Subsequently applying adequate numerical techniques such as the method of moments (MoM), yields the surface current density. This approach has been applied to 2-D scattering problems [5] and was recently proposed in 3-D for scattering at relatively simple geometries in [6]. In this contribution, the Dirichlet-to-Neumann operator [4] is applied in 3-D to assess the impact of conductivity on the characteristics of a realistic antenna structure. In Section II, the differential surface admittance operator concept is introduced for 3-D problems and an expression composed of the eigenvectors of the volume is presented. Section III features the validation of the novel method by means of a lossy Yagi-Uda antenna's analysis.

II. THEORY

We study the geometries in Fig. 1 assuming time-harmonic ($e^{j\omega t}$ dependence) electromagnetic fields. In Fig. 1(a) a homogeneous, non-magnetic material with wavenumber k is confined to the volume $\mathcal{V}$ with boundary surface $\mathcal{S}$ while the remaining, surrounding space is denoted $\mathcal{V}_0$ and defined by the

Fig. 1. (a) A homogeneous volume $\mathcal{V}$ with boundary $\mathcal{S}$ is embedded in another homogeneous volume $\mathcal{V}_0$. In (b), the inner material is replaced by the background medium and a surface current density $\mathbf{J}_s$ is introduced on $\mathcal{S}$.

wavenumber k_0 . This outside region is governed by the fields $(\mathbf{E}_0, \mathbf{H}_0)$ while the inner region's fields are $(\mathbf{E}_1, \mathbf{H}_1)$. On $\mathcal{S}$, these quantities are related through the Dirichlet-to-Neumann (DtN) operator $\mathcal{D}_k$ as follows:

$$\mathbf{u}_n \times \mathbf{H}_1 = \mathcal{D}_k \mathbf{E}_1^t, \quad (1)$$

with $\mathbf{u}_n$ the outward pointing normal to $\mathcal{S}$ and the superscript t indicating the tangential component of a field. In the different, but similar case of Fig. 1(b), the inner material is replaced by the background material. The field quantities inside $\mathcal{V}$ are now given by $(\mathbf{E}, \mathbf{H})$. As before, a DtN operator on the boundary surface links these fields:

$$\mathbf{u}_n \times \mathbf{H} = \mathcal{D}_{k_0} \mathbf{E}^t. \quad (2)$$

In order to retain the same field distribution in $\mathcal{V}_0$, a surface current density $\mathbf{J}_s$ is introduced. Given the boundary condition

$$\mathbf{u}_n \times (\mathbf{H}_1 - \mathbf{H}) = \mathbf{u}_n \times (\mathbf{H}_0 - \mathbf{H}) = \mathbf{J}_s \quad (3)$$

and (1)–(2), we introduce the differential surface admittance operator $\mathcal{Y}$ in the following expression for the surface current density:

$$\mathbf{J}_s = \mathcal{Y} \mathbf{E}_1^t = (\mathcal{D}_k - \mathcal{D}_{k_0}) \mathbf{E}_1^t, \quad (4)$$

where the tangential continuity of the electric field at $\mathcal{S}$ is exploited. An expression based on the eigenfunctions of the volume $\mathcal{V}$ is derived in [6] and is given by

$$\mathbf{J}_s = \mathcal{Y} \mathbf{E}_1^t = \eta \sum_l \left[\frac{|k_l|^2 \int_{\mathcal{S}} (\mathbf{u}_n \times \mathbf{E}_1^t) \cdot \mathbf{h}_l^* dS}{(k_l^2 - k^2)(k_l^2 - k_0^2) \mathcal{N}_l^2} \right] (\mathbf{u}_n \times \mathbf{h}_l). \quad (5)$$

Fig. 2. Yagi-Uda wire antenna with dipole feed, one reflector and three directors. Dimensions for operation at 110 MHz can be found in Table I.

TABLE I
DIMENSIONS (IN m) OF THE ANTENNA IN FIG. 2.

Parameter	Value	Parameter	Value
l_f	1.332	r	0.001
l_r	1.348	d_r	0.8
l_d	1.12	d_d	0.637

The contrast parameter η is defined as $(k^2 - k_0^2) / j\omega\mu_0$, k_l is the wavenumber of the l -th eigenmode $\mathbf{h}_l$, the solenoidal magnetic eigenmodes of the volume $\mathcal{V}$ with normalization constant $\mathcal{N}_l^2$. These eigenvectors satisfy equations (10.13) and (10.14) in [7]. For general shapes, the eigenmodes have to be calculated numerically but for some simple, omnipresent shapes such as the cuboid or the circular cylinder, closed expressions can readily be found in literature. In order to solve for $\mathbf{J}_s$, the exterior problem has to be solved as well. A boundary integral equation method (BIE) such as MoM is ideally suited for this type of situation. Employing a standard discretization scheme to both the selected BIE and (5) results in a set of matrix equations that can be solved using regular techniques. A more detailed description of this procedure can be found in [6].

III. EXAMPLE

Consider the Yagi-Uda antenna displayed in Fig. 2. The driving component of this structure is a dipole antenna with length l_f . The remaining elements are of parasitic nature: one reflector with length l_r at a distance d_r behind the central dipole and three directors with length l_d which are placed in front with a spacing of d_d . All wires have the same radius r . Values for these parameters can be found in Table I for the Yagi-Uda operating at 110 MHz, i.e., the center frequency of the localizer in an instrument landing system (ILS). The gain of this configuration is calculated numerically by means of the differential surface admittance operator for circular cylinders for varying inclination angles θ in the vertical plane (xz -plane). This gain pattern is plotted in Fig. 3 for decreasing values of the skin depth $\delta = \sqrt{2/(\omega\mu\sigma)}$, i.e., for increasing conductivity σ . A reference solution for a perfectly electrically conducting (PEC) Yagi-Uda is obtained through 4nec2 [8], a free NEC based antenna modeler. Due to the inherent symmetry of the structure under study, only half the range of θ is shown. It is apparent that the antenna response evolves towards that of the PEC solution for decreasing skin

Fig. 3. Gain of the Yagi-Uda antenna at 110 MHz for various values of the skin depth. The reference PEC solution is obtained through 4nec2 [8].

depths. Moreover, it is shown that an insufficient conductivity detrimentally affects the antenna's characteristics. The remaining discrepancy between the solution for $\delta/r = 0.01$ and the PEC reference can be attributed to both the fundamental approximations of the reference solution, viz., thin-wire MoM, and the finite mesh of the employed method.

IV. CONCLUSION

In this paper, we have presented a 3-D differential surface admittance operator for the analysis of a realistic antenna configuration, i.e., a Yagi-Uda antenna used in ILS systems. Through incorporation into a BIE-MoM structure, the 3-D operator's appositeness to the assessment of material properties on antenna performance is demonstrated.

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