Summary:
The Thermal Lag Engine (TLE) was patented by Peter Tailer in 1995 proposing an ultimately simple engine with a single moving part, the piston. During the return of the piston the engine gas temperature and pressure increase, because of the energy input from two processes: heat transfer from the hot source and compression work from the engine’s return mechanism. These processes happen at different speeds; energy gain from heat transfer happens slower than that from compression work. The TLE uses this time difference to diphase the engine’s heat gain from the compression work, which is the displacers function in conventional Stirling engines. Further the piston’s motion exposes and hides the cold heat exchanger creating the intermittency needed between cooling and heating for the engine to work. Even though this engine follows a Stirling like cycle, due to the time dependency of the engines return mechanism, the geometric dependence of the engine cooling mechanism, and the fact that these are also coupled, makes the modeling of this engine complex and only possible through a dynamical modeling approach. A previous attempt to model this engine has been done by Prof. Frank Wicks and Carlos Caminero from Union Collage, NY, and the results of this model were presented. This paper proposes a control volume energy based approach to modeling this engine, states a model and the results of its simulations. Further it presents a physical explanation of the TLE workings and states are of improvement. It concludes that the TLE could be a competitive engine in the future.

Stirling Engines, Thermal Lag Engine, Solar Energy.
Introduction

Air engines were invented by Sir. George Caley in 1807 and later improved by the Stirling brothers through the addition of the economizer or regenerator in 1816. [1] For the best part of the 200 years that have gone by, Stirling engines have been constructed in three configurations, Alfa, Beta and Gama types. These all consist in shifting a gas between cold and hot spaces in the engine using a mechanical devise, the displacer, and harnessing the pressure fluctuations that emerged from this displacement. The displacer’s motion was engineered to be a quarter of a cycle apart from the piston allowing the gas to expand and compress approximating isothermal processes. These implied that the working gas remained hotter throughout the expansion than the compression, making regeneration from the hot expansion stroke to the cold compression stroke possible and thus improving the engine’s efficiency. This increase in efficiency and the fuel flexibility provided by an external combustion devise makes Stirling machines very competitive with small scale hydrocarbon generation technology, especially considering the growing concerns about fuel scarcity and the environment.

Stirling engines and refrigerators are still an active area of research. Researchers are constantly proposing improvements to theses machines and better ways to describe them, mostly concentrating on the classical configurations. The three most challenging practical aspects of Stirling machines are: that its materials must work at the hottest temperature of the engine making lubrication and material selection difficult, the requirement for a constant mass process and designing heat exchangers that can accelerate heat transfer processes to increase the engine’s frequency. The mechanical coupling between the piston and the displacer introduced further difficulties in addressing these problems. In order to tackle these difficulties three different variants of Stirling like machines have been proposed and developed since the 1970’s, the thermoacoustic hybrid Stirling Engine (TASHE) developed by Swift and his team [2], the free piston Stirling Engine developed by W.T. Beale an others [3] and finally the Thermal Lag Engine (TLE) proposed by Peter Tailer in 1995 [4]. These three configurations eliminated the mechanical coupling between the displacer and the piston,
reduced the moving parts and addressed the sealing problem. Whilst the free piston engine uses gas springs to create the coupling between the piston and the gas’s displacement and the TASHE uses thermoacoustic oscillations for the same purpose, the TLE works on a different principle harnessing the time lag created by the gas’s thermal inertia to produce a lower pressure compression and higher pressure expansion. Harnessing this effect allowed Peter Tailer to construct an ultimately simple engine with a single moving part, the piston. The working principles of the TLE were laid down by Peter Tailer [5] and a model for a similar engine was proposed by Collin West [6]. Fran Wicks presented the results of a model for the TLE in 1994 [7]. The purpose of this effort is to build upon their work and present an approach to further understanding and modelling the TLE, as part of an ongoing investigation to obtain a competitive solar powered engine.

The thermoacoustic Stirling Hybrid Engine (TASHE), a well studied cousin of the Thermal Lag Machine (TLE) [8], follows a thermodynamic cycle that is extremely dependent on the time phase shift between its velocity oscillation and acoustic pressure oscillations [9]. Therefore solving the flow field in the TASHE engine becomes a crucial problem to study and design these machines. On the other hand the TLE depends on the time lag that exists between heat transfer originated and mechanically originated pressure changes in the engine arising from the gas’s thermal inertia. [4] Consequently solving the velocity flow field in the engine is not essential to describe its thermodynamic workings. This assumption then implies that mass and energy transport and dissipation from flow effects in the TLE are negligible, only adding undesired energy dissipation effects within the system. The modeling approach described bellow makes this assumption. These effects could be incorporated as loss terms in later versions of the modeling as well.

The model follows a zero-dimensional control volume approach, using time dependent energy and mass balance relationships to describe the engine’s behavior. It assumes that the engine is composed of a fixed volume hot space and a varying volume cold space. These spaces are coupled by overall energy and mass balance relationships, yet their corresponding gas temperatures are allowed to vary separately. The pressure
equalizes throughout the system at the speed of sound and thus is assumed to be uniform throughout the engine as the integration of the model occurs, yet because the temperatures are different in the hot and cold spaces, there are density variations along the engine.

The model consists of 3 control volumes (CV$_{tot,h,c}$). CV$_h$ encloses the hot section of the engine, and has constant dimensions. CV$_c$ corresponds to cold section of the engine and has variable dimensions along its axial direction; this dimension is varied by the displacement of the piston. CV$_{tot}$ is a control volume that engulfs the other 2. This is illustrated in Figure 1.

The model consists on mass and energy balances for each of these CVs. The TLE engine follows an “ideal” cycle composed of the following sequence of processes: a polytropic expansion, a small adiabatic expansion, isochoric cooling, a small adiabatic compression, a polytropic compression and finally isochoric heating. This cycle is illustrated in Figure 2. The index of the polytropic processes “n” varies in time and this variation is coupled to the ratio of heat exchange to work exchange, which is also not constant in time as this varies with the piston dynamics. Therefore a dynamic model is required to describe this engine’s behavior. This is not surprising as so much of this engine’s workings happen because of the effect of time.
The Physics

In order to understand decisions made about the modeling later, it is pertinent to explain how the physics of each process works and some of the fundamental assumptions. This will be done next. For 1-2 it is assumed the expansion of the working gas drives the engine, only displacing the piston to do as much work as the energy balances permit from the heat input at $CV_h$, the heat loss at $CV_c$ and the internal energy change. Energy is passed between $CV_h$ and $CV_c$ as the pressure in the $CV_h$ drives hot mass into the other control volume. If left be, this expansion would take place until the gas could not produce enough force on the piston to overcome the engine’s load, the point at which this would occur would depend on the rate of heat input to output. As the piston displaces outward more cooling surface is exposed to the gas increasing the heat transfer out of the engine. The expansion can happen quasi-isothermally as long as $Q_h$ is much greater than $Q_c$, when they become equal the expansion can only be adiabatic. From the piston position where $Q_h$ equals $Q_c$ the gas would continue to expand doing work at the expense of its internal energy. This further piston displacement changes the heat exchange surfaces such that $Q_c$ becomes greater than $Q_h$, this happens in process 2 – 3. Then isochoric cooling can take place, which is process 3 – 4.

At 4 the return mechanism of the engine kicks in, so external work is provided in order to bring the piston back to 6. Initially an adiabatic compression takes place modifying
the heat exchange ratios. This is process 4 – 5. Then a polytropic compression takes place, 5 – 6, through this process the engine pressure is increasing due to heating and compression work, therefore the polytropic index is greater than the adiabatic index throughout this process. As the piston compresses CV_c, cold working gas is pushed into CV_h. This gas then heats up increasing the internal energy of the working fluid overall. Still the compression process pushes cold mass into CV_h quicker than it can gain energy from the source by heat exchange. This is precisely the thermal lag effect. This delay in the heating process implies that the compression occurs at a lower mean temperature and pressure than the expansion, meaning net work is produced. The quicker the piston is driven back the greater the thermal lag effect and thus the larger the power output of the engine per cycle. The occurrence of this effect was observed by Peter Tailer in his experiments with Stirling engines, and it is what gave birth to the TLE concept [5].

The thermal lag effect is also present during the expansion process, when the hot gas is coming into CV_c it also experiences a delay in the cooling compared to a purely adiabatic expansion, this is because the gas is heating up as the engine is expanding. If the expansion and compression happened infinitely fast the theoretical maximum power output of this engine would be that of an ideal Otto cycle. Yet if the effect of time is considered and the expansion is allowed to be slower than the compression, in the ideal limit the TLE expansion would be isothermal and the compression adiabatic, yielding the maximum work output per cycle for a TLE machine as described by P. Tailer and C. West. [10] In a slower expansion driven by the rate of heat input and a quick compression driven by the piston, the work output per cycle of the ideal TLE would be greater than that of the ideal Otto cycle. This is illustrated in Figure 3.
Thus the TLE cycle is a Hybrid between the Otto cycle and the Stirling cycle. The rate of heat exchange to and from the engine is dependent on the engine geometry and piston’s displacement. Thus it is possible to devise a TLE engine configuration, by dumping extra heat during the compression, so that it returned by an isothermal, rather than an adiabatic process. This would then permit the TLE to approach the Stirling cycle. Having the TLE return via an isotherm would yield a larger cycle than in its classic configuration as illustrated in Figure 4, yet in order to do so extra heat needs to be dumped thus reducing the cycle’s efficiency. This could be ameliorated by regenerating from the expansion to the compression, as illustrated in Figure 4 the temperatures of the processes could permit this.
The Regenerator

The regenerator or economizer in Stirling cycles takes some of the energy that otherwise would have to be dumped throughout the expansion and returns it during the compression. This is possible because the expansion takes place at a higher temperature overall than the compression, such is also true for the TLE cycle. Peter Tailer, suggests that including a regenerator between the cold and hot section of then engine would reduce its efficiency. Considering that in the TLE a regenerator between the hot and cold sections would aid in the cooling when you want heat input during the expansion and assists in the heating when heat is required to be dumped during the compression, this seems like a correct assertion as the area enclosed by each cycle would be reduced. Yet efficiency is about the rate of work done to the heat supplied, and even though work output per cycle would be reduced this rate could become smaller in a TLE with a regenerator. In other words for the TLE there seems to be a compromise between net work output, the heat required to produce it and the heat that could be recovered in a regenerator.

Many of the model TLE engines built include some form of metal mesh in the hot section of the engine. Peter Tailer in his experimental TLE engines added chain links in the hot section of the engine in order to improve heat transfer in that space, it was doing this that made his original TLE produce a significant amount of work. [5] Colin West in his theoretical paper about a TLE like engine also suggests the presence of a wire mesh in the hot section of the Engine.[6] This seems counter intuitive with the previous statements about the regenerator, but the location of this wire mesh inside the engine this will be very significant. Such a mesh in the hot section would add thermal inertia in the hot part of the engine making it more difficult to cool down the engine during the expansion. Further it would improve considerably the heat transfer to the gas in that region, thus accelerating the speed at which the isothermal expansion can take place. This effect would increase the potential power output of the engine.

The effects of the hot mesh during the compression would not be significant because the return of the engine would be much quicker than the expansion. It is well understood that the variation of temperature because of compression is many times faster than the
practical variations attainable through heat transfer in any heat exchanger. As the gas is compressed the temperature increases and the rate of heat transfer into the gas is then reduced. The increase in temperature from compression is so quick that the enhanced heat exchange capacity of the mesh would be brought almost immediately to its value before the isochoric heating starts, well before much heat could be added from the mesh to the gas. Further, during the compression cold gas is added to the hot section, which is heated quicker from the compression than the heat exchange from the mesh. The implication of these combined effects is that even with the enhanced heat exchange in the hot section the piston would still be able to compress the gas in a quasi-adiabatic way.

It seems that this metal mesh serves as a form of regenerator and this should become apparent in a detailed modeling of the machine, which considers thermal variation in the gas and the engines body together and captures accurately the longitudinal temperature profile of the engine. The effects and location of this mesh would not be considered at this point in the model. In future work this will be investigated.

The TLE Model

<table>
<thead>
<tr>
<th>Variables:</th>
<th>K = constant belonging to Fourier equation including the overall heat transfer coefficient and the area, or the specific area in the case of the variable heat exchanger. (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$ = Internal Energy (J)</td>
<td>Subscripts:</td>
</tr>
<tr>
<td>$Q$ = Heat transfer (J)</td>
<td>$c$: corresponds to cold control volume</td>
</tr>
<tr>
<td>$W$ = work (J)</td>
<td>$h$: corresponds to the hot control volume</td>
</tr>
<tr>
<td>$H$ = Enthalpy flux (J)</td>
<td>$tot$: corresponds to the cold control volume</td>
</tr>
<tr>
<td>$T$ = Temperature (K)</td>
<td>$tr$: corresponds to the mass of fluid that is in transition between the CV$_h$ and CV$_c$</td>
</tr>
<tr>
<td>$V$ = volume (m$^3$) (Also for subscripts)</td>
<td>$p$: corresponds to the piston</td>
</tr>
<tr>
<td>$P$ = pressure (Pa) (Also for subscripts)</td>
<td>$f$: corresponds to the fins</td>
</tr>
<tr>
<td>$m$ = mass (kg)</td>
<td>$s$: corresponds to the source</td>
</tr>
<tr>
<td>$\eta$ = efficiency</td>
<td>$amb$: corresponds to ambient values</td>
</tr>
<tr>
<td>$A$ = area (m$^2$)</td>
<td>$0$: implies initial</td>
</tr>
<tr>
<td>$x$ = the displacement of the piston (m)</td>
<td></td>
</tr>
<tr>
<td>$c$ = specific heat capacity (J/kgK)</td>
<td></td>
</tr>
<tr>
<td>$C(t)$ = function describing the process</td>
<td></td>
</tr>
<tr>
<td>$D$ = Piston displacement amplitude (m)</td>
<td></td>
</tr>
<tr>
<td>$N$ = normalization constant for displacement</td>
<td></td>
</tr>
</tbody>
</table>

Referring to Figure 1 the model consist on energy and mass balances between the CVs. The engine modelled is an approximation of the engines observed to work on the internet and to Peter Tailer's experiments. [11] It only seeks to capture the thermodynamic essence of the thermal lag concept. In the future a more detailed model
will be developed including momentum effects inside the engine, the heat capacity of engines body and other effects. Experiments are being devised to validate these and future results. This model consists of two spaces $CV_h$ and $CV_c$. $CV_h$ receives energy by heat transfer from the source and gives energy to $CV_c$ by enthalpy flux, so only flow work is done by $CV_h$. $CV_c$ receives the enthalpy flux from $CV_h$ in order to perform shaft work and dumps heat to the sink in order to achieve a lower pressure return and thus produce network output. The mass in the system is conserved, thus all the mass that leaves $CV_h$ enters $CV_c$ and vice-versa. A displacement function has been devised and imposed to capture the thermal lag effect; this function is asymmetrical meaning that the compression happens faster than the expansion. This allows for the cycle to give a net work output, with a symmetrical displacement function this machine behaves like a refrigerator. In future models, an inertial displacement model will be used instead of imposing the displacement; this will allow studying better the dynamics of the system. At this point this has not been done because of complex couplings in the mathematics, which has made the integration of such a model impossible until now. Thus the model proposed is:

\[
\begin{align*}
CV_h & \quad CV_c & \quad CV_{tot} \\
\dot{Q}_h &= \dot{H}_{\text{in}} (T_s - T_h) & \dot{Q}_c &= K_c \eta_c \alpha (T_{\text{in}2} - T_c) \\
\dot{U}_h &= \dot{Q}_h - H_{\text{in}} & \dot{U}_c &= Q_c + H_{\text{in}} - W_c \\
U_h &= m_h c_v T_h & W_c &= \rho_c \dot{V}_c \\
\dot{P}_h &= \frac{m_h R T_h}{V_h} & U_c &= m_c c_v T_c \\
V_c &= \rho_c \dot{x} & \dot{P}_c &= \frac{m_c R T_c}{V_c} \\
\end{align*}
\]

Isobaric expansion of $CV_c$ followed by an isochoric pressure equalization of $CV_c$ and $CV_h$. 
Assumptions:

- The overall heat transfer coefficient is constant and governed by the biggest film resistance, which would be in that of the air outside. The value chosen for this coefficient is a standard value for free convection in air 20 W/m².
- The gas is assumed to behave as a perfect gas and to be air.
- The displacement function of the engine is imposed by an external mechanism.
- Momentum and pressure fluctuations effects are considered negligible.
- $T_s$ and $T_{amb}$ are constant.
- The thermal inertia in the engine’s body is assumed negligible (The gas is changing temperature very quickly thus the thermal inertia of the body will be relevant, this will be considered in later work).
- By convention the mass transfer out of $CV_h$ is considered negative.
- The reference state for the enthalpy calculation is 0K.
- The fin surface efficiency is given by Schmidt’s formula.

Integration strategy

In order to integrate the above model the following integration strategy was followed. The model was discretized using a fixed time step. The dimension of this time step was explored and numerical stability is achieved around 1000 cuts per cycle.

In order to close the model a process would have to be defined within a time steps. This process is unknown as it would have a different polytropic index for every time step. Thus it was decided to approximate this process through other known processes, if the time step were small enough this would provide and adequate approximation. The process between states across a time step was approximated by an isobaric expansion of $CV_c$ followed by an isochoric pressure equalization between the CVs, assuming that both CVs reach an equal pressure at the end of every time step. This corresponds to the assumption that pressure fluctuations are occurring many times faster than the bulk movement of the engine and thus the engine follows a process such that the pressure in both CVs is equal throughout the cycle. This isobaric volume change followed by and isochoric pressure equalization, would allow mapping the process through which the engine is actually going, along the equal overall pressure line. This is illustrated in Figure 5. An algorithm was programmed to perform this integration.
Displacement function

A key factor in achieving a network output from this machine was the choice of displacement function. If the displacements function was a symmetrical one, even if a delay was included at the ends, both the expansion and compression would happen very quickly. The gas would then be cooled by expansion below the sink temperature causing it to take heat in and remain there for some time causing it to heat up further. Then the compression would pump heat back into the source as it would raise the temperature above $T_s$ in $CV_h$. If the displacement function was selected to be asymmetrical such that the compression happened very quickly and the expansion slower, the gas would have time to heat up while it expands and not while it compresses. This allows for a compression at a lower pressure than the expansion. The more asymmetrical the displacement function, the better the work output per cycle of the engine. This would be the case because it would perform an isothermal expansion and spend a longer time cooling before the compression, allowing for a lower pressure compression. Playing with the displacement function and the rate of heat transfer in the cold side of the engine holds the key to a competitive TLE. The displacement function for this model is illustrated in Figure 6. The results of the simulation with this
displacement function and free convection overall heat transfer coefficients for both CVs are shown in Figure 7.

![Figure 6: Displacement Function](image1)

![Figure 7: Air cooled TLE PV-diagram](image2)

### Work output per cycle: 0.06 J
### Efficiency: 4.4%
### Frequency: 30 Hz (1800 rpm)
### Power output: 1.8 W
### Heat transfer coefficients: 20 W/m²
### K ratio: 12.8 (K_c/(K_h/L_1))
### Compression Ratio: 2 (V_{min}/V_{max})
### Source Temperature: 600 K
### Sink Temperature: 302 K
### Surface efficiency for fins: 76%

**Dimension illustrated in Figure 1.**

**Potentials for improvement**

Figure 7 illustrates a very small work output and efficiency. The intention was to model a machine that worked like the small TLEs that have been described and observed, which work with the ambient as the heat sink and produce enough work to simply move their flywheels. The cycle above would be capable of such a feat, leaving very little room for producing useful work. Thankfully there is lots of room for improvement for a TLE designer to produce a competitive engine. Doing so will require a complex optimization process of the many parameters involved, which effects are all coupled together in
nontrivial ways. This would allow finding an acceptable compromise between all these parameter and thus a good TLE.

Some experiments have been performed through simulation and it has been observed that there is a compromise between work output per cycle, the engines efficiency and the frequency. For example increasing the frequency reduces the work output per cycle, but increases the engine’s efficiency as more work is done over time with a similar heat input. There must be an optimum compromise between these variables.

Yet, the most significant space for improvement in this mechanical configuration of the engine will be from improving the heat exchangers design both in the hot and cold section. The rate of temperature raise from heating, to temperature drop from expanding is a fundamental relationship in achieving a larger power output for the engine. The quicker the heating the faster the expansion can be, and thus the engine’s frequency can be increased without compromising work output per cycle, thus improving both, the efficiency and power output per unit of volume of the engine.

A more important relationship is the ratio of the Fourier equation constants $K_s$. The larger $K_c$ can be with respect to $K_h$, without increasing the diameter of $CV_c$ which would increase the rate of temperature drop from expansion, the greater the power output per cycle of the engine. The values of $K$ are limited by physical and practical parameters, but they can be improved through heat exchanger design and/or cooling the $CV_c$ with a liquid jacket. Figure 8 illustrates the cycle’s properties if $CV_c$ was cooled using a water jacket leaving all other parameters the same. This would change the overall heat transfer coefficient there to an approximate value of 100 W/m² starting to postulate an attractive TLE.
Conclusions

A model of the TLE has been presented as well as a physical explanation of its workings. The factors leading to potential areas of improvement to achieve a competitive TLE have been discussed and illustrated. Ideal limits of such a cycle have also been explored based on the previous work of Peter Tailer, Collin West and Frank Wicks. Limitations of the modeling done have been pointed out and areas for improvements discussed. This work preludes with encouraging results a further investigation of the TLE as a potential very competitive engine to harness the heat from the sun.

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