Dedicated Stochastic Framework for the Variability Analysis of Compressible Textile Antennas

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Abstract—A dedicated stochastic framework to address the impact of substrate compression on the performance of textile antennas is presented. The joint probability distribution of the height and the permittivity of a compressible substrate is experimentally estimated. Then, a generalized polynomial chaos technique accurately quantifies the effect of the random variations on both substrate parameters on the figures of merit of a dual-polarized ISM textile antenna.

I. INTRODUCTION

Among the challenges posed by the deployment of wearable components in critical scenarios, the effect of substrate compression on the performance of textile antennas still represents an open problem for designers. Up to now, this effect is accounted for in the design phase by checking the performance of the antenna for different values of the thickness of the substrate and by adjusting the permittivity accordingly, since the material density and, hence, its dielectric constant, increases under compression. However, this approach is not rigorous nor supported by experimental results on the variations of height and permittivity and on the correlation between them. Therefore, we present a stochastic framework that combines the measurements performed on several samples of a compressible substrate with a non-intrusive stochastic collocation method (SCM) based on generalized polynomial chaos (gPC) [1] to quantify the impact of variations in substrate permittivity and thickness on the antenna's figures of merit. An example demonstrates a speedup factor of 83 compared to Monte Carlo.

II. STOCHASTIC FRAMEWORK FOR COMPRESSIBLE ANTENNAS

A. Stochastic Collocation Method

Consider the textile dual-polarized probe-fed compressible ISM patch antenna shown in Fig. 1, which is designed (see Table I) to resonate at the frequency f_r=2.45 GHz. Our aim is to assess the effect of stochastic variations in the substrate permittivity ϵ_r and height h on the magnitude of the return loss S_{11} at the operating frequency f_r, which is required to be lower than 0.316 (-10dB). Therefore, we introduce a SCM and we define $|S_{11}|$ as a function of two normalized independent input random variables x_1 and x_2, corresponding to h and ϵ_r, respectively, by the following gPC expansion of total order $2Q$:
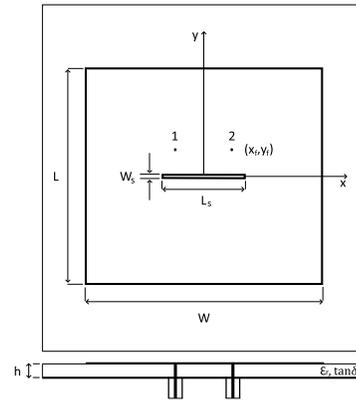

Fig. 1. Schematic of the dual polarized ISM textile antenna under study.

TABLE I
NOMINAL VALUES OF THE ANTENNA'S GEOMETRICAL PARAMETERS
(FIG. 1).

parameter	value
patch length L	44.46 mm
patch width W	45.32 mm
slot length L_S	14.88 mm
slot width W_S	1 mm
feed points (x_f, y_f)	$(\pm 5.7, 5.7)$ mm

$$|S_{11}| = f(x_1, x_2) = \sum_{k_1=0}^{Q} \sum_{k_2=0}^{Q} t_{k_1 k_2} \phi_{k_1}^{X_1}(x_1) \phi_{k_2}^{X_2}(x_2), \quad (1)$$

where $\phi_{k_i}^{X_i}(x_i)$ are polynomial basis functions of degree k_i corresponding to the i-th variable ($i = 1, 2$). Since x_1 and x_2 are mutually independent, the polynomials $\phi_{k_i}^{X_i}(x_i)$ are introduced as functions of a single random variable. Moreover, they are orthonormal with respect to the following inner product:

$$< u(x_i), v(x_i) >= \int_{\Gamma_i} u(x_i)v(x_i)p_{X_i}(x_i)dx_i, \quad (2)$$

where Γ_i is the support of the PDF $p_{X_i}(x_i)$ of the random variable x_i, and are selected according to the Wiener-Askey scheme for commonly used PDFs [1]. Any existing correlation among the variations on the antenna's design parameters x'_1, x'_2 is related to the normalized random variables x_1, x_2 in (1) by

$$[x]' = [T][x] + [\mu], \quad (3)$$

where $[T]$ is the upper triangular matrix resulting from the Cholesky decomposition of the covariance matrix $[\Sigma]$ corresponding to the joint PDF of x_1', x_2', whereas $[\mu]$ is the vector of the mean values of x_1', x_2'. The coefficients $t_{k_1 k_2}$ are calculated by applying Galerkin weighting to (1):

$$t_{k_1 k_2} = <f, \phi_{k_1}^{X_1}\phi_{k_2}^{X_2}> = \int\int_\Gamma f\phi_{k_1}^{X_1}\phi_{k_2}^{X_2}P_{X_1 X_2}dx_1 dx_2 \quad (4)$$

where $\Gamma = \bigcup_i \Gamma_i$ is the 2D support of $P_{X_1 X_2} = p_{X_1}(x_1)p_{X_2}(x_2)$. Integrals (4) are then evaluated by the following $V_1 \times V_2$-point Gaussian quadrature rule:

$$t_{k_1 k_2} \approx \sum_{l_1=0}^{V_1}\sum_{l_2=0}^{V_2} w_{l_1}w_{l_2}\phi_{k_1}^{X_1}(x_{l_1})\phi_{k_2}^{X_2}(x_{l_2})f(x_{l_1}', x_{l_2}') \quad (5)$$

where the quadrature points x_{l_1}, x_{l_2} and weights w_{l_1}, w_{l_2} are derived by the Golub-Welsch algorithm [2]. The values $f(x_{l_1}', x_{l_2}')$ are computed with ADS Momentum, after the quadrature points x_{l_1}, x_{l_2} are modified according to (3) to embed the information about the correlation among the antenna's design parameters into the coefficients $t_{k_1 k_2}$.

B. Estimation of the input PDF

A resonance-perturbation technique based on an aperture-coupled patch antenna is devised to estimate the PDF of the relative permittivity ϵ_r and the thickness h of compressible substrates. The structure consists of two rigid planar circuit boards, containing the feed of the antenna and the conductive patch, respectively, with the material under test (MUT) in between. The structure was originally designed to resonate in the vicinity of the GPS-L1 frequency band. However, when the thickness and permittivity of the MUT change, this resonance frequency is shifted. Therefore, if the thickness of the substrate is known, the value of the measured resonance frequency can be used to estimate the permittivity of the sample. This procedure is carried out by simulating the structure in CST Microwave Studio's frequency domain solver with the measured value of thickness of the MUT and by varying the value of the permittivity of the MUT until the values of the simulated and measured resonance frequencies match.

III. EXAMPLE

The proposed stochastic framework is applied to the textile antenna shown in Fig. 1 [3], which is designed for a flexible closed-cell expanded rubber protective foam with height $h = 3.94$ mm, permittivity $\epsilon_r = 1.53$ and a loss tangent $tan\delta = 0.012$. 25 different samples of this material were compressed up to 70% of their nominal value of h by means of a Hotronix Air Fusion Heat Press to achieve 10 different values of thickness and the corresponding values of relative permittivity were estimated. By fitting a bivariate normal distribution to the measured data, we obtained the mean values $\mu_h = 3.36$ mm and $\mu_{\epsilon_r} = 1.58$ of the substrate thickness h and permittivity ϵ_r, respectively, as well as the covariance matrix $[\Sigma]$ of their joint PDF. The correlation value ρ between h and ϵ_r is found to be equal to -0.68, thus indicating strong correlation between the two parameters.

The sets of orthonormal polynomials $\phi_{k_1}^{X_1}(x_1)$, $\phi_{k_2}^{X_2}(x_2)$ simply comprise conventional Hermite polynomials, as the estimated input PDF is a bivariate normal distribution. The gPC expansion (1) converges for a maximum order of expansion $Q = 10$. In order to compute the expansion coefficients $t_{k_1 k_2}$, $(Q+1)^2 = 121$ quadrature points are evaluated, requiring 24 min (Intel i7 CPU, 16 GB RAM). Fig. 2 shows $|S_{11}|$ as a function of the variables x_1 and x_2, in a span of ± 3, corresponding to a normalized interval of $\pm 3\sigma$. The limit of 0.316 (-10dB) is indicated by means of a grey plane.
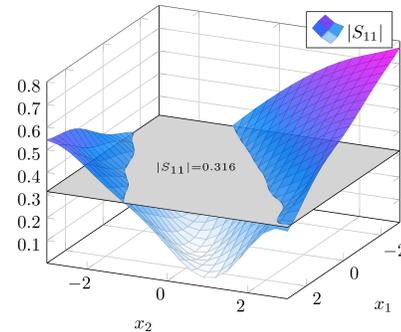

Fig. 2. $|S_{11}|$ as a function of the normalized thickness h (x_1) and the normalized relative permittivity ϵ_r (x_2) according to SCM.
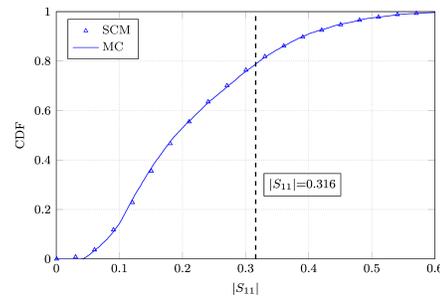

Fig. 3.   Comparison between the CDFs of $|S_{11}|$ constructed with SCM and the Monte Carlo (MC) simulations.

To validate our analysis we perform a Monte Carlo simulation, evaluating 10000 realizations of h and ϵ_r in ADS Momentum, which takes about 33 h. The corresponding cumulative distribution functions (CDF) for $|S_{11}|$ and the one computed from the expansion (1) are shown in Fig. 3. Both CDFs are perfectly overlapping. We notice that $|S_{11}|$ is within the threshold value of 0.316 in only 79.24% of the cases.

ACKNOWLEDGEMENT

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