Full-Wave Modeling of Interacting Multiport Devices with Arbitrary Relative Positions and Orientations for Efficient EMI Assessment

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Abstract—A novel method to accurately and efficiently model the interaction between radiating devices is proposed. Whereas previous work of the authors dealt with *singleport* devices (antennas), this paper constitutes an important extension to actual *multiport* devices, as such paving the way for electromagnetic interference assessment of real-life (sub)systems early in their design cycle. The method solely relies on the knowledge of the radiation patterns of the different ports of the devices, which can either be measured or simulated using a solver of choice. These patterns are then used to compute the electromagnetic interaction between the devices that may be positioned in each other's radiative near-field (Fresnel region) or far-field (Fraunhofer region). Furthermore, in the model, their relative positions and orientations can be altered at a very low computational cost. The technique is thoroughly validated and illustrated, demonstrating its appositeness to study the electromagnetic compatibility behavior of the multiport devices.

Index Terms—Electromagnetic Interference (EMI) Modeling, Radiated Immunity and Emission, Rotating Multiport Devices

I. INTRODUCTION

KNOWLEDGE of the Electromagnetic Compatibility (EMC) behavior of devices and systems is of the utmost importance in high-speed, high-sensitivity and also general electronic applications. Concerning Electromagnetic Interference (EMI) and EMC aspects, it is critical to design reliable and robust products, and also to pass legislative and regulatory requirements. Traditionally, the radiated emission or immunity behavior is characterized in an anechoic or semi-anechoic chamber, where the Device Under Test (DUT) is rotated, and a new measurement is performed for every angular position. Nowadays, it is well-known that the success or failure of a product regarding EMC, is already partially determined during the design phase (or precompliance phase). Computer-aided design can take radiated emission and immunity into account numerically early enough during the development of the product, but this is not a trivial task since even a single simulation often requires a large amount of computational resources in order to achieve reliable results. Moreover, to study the entire EMC behavior, the analysis needs to happen for many angular positions of the DUT, and a lot of (computationally expensive) simulations are needed.

Of course, several numerical techniques exist that relax these high computational requirements. First of all, to analyze

emission, equivalent model or model-reduction techniques may reduce the complexity of the problem [1], [2]. Furthermore, when looking at susceptibility, various hybrid techniques have been proposed that combine full-wave methods with model-reduction techniques [3], [4] or that use an extended *S*-parameter model [5], [6]. Another way to deal with the high computational demands is by using Domain Decomposition Methods (DDMs). These methods tackle a problem by dividing it into several subproblems, which are individually solved in an efficient way, and afterwards recombined to assess the global behavior [7]–[11].

In previous work [12], [13], a formalism to efficiently model electromagnetic (EM) interactions between two *singleport* devices in (semi-)anechoic conditions was presented. This approach is valid for devices that are positioned in each other's far-field (Fraunhofer region) or radiative near-field (Fresnel region). In [14], this approach was extended to allow for any number of *singleport* devices.

In this paper, we extend the formalism for *singleport* to *multiport* devices. Such a multiport device may consist of physically separate devices, e.g. an array of antennas, or it can just as well be an electronic circuit (with multiple ports) on a Printed Circuit Board (PCB). The main difference with previous approaches is that, now, crosstalk between ports can be taken into account. This enables efficient intra-system modeling of the EMI (both emission and susceptibility) of several subsystems constituting an entire system, and also inter-system modeling of real-life complex products, thus characterizing their EMC behavior early in the design cycle. The novel method requires only a single simulation (or measurement) of the radiation patterns of each multiport. Moreover, rotation of these multiports is made tractable by leveraging a spherical harmonics expansion together with Wigner-D matrices. Furthermore, efficient repositioning of devices relative to each other is possible, also without a high computational cost. The advocated method is carefully validated concerning its accuracy and efficiency, and a state-of-the-art application example shows its appositeness.

This paper is organized as follows. In Section II the novel extended formalism which describes the EM interaction between multiports as well as their rotation over arbitrary angles, is explained. Section III gives a thorough validation and illustration. Conclusions are presented in Section IV. In the sequel, all sources and fields are assumed to be time harmonic with angular frequency ω and time dependencies $e^{j\omega t}$ are suppressed. Unit vectors are denoted with a “hat”, e.g. $\hat{v}$.

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II. FORMALISM

A. Problem description

Consider the general problem geometry of two multiport devices (Fig. 1). An N -port and an M -port are arbitrarily po-

Fig. 1: Schematic representation of an N -port device interacting with an M -port device. Each multiport device may be rotated over arbitrary angles (θ_{TX}, ϕ_{TX}) and (θ_{RX}, ϕ_{RX}) about their respective phase centers $\mathcal{O}_{TX}$ and $\mathcal{O}_{RX}$.

sitioned in space and they are electromagnetically interacting with each other. Each multiport device is assigned a phase center, $\mathcal{O}_{TX}$ or $\mathcal{O}_{RX}$, at positions $\mathbf{r}_{TX}$ and $\mathbf{r}_{RX}$ respectively. These phase centers are the points from/about which translations and rotations of the multiport devices will be performed later on.

When considering, e.g., the N -port as a transmitting (TX) device, i.e., the source of the disturbance, the goal is to efficiently and accurately compute its influence at all ports of the receiving (RX) M -port, i.e. the victim of the disturbance.

B. Electromagnetic interaction between multiport devices

In [12] and [13] it was shown, for *singleport* devices ($N = M = 1$), placed in each other's radiative near-field (Fresnel zone), that the short-circuit current induced at the port of the receiving device is given by

$$I_{sc} = \frac{-1}{Z_c V_0} \iint_{\Omega} T(\mathbf{r}_{TX,RX}, \hat{\mathbf{k}}) \mathbf{F}_{TX}(\hat{\mathbf{k}}) \cdot \mathbf{F}_{RX}(-\hat{\mathbf{k}}) d\hat{\mathbf{k}}. \quad (1)$$

Here, Z_c is the wave impedance of the background medium and V_0 is a pertinent normalization factor. The integration in (1) is performed over the Ewald sphere Ω . $\mathbf{F}_{TX}(\hat{\mathbf{k}})$ and $\mathbf{F}_{RX}(\hat{\mathbf{k}})$ represent the radiation patterns of the transmitting and receiving device in the direction $\hat{\mathbf{k}} = \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}}$, i.e. the unit wave vector in spherical coordinates. The translation operator $T(\mathbf{r}_{TX,RX}, \hat{\mathbf{k}})$ is given by:

$$T(\mathbf{r}_{TX,RX}, \hat{\mathbf{k}}) \approx \sum_{l=0}^L (2l+1) j^{-l} h_l^{(2)}(k|\mathbf{r}_{TX,RX}|) P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}_{TX,RX}). \quad (2)$$

Here, $h_l^{(2)}(\cdot)$ is the l -th order spherical Hankel function of the second kind and $P_l(\cdot)$ the Legendre polynomial of degree l . The number of multipoles L truncates the sum and determines its accuracy. Traditional guidelines to select this number can

be used [15]. The vector $\mathbf{r}_{TX,RX} = \mathbf{r}_{RX} - \mathbf{r}_{TX}$ represents the relative positions between the devices' phase centers.

Consequently, for interacting *multiports* as shown in Fig. 1, the short-circuit current $I_{sc}^{(m)}$ induced at port P_m^{RX} of the receiving multiport is given by:

$$I_{sc}^{(m)} = \frac{-1}{Z_c V_{0,m}} \iint_{\Omega} T(\mathbf{r}_{TX,RX}, \hat{\mathbf{k}}) \mathbf{F}_{TX}(\hat{\mathbf{k}}) \cdot \mathbf{F}_{RX}^{(m)}(-\hat{\mathbf{k}}) d\hat{\mathbf{k}}. \quad (3)$$

It is very important to note that in (3), the radiation vector $\mathbf{F}_{TX}(\hat{\mathbf{k}})$ is the *total* radiation pattern of the transmitting multiport device, taken into account the excitation at each of its N ports. In contrast, $\mathbf{F}_{RX}^{(m)}(-\hat{\mathbf{k}})$ is the m -th *embedded* radiation pattern of the multiport. As this is quite different and more intricate than was the case for singleport devices [12], [13], the entire Sections II-D and II-E will be devoted to the computation of these radiation patterns.

C. Translation and rotation of the devices

The translation operation $T(\mathbf{r}_{TX,RX}, \hat{\mathbf{k}})$ enables the repositioning of devices relative to each other, by simply altering their phase centers and thus $\mathbf{r}_{TX,RX}$. For each repositioning, only the translation operator has to be recalculated. This does not entail a high computational cost, as compared to a new simulation (needed by other full-wave methods) or a new measurement. This will be demonstrated further in Section III.

Furthermore, rotation of devices, typically needed during EMI assessment, can be performed at a low cost as well, by making use of a spherical harmonics expansion. Consider a multiport device with an embedded radiation pattern $\mathbf{F}^{(m)}(\hat{\mathbf{k}})$ ($m = 1, \dots, M$). Then, the rotated embedded radiation pattern $\mathbf{F}^{(m),\mathcal{R}}(\hat{\mathbf{k}})$, for any arbitrary orientation of the multiport, is efficiently computed by going to the spherical harmonics domain, following the scheme shown in Fig. 2. The

Fig. 2: Rotation of $\mathbf{F}^{(m)}(\hat{\mathbf{k}})$ to $\mathbf{F}^{(m),\mathcal{R}}(\hat{\mathbf{k}})$ using the spherical harmonics domain.

transformation $\mathcal{R}$ mapping $\mathbf{F}^{(m)}(\hat{\mathbf{k}})$ onto its rotated version $\mathbf{F}^{(m),\mathcal{R}}(\hat{\mathbf{k}})$ in the spatial domain, is equivalent to a transformation $\mathcal{R}_{SH}$, converting a set of coefficients (A_{pq}, B_{pq}) into a new set of coefficients $(A_{pq}^{\mathcal{R}}, B_{pq}^{\mathcal{R}})$ in the spherical harmonics domain. The rotation $\mathcal{R}_{SH}$ itself is performed using Wigner-D matrices [16], [17]:

$$\mathcal{R}_{SH} \longleftrightarrow \begin{Bmatrix} A_{pq}^{\mathcal{R}} \\ B_{pq}^{\mathcal{R}} \end{Bmatrix} = \begin{Bmatrix} A_{pq} \\ B_{pq} \end{Bmatrix} \sum_{|r| \leq p} e^{-jq\gamma} d_{pq}^r(\beta) e^{-jr\alpha}, \quad (4)$$

with $d_{pq}^r(\beta)$ the Wigner small d-matrix, given by

$$d_{pq}^r(\beta) = (-1)^{r-q} \sqrt{(p+r)!(p-r)!(p+q)!(p-q)!} \cdot \sum_s (-1)^s \frac{(\cos \frac{\beta}{2})^{2(p-s)+q-r} (\sin \frac{\beta}{2})^{2s-q+r}}{(p+q-s)!s!(r-q+s)!(p-r-s)!}. \quad (5)$$

Here, the range of s is determined by the condition that all factorials are nonnegative, thus $s \in [\max(0, q-r), \min(p+q, p-r)]$. α , β and γ are the standard Euler angles that define the rotation using the $z-y-z$ convention in a right-handed frame (Fig. 3). To go from an

Fig. 3: Euler angles α , β and γ in the $z-y-z$ convention when rotating from coordinate system (x, y, z) to (x^R, y^R, z^R) .

original radiation pattern $\mathbf{F}^{(m)}(\hat{\mathbf{k}})$ in the coordinate system (x, y, z) to a rotated version $\mathbf{F}^{(m),R}(\hat{\mathbf{k}})$ in the coordinate system (x^R, y^R, z^R) , we first rotate by an angle α about the z -axis, then by an angle β about the new y' -axis and finally by an angle γ about the new z'' -axis. Starting from a proper reference orientation, any target orientation can be reached. For example, in the case of Fig. 3, the Euler angles (α, β, γ) are readily related to the inclination and azimuthal angles θ and ϕ , by choosing $\alpha = \phi$, $\beta = \theta$ and $\gamma = 0$.

To switch between the spatial domain and the spherical harmonics domain, and thus, to obtain the spherical harmonics coefficients A_{pq} and B_{pq} from a radiation pattern $\mathbf{F}^{(m)}(\hat{\mathbf{k}}) = F_\theta^{(m)}(\hat{\mathbf{k}})\hat{\boldsymbol{\theta}} + F_\phi^{(m)}(\hat{\mathbf{k}})\hat{\boldsymbol{\phi}}$ (in a spherical coordinate system) or vice versa, the transformations $\mathcal{F}$ and its inverse $\mathcal{F}^{-1}$ are used [18]:

$$\mathcal{F} \longleftrightarrow \begin{Bmatrix} A_{pq} \\ B_{pq} \end{Bmatrix} = \frac{-1}{p(p+1)} \int_0^{2\pi} \int_0^\pi \left[q \begin{Bmatrix} jF_\phi(\theta, \phi) \\ -F_\theta(\theta, \phi) \end{Bmatrix} Y_{pq}^*(\theta, \phi) + \sin \theta \begin{Bmatrix} -F_\theta(\theta, \phi) \\ jF_\phi(\theta, \phi) \end{Bmatrix} \frac{dY_{pq}^*(\theta, \phi)}{d\theta} \right] d\theta d\phi, \quad (6)$$

$$\mathcal{F}^{-1} \longleftrightarrow \begin{Bmatrix} F_\theta(\theta, \phi) \\ F_\phi(\theta, \phi) \end{Bmatrix} = \sum_{p=0}^P \sum_{|q| \leq p} \left[\begin{Bmatrix} A_{pq} \\ jB_{pq} \end{Bmatrix} \frac{dY_{pq}(\theta, \phi)}{d\theta} + \begin{Bmatrix} B_{pq} \\ jA_{pq} \end{Bmatrix} \frac{qY_{pq}(\theta, \phi)}{\sin \theta} \right]. \quad (7)$$

Here, P is a parameter that determines the accuracy and for practical purposes it can be chosen equal to L [19]. Furthermore, $Y_{pq}(\theta, \phi)$ are the orthonormalized scalar spherical harmonics [20]:

$$Y_{pq}(\theta, \phi) = \sqrt{\frac{(2p+1)(p-q)!}{4\pi(p+q)!}} P_p^q(\cos \theta) e^{jq\phi}, \quad (8)$$

where $P_p^q(\cdot)$ is the associated Legendre polynomial of degree p and order q .

As will be illustrated in Section III, leveraging (6) and (7) together with (4) allows for the efficient rotation $\mathcal{R}$ of multiport devices without the need for a completely new (expensive) simulation or measurement.

D. Circuit description

In Fig. 4 the equivalent circuit models of a multiport device in transmit mode (Fig. 4a) as well as in receive mode (Fig. 4b) are shown. The gray area represents the device itself, similar as in Fig. 1. In this paper, the equivalent Thévenin model is used in transmit mode, while the Norton equivalent is chosen in receive mode, but other choices could be made as well. Denote

$$\mathbf{V} = \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} \quad \text{and} \quad \mathbf{I} = \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} \quad (9)$$

the vectors of voltages V_n at port P_n and currents I_n flowing into port P_n ($n = 1, \dots, N$). It is seen from Fig. 4a that an N -port device in transmit mode is characterized by an N -port impedance matrix $\mathcal{Z}$:

$$\mathcal{Z} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix}, \quad (10)$$

where $\mathbf{V}$ and $\mathbf{I}$ are linked through $\mathcal{Z}$ as:

$$\mathbf{V} = \mathcal{Z} \cdot \mathbf{I}. \quad (11)$$

In receive mode, using the Norton equivalent formulation, the N -port device is characterized by the admittance matrix $\mathcal{Y}$, which is the inverse of the impedance matrix $\mathcal{Z}$, together with an extra term $\mathbf{I}_{sc}$ containing the induced short circuit currents:

$$\mathcal{Y} = \mathcal{Z}^{-1} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix}, \quad \mathbf{I}_{sc} = \begin{bmatrix} I_{sc}^{(1)} \\ \vdots \\ I_{sc}^{(N)} \end{bmatrix}, \quad (12)$$

which are linked through:

$$\mathbf{I} = \mathbf{I}_{sc} + \mathcal{Y} \cdot \mathbf{V}. \quad (13)$$

The elements of the device's impedance matrix $\mathcal{Z}$ are found by exciting each port successively by a 1 A current source,

(a) Equivalent Thévenin model of an N -port in transmit mode. (b) Equivalent Norton model of an N -port in receive mode.Fig. 4: Equivalent circuit models of an N -port in transmit and receive mode.

leaving all other ports open, and measuring the voltage at all ports. In this manner Z_{nm} represents the voltage induced in the open-circuited port P_n , due to a 1 A current excitation at port P_m ($n, m = 1, \dots, N$).

A radiation vector $\mathbf{F}^{(n)}(\hat{\mathbf{k}})$ is obtained by exciting port P_n by a 1 V voltage source and short-circuiting the other ports, as shown in Fig. 5. This radiation pattern is called the *embedded element pattern* of port P_n of the multiport device, and the coupling with all other ports is correctly taken into account. Note that, when a transmitting N -port is excited simultaneously at several of its N ports, yielding several nonzero embedded radiation patterns $\mathbf{F}^{(n)}(\hat{\mathbf{k}})$, the influence on a receiving M -port, and in particular its induced short-circuit current at port m , is given by (3), where the *total* element pattern $\mathbf{F}_{\text{TX}}(\hat{\mathbf{k}})$ has to be used, as further explained in the next section.

E. Active element pattern

To determine the radiation patterns of a particular port of a multiport device, one has to carefully deal with the terminations at the other ports. Whereas $\mathbf{F}^{(n)}(\hat{\mathbf{k}})$ at port P_n is defined when all other ports are short-circuited, sometimes other formulations are more suitable. For example, to measure radiation patterns over a broad frequency range, accurate 50Ω terminations are more readily available than short-circuits. It is thus useful to define the so-called *active element pattern*, which represents the radiation pattern $\mathbf{F}^{(n),a}(\hat{\mathbf{k}})$ for excitation of port n with a Thévenin generator, while all other ports are terminated by a nonzero (typically 50Ω) load, as is seen in Fig. 6. If only the active radiation patterns are available, a

conversion to the embedded patterns is found from Figs. 5 and 6 by using the superposition principle:

$$\mathbf{F}^{(n),a}(\hat{\mathbf{k}}) = \frac{V_1^{(n),a}}{V_1^{(1)}} \mathbf{F}^{(1)}(\hat{\mathbf{k}}) + \dots + \frac{V_N^{(n),a}}{V_N^{(N)}} \mathbf{F}^{(N)}(\hat{\mathbf{k}}), \quad (14)$$

where

$$\begin{cases} V_m^{(n),a} = V_{g,n}^a \delta_{nm} - Z_{g,n} I_m^{(n),a} \\ V_n^{(n)} = V_{g,n} \end{cases} \quad (15)$$

and with δ_{nm} the Kronecker delta. Substitution of (15) into (14) yields:

$$\mathbf{F}^{(n),a}(\hat{\mathbf{k}}) = \sum_{m=1}^N \frac{V_{g,n}^a \delta_{nm} - Z_{g,n} I_m^{(n),a}}{V_{g,n}} \mathbf{F}_m^{(n)}(\hat{\mathbf{k}}). \quad (16)$$

In vector notation, we express the relation between the active radiation patterns $\mathbf{F}^{(\cdot),a}(\hat{\mathbf{k}})$ and the embedded radiation patterns $\mathbf{F}^{(\cdot)}(\hat{\mathbf{k}})$ as

$$\begin{bmatrix} \mathbf{F}^{(1),a}(\hat{\mathbf{k}}) \\ \vdots \\ \mathbf{F}^{(N),a}(\hat{\mathbf{k}}) \end{bmatrix} = \text{diag} \left(\frac{1}{V_{g,i}} \right) \cdot \left(\text{diag} (V_{g,i}^a) - \text{diag} (Z_{g,i}) \cdot \mathcal{I}^a \right) \begin{bmatrix} \mathbf{F}^{(1)}(\hat{\mathbf{k}}) \\ \vdots \\ \mathbf{F}^{(N)}(\hat{\mathbf{k}}) \end{bmatrix}, \quad (17)$$

Fig. 5: Generation of the n -th *embedded* radiation vector $\mathbf{F}^{(n)}(\hat{\mathbf{k}})$ of an N -port ($n = 1, \dots, N$).

Fig. 6: Generation of the n -th *active* radiation vector $\mathbf{F}^{(n),a}(\hat{\mathbf{k}})$ of an N -port ($n = 1, \dots, N$).

where $\text{diag}(\cdot)$ represents a diagonal matrix. $\mathcal{I}^a$ is the active current matrix given by

$$\mathcal{I}^a = \begin{bmatrix} I_1^{(1),a} & \dots & I_N^{(1),a} \\ \vdots & \ddots & \vdots \\ I_1^{(N),a} & \dots & I_N^{(N),a} \end{bmatrix}, \quad (18)$$

and governed by Kirchoff's law:

$$\text{diag} \left(V_{g,i}^a \right)_{i=1}^N = \left(\mathcal{Z} + \text{diag} \left(Z_{g,i} \right)_{i=1}^N \right) \cdot \mathcal{I}^a. \quad (19)$$

Substitution of (19) into (17) yields the embedded radiation vectors $\mathbf{F}^{(\cdot)}(\hat{\mathbf{k}})$ as a function of the active radiation vectors

$\mathbf{F}^{(\cdot),a}(\hat{\mathbf{k}})$:

$$\begin{bmatrix} \mathbf{F}^{(1)}(\hat{\mathbf{k}}) \\ \vdots \\ \mathbf{F}^{(N)}(\hat{\mathbf{k}}) \end{bmatrix} = \text{diag}_{i=1}^N \left(\frac{V_{g,i}^a}{V_{g,i}^a} \right) \cdot \left[\mathcal{E} - \left(\text{diag}_{i=1}^N \left(\frac{1}{Z_{g,i}} \right) \cdot \mathcal{Z} + \mathcal{E} \right)^{-1} \right]^{-1} \begin{bmatrix} \mathbf{F}^{(1),a}(\hat{\mathbf{k}}) \\ \vdots \\ \mathbf{F}^{(N),a}(\hat{\mathbf{k}}) \end{bmatrix}, \quad (20)$$

where $\mathcal{E}$ is the $N \times N$ identity matrix.

The *total* radiation $\mathbf{F}_{\text{TX}}(\hat{\mathbf{k}})$, used in (3), emitted by a transmitting device that is excited by multiple Thévenin generators at several or at all of its N ports, is now easily found by adding up the pertinent active radiation patterns.

For completeness, we mention that (the crosstalk between the ports of) a multiport device is often more conveniently described by a scattering matrix, again typically with respect to 50Ω reference impedances. Nonetheless, the multiport's

scattering matrix $\mathcal{S}$ and its impedance matrix $\mathcal{Z}$ (11) can be converted into one another as follows [21]:

$$\mathcal{S} = (\mathcal{Z} + Z_0 \mathcal{E})^{-1} (\mathcal{Z} - Z_0 \mathcal{E}) \quad (21)$$

$$\mathcal{Z} = Z_0 (\mathcal{E} + \mathcal{S}) (\mathcal{E} - \mathcal{S})^{-1}, \quad (22)$$

where Z_0 is the reference impedance (e.g. 50Ω).

III. VALIDATION AND APPLICATION EXAMPLES

A. Validation example

To validate the proposed method, a numerical example is proposed where the transmitter and the receiver are both antenna arrays, each consisting of two half-wavelength dipoles spaced $\lambda/10$ apart (wavelength $\lambda = 1$ m). The distance between the antenna arrays is $|\mathbf{r}_{\text{TX,RX}}| = 3\lambda$ and the antennas' wire thickness is chosen to be $10^{-4}\lambda$. The receiving antenna array is rotated over Euler angles $\beta = \frac{2}{3}\alpha = \frac{2}{3}\gamma$, with β varying from 0 to 2π , while the transmitting array remains fixed. In Fig. 7, a snapshot of this setup for $\beta = 3\pi/4$ is shown.

The short-circuit currents $I_{sc}^{(1)}$ and $I_{sc}^{(2)}$, calculated with our proposed formalism, are compared to a reference solution. This reference solution is obtained from a Method of Moments (MoM) solver for arbitrary thin wires [22]. In the MoM simulation, one half-wavelength dipole is modeled using five segments and the short-circuit currents are calculated by directly inverting the full MoM matrix equation. The dipoles in the transmitting array are excited with $V_{g,1} = V_{g,2} = 1$ V between their terminals, and $Z_{g,1} = Z_{g,2} = 0 \Omega$.

Fig. 7: Configuration of the validation example where the receiving array (right) is rotated over Euler angles $\beta = \frac{2}{3}\alpha = \frac{2}{3}\gamma$ with $\beta = 3\pi/4$ w.r.t. the transmitting array (left).

Contrary to the MoM, our method does not require the computation and inversion of a complete matrix equation for every new relative orientation between transmitting and receiving array. The novel method only requires a single computation of the embedded radiation patterns $\mathbf{F}^{(1)}(\hat{\mathbf{k}})$ and $\mathbf{F}^{(2)}(\hat{\mathbf{k}})$ of the dipole array. Here, we obtained these radiation patterns using a MoM simulation for a single dipole array. Once the radiation patterns are available, they are decomposed into spherical harmonics. After this initial setup, simulations

can be performed for different relative orientations or positions without a large computational cost as compared to a full MoM simulation.

The results obtained from the MoM simulation and from our novel multiport method are compared in Fig. 8 for 361 samples of β to obtain a one-degree resolution. The short-circuit currents $I_{sc}^{(1)}$ and $I_{sc}^{(2)}$ are equal due to the symmetry of the problem geometry. For this example we have used $P = L = 5$ in the calculation of the translation operator and the spherical harmonics. An excellent agreement between the novel method and the MoM simulation is observed. The MoM simulation for all 361 angles took 624 s, whereas the novel method only took 1.3 s of setup time plus 14.9 s of calculation time for all 361 rotated configurations. In total, this corresponds to a speed-up factor of about 38. All simulations have been carried out on an Intel® Core™ i7-2600 processor running at 3.40 GHz and with 16 GB of memory.

B. Application example

In this application example, the two interacting multiport devices in the system are a transmitting Substrate Integrated Waveguide (SIW) three-element antenna array (culprit) and a receiving two-port slotted microstrip line (victim). The three-element antenna array was designed to set up a stable, high data rate ultra-short-range 3×3 multiple-input multiple-output (MIMO) wireless communication link between an access point integrated inside or underneath the worktop of a desk, and a mobile user (MU) positioned on top of that worktop [23]. Such an approach could be adopted in a meeting room, to provide an ultra-high data-rate wireless connection at each seat, replacing wired docking stations. The three-element antenna array consists of three identical ultra-wideband antenna elements, arranged such that the array exhibits threefold rotational symmetry. This specific geometry exploits both spatial and

Fig. 8: Amplitude of the short-circuit currents $I_{sc}^{(1)} = I_{sc}^{(2)}$ at the ports of the receiving dipole, induced by a transmitting dipole array, rotated over Euler angles $\beta (= \frac{2}{3}\alpha = \frac{2}{3}\gamma)$ (see also Fig. 7).

polarization diversity to provide a high multiplexing gain in proximity of the designated working area of the MU.

The microstrip line in the system represents a part of a PCB inside the MU's device (e.g. a laptop) that is put on the table above the three-element antenna. In this manner, a real-life EMI situation, where circuitry inside the laptop is affected by the wireless data link, is considered. Using our novel method, potential EMC problems can now be detected very efficiently during design. The microstrip has dimensions of $17.5 \text{ mm} \times 1.6 \text{ mm}$ and is placed on a substrate ($\epsilon_r = 3.66$ and $\tan \delta = 0.004$) with 1.6 mm thickness. There is a slot present in the ground plane that has the same dimensions as the microstrip, but it is perpendicular to it. The microstrip is directed toward, and the slot is directed away from the three-element array as shown in Fig. 9.

In a real-life scenario, the laptop (and thus the microstrip) may assume several positions w.r.t. the three-element array. This is here reproduced in simulation by rotation of the microstrip about the z -axis or thus by varying α in (4) (see also Figs. 3 and 9). In this example 361 samples for α between 0 to 2π are chosen, to obtain a one-degree resolution. The interaction between the three-element array and the microstrip for each of these samples is calculated for a frequency of $f = 5 \text{ GHz}$ using the formalism explained in this paper, and compared with full-wave simulations in Computer Simulation Technology Microwave Studio (CST MWS). The radiation patterns themselves, needed in our approach, are also obtained using CST MWS to make a fair comparison of the simulation times. The simulated radiation patterns are the active radiation patterns and for the receiving device (microstrip), they are converted using (20) to obtain the embedded radiation patterns required in (3). Further, the impedance matrices $\mathcal{Z}$ of the two multiport devices are obtained via (22) after an S -parameter simulation, using again CST MWS. The two multiport devices are placed 100 mm apart, which is less than 2λ , and the number of multipoles is chosen $P = L = 8$.

Fig. 9: Three-element SIW antenna array interacting with a two-port slotted microstrip line for $\alpha = 0$.

With our novel method, we can now easily compute the interaction between the two devices. In order to compare it to the results of the CST MWS full-wave simulations, we convert our results (i.e. the short-circuit current) to S -parameters as well, using standard circuit theory techniques [21]. In particular, a 5×5 S -parameter matrix is obtained, describing the interaction between all 5 ports in the system. In Fig. 10, a good agreement for $|S_{14}|$, $|S_{24}|$ and $|S_{34}|$ between our method and the full-wave simulations is shown for all angles α . The

Fig. 10: Scattering parameters of the link between the three-element array and the two-port microstrip (see Fig. 9 for port definitions). Lines: novel method; Markers: CST MWS.

port numbering is indicated on Fig. 9. Note that the other S -parameters may be derived from these results based on symmetry. The total simulation time in our novel method consists of about 30 min setup time (calculation of the radiation patterns and scattering parameters) and then it takes on average 0.21 s per sample of α . Calculating the scattering matrix for one rotation sample in CST MWS takes about 40 min implying a speed up factor compared to the novel method of about 460. This shows the efficiency of our method.

Also a displacement relative to the three-element array can be performed by varying $\mathbf{r}_{\text{TX,RX}}$ in (2). Here, first we align the microstrip line with the first antenna element P_1 ($\alpha = \pi/2$) and then the position of the microstrip is varied over the y -axis. This is done for Δy ranging from -100 mm to 100 mm , mimicking a realistic displacement of a user's laptop. The result for 1000 sample points of $|S_{15}|$ is shown in Fig. 11. A good agreement between full-wave simulations and our approach is observed. For this simulation, no new setup time was needed and calculating the 5×5 scattering matrix for one sample takes on average 2 ms, which shows that translations can be performed very efficiently.

IV. CONCLUSIONS

In this paper, a new approach was presented that enables the efficient and accurate modeling of the interaction between multiport devices. The method solely relies on the knowledge of the radiation patterns at each port of the devices to model

Fig. 11: $|S_{15}|$ when the microstrip line is translated on the y -axis from -100 mm to 100 mm for $\alpha = \pi/2$ (see Fig. 9 for port definitions). Line: novel method; Marker: CST MWS.

their EMC behavior when positioned in each other's radiative near-field (Fresnel region) or far-field (Fraunhofer region). Whereas in conventional simulation tools a repositioning or rotation of the devices would require a completely new (expensive) simulation, the advocated approach allows translating the devices without a large computational cost. Also rotations are performed very efficiently in the spherical harmonics domain. It is important to stress that, in contrast to previous work dealing with *singleport* devices (i.e. single antennas), the novel method can account for intricate *multiport* devices. However, thereto, the devices' *embedded* and *active* radiation patterns need to be constructed and exploited appropriately, as was detailed in this paper. The novel method was thoroughly validated by comparison with a full-wave MoM solver, demonstrating its accuracy and efficiency. Moreover, a real-life scenario of an SIW three-element antenna array, being a subsystem of a MIMO transmitting device, disturbing a slotted microstrip line, being a part of a MU's laptop, was examined, as such illustrating the appositeness of the technique for EMI assessment early in the design cycle of new products.

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