PREDICTING RESIDUAL STRESS IN WELDS USING EMPIRICAL AND RECONSTRUCTIVE METHODS

J. Ni and M. Abdel Wahab
Ghent University, Laboratory Soete, Belgium

Abstract: The prediction of residual stress in the welding process has been improved during the last decade. Two groups of methods can be roughly classified. The first group derives the stress field by simply analysing and fitting the measured values to achieve a whole description of stress in structural components. In the second group, the residual stress field is reconstructed based on the concept of eigenstrain and mechanics theories. It provides an improved treatment of experimental results and can be regarded as an improved version of the first. This paper reviews theories, formulations and corresponding results of the two methods.

Keywords: Residual stress, Welding, Reconstructive methods

1 INTRODUCTION

Residual stress is a typical phenomenon that occurs during manufacturing processes, such as quenching, forging, welding, etc. It occurs as a result of inhomogeneous dimensional changes, heat treatment or/and phase transformation, and is defined as a self-equilibrating internal stress existing in a body without any external forces or constraints [1]. Residual stress has various impacts on the behaviour of welded structures. The yield stress of components may be changed due to the existence of residual stress. Furthermore, the sensitivity to corrosive environment may be affected, cracks may form without external load, and fatigue life and elastic stability may be influenced.

In this review, only macro residual stress is considered. The term ‘macro’ refers to the residual stress as an averaged value over a long distance of the order of grains or precipitates [2]. Residual stress at the level of material imperfections (crystal inhomogeneity, misalignment of lattice plane, interstitial defects, etc.) is not taken into account due to the fact that most approaches presented here treat welds as continuum. Among numerous methods, the one that simply fits stress data is the most practical. This method does not require much input and can be applied to many situations. There also exist several handbooks [3-5] providing the formula to predict residual stress based on polynomial series. But sometimes it is supplemented by simulation results acting as upper bound for safety reason [6]. The first approach is named empirical methods in the paper. The second approach is referred to as reconstructive method, in which the experimental data used to reconstruct the stress/strain field are treated based on theories of mechanics instead of curves fitting. The main idea of the second approach lies in the concept of eigenstrain, which is the only factor causing residual stress in simulation models. Once a suitable eigenstrain field is found, both the strain and stress states can be fully characterized. The third section of this review focuses on this method.

2 EMPIRICAL METHOD

The empirical method predicts the stress field from the experimental results. By analysing the influencing factors, the distribution of residual stress can be obtained by simply fitting the results using empirical equations. Mohr et al. [7] investigated the stress distribution in girth butt welding of pressure vessels and piping as shown in Fig. 1. Two parameters, the thickness of work pieces and heat input were considered to be the main factors affecting the results. It was found that the majority of axial residual stresses fell between 20% and 80% of yield stress. When the heat input is smaller than 20 kJ/in per pass, the axial residual stress at internal surface mainly depends on the wall thickness.
In general, Mohr et al. [7] proposed an improved empirical treatment for estimating residual stresses in terms of heat input and wall thickness. Since the heat input is proportional to the number of weld passes and wall thickness, a further step was made to express residual stresses $\sigma_{\text{residual}}$ at internal surface in terms of the number $n$ weld passes and yield stress $\sigma_{ys}$ of base material as follows:

If $n \leq 5$

$$\sigma_{\text{residual}} = \sigma_{ys}$$  \hspace{1cm} (1)

If $6 < n \leq 20$

$$\sigma_{\text{residual}} = \sigma_{ys} \left( 1.33 - \frac{n}{15} \right)$$  \hspace{1cm} (2)

If $n > 20$

$$\sigma_{\text{residual}} = 0.0$$  \hspace{1cm} (3)

The correlation between the number of weld passes and the longitudinal residual stress at the weld centreline from experiments, as well as, the fitted equations are shown in Fig. 2. For safety reason, the results from empirical equations should be checked within the bound of available data.

A similar approach suggested a polynomial curve fit of residual stress measurements on selected structural components [9]. Dong et al. [10] calculated the longitudinal and transverse residual stress of circumferential
girth welds as in Fig. 1 empirically. The results were compared to finite element solution and experimental data. In his approach the longitudinal residual stress $\sigma'$ is prescribed as a linear function of position $x$ through the thickness of work piece as:

$$\sigma' = \sigma_y + \left(\sigma_o' - \sigma_i'\right) \frac{x}{t}$$

(4)

where $x$ is measured from inner surface to the position of interest and $t$ is the thickness of pipe as shown in Fig. 1. $\sigma_o'$ and $\sigma_i'$ are residual stress at the outer and inner surface of pipes respectively, and are given as:

$$\sigma_o' = \sigma_y$$

(5)

For $t \leq 15\text{mm}$

$$\sigma_i' = \sigma_y$$

(6)

For $15\text{mm} < t \leq 85\text{mm}$

$$\sigma_i' = \sigma_y [1.0 - 0.0143(t - 15)]$$

(7)

For $t > 85\text{mm}$

$$\sigma_i' = 0$$

(8)

$\sigma_y$ takes the larger value of yield strengths from base $\sigma_{yB}$ and weld material $\sigma_{yw}$.

On the other hand, the transverse residual stress $\sigma'$, depends not only on the type of steels (ferritic or austenitic steel), but also on the magnitude of heat input. The stress distribution for ferritic steels are given as:

For $\frac{q}{t} \leq 50\text{J/mm}^2$ (low heat input)

$$\sigma' = \sigma_y \left[1 - 6.80 \frac{x}{t} + 24.30 \frac{x^2}{t^2} - 28.68 \frac{x^3}{t^3} + 11.18 \frac{x^4}{t^4}\right]$$

(9)

For $50 < \frac{q}{t} \leq 120\text{J/mm}^2$ (medium heat input)

$$\sigma' = \sigma_y \left[1 - 4.43 \frac{x}{t} + 13.53 \frac{x^2}{t^2} - 16.93 \frac{x^3}{t^3} + 7.03 \frac{x^4}{t^4}\right]$$

(10)

For $\frac{q}{t} > 120\text{J/mm}^2$ (high heat input)

$$\sigma' = \sigma_y \left[1 - 0.22 \frac{x}{t} - 3.06 \frac{x^2}{t^2} + 1.88 \frac{x^3}{t^3}\right]$$

(11)

where $x$ and $t$ are defined as the same in the case of longitudinal residual stress while $\sigma_y$ takes the minor value of $\sigma_{yB}$ and $\sigma_{yw}$. $\dot{q}$ is the linear heat input and expressed as a function of welding current $I$, voltage $V$ and travel speed $u$:

$$\dot{q} = \frac{I \cdot V}{u}$$

(12)
The stress distribution at high heat input for autenitic steels holds the same as ferritic steels. The stresses in medium and low heat input have the same expression as Equation (9). Comparison with experimental data showed that the results from empirical equations were relatively conservative. Improvement can be done in several aspects. For example, the heat input can be treated as continuous rather than discretely as low, medium and high. Important geometrical parameters, such as pipe radius can be considered in expressions as well.

3 RECONSTRUCTIVE METHOD

An improvement is made with respect to the treatment of measured and simulated results using a reconstructive method. Unlike the empirical method, this method solves residual stress field based on mechanics theories. By implementing theories of shells or micromechanics, the problem is simply reduced to figuring out a stress field inherited with non-elastic strains. The non-elastic strain refers to the total plastic strain due to phase transformation, thermal expansion, geometrical mismatch, etc. In many researches, non-elastic strain is also named eigenstrain [11] or inherent strain [12]. It should be noted that the eigenstrain accounts for all permanent strains that give rise to residual stresses, but not a simply addition of the various non-linear strains. A general decomposition of the total strain \( \varepsilon_{\text{total}} \) can be decomposed as:

\[
\varepsilon_{\text{total}} = \varepsilon^e + \varepsilon^* 
\]

where \( \varepsilon^e \) is elastic strain and \( \varepsilon^* \) is eigenstrain.

Masubuchi [13] considered four steps in this method, namely a) heat flow analysis, b) transient thermal stresses analysis, c) determination of non-elastic strain and total strain, and finally d) residual stresses analysis. Among those steps the third is the most important for the analysis of the distribution and magnitude of residual stress. Mura [11] developed a mathematical framework determining residual stresses considered in an infinite 3-D body. In his approach, the phases were supposed to transform within a domain of sphere or ellipsoid, and the resultant volume change was assumed to be uniform within that domain. Then, corresponding eigenstrain was uniformly distributed as well and constant throughout the domain according to Eshelby [14]. At a long distance outside the domain it decreased to zero.

With the knowledge of eigenstrain, the corresponding elastic and total strain can be determined. If the whole eigenstrain field is known, the residual stress and strain field can be deduced directly, which is known as the direct problem of eigenstrain theory [15]. The fundamental equations in the case of infinitely extended material are given as [11]:

\[
C_{ijkl} u_{k,j} = C_{ijkl} \varepsilon^*_{kl,i} 
\]

\[
\varepsilon^*_{ij}(x) = \tilde{\varepsilon}^*_{ij}(\xi) \exp (i \xi \cdot x) 
\]

where \( C_{ijkl} \) is elastic moduli and \( u \) the displacement. The eigenstrain \( \varepsilon^* \) is supposed to be in the form of a single wave of amplitude \( \tilde{\varepsilon}^* \) and \( \xi \) is the wave vector corresponding to the given period of distribution (15). However, the direct solution of eigenstrain theory is not practical when applied to other geometries like plane-bounded semi-infinite space or when the information of eigenstrain is incomplete.

To avoid the limitation of direction solution, an inverse method was developed. The basis of this method lies in the quantitative comparison between experimental data and predicted values of reconstructed strain field at the same location, which is called the inverse problem of eigenstrain theory [16]. In the inverse method, the unknown eigenstrain distribution is expressed as a summation of series of basic functions with coefficients to be determined. Those unknown coefficients are deduced by the least square method. Ueda et al [17] proposed the relations between residual \( \sigma_{\text{residual}} \), elastic strains \( \varepsilon^e \) and eigenstrain \( \varepsilon^* \) as:

\[
\varepsilon = H^* \varepsilon^* 
\]

\[
\sigma_{\text{residual}} = D \varepsilon 
\]

where \( H^* \) is elastic response matrix and \( D \) the stress-strain matrix.

The choice of basic function for eigenstrain varies for different researchers. Cao et al. [18] and Qian et al. [19] adopted a series of smooth functions, such as two-term or trigonometric polynomials. Kartal et al. [20]
used the polynomials of Legendre and Korsunsky et al. [21] and the Chebyshev polynomials to present the nature of eigenstrain distribution. A general expression of eigenstrain is:

$$\varepsilon^*_{ij} = \sum_{m=1}^{M} \alpha^m_{ij} N_m$$

where $\alpha^m_{ij}$ is the unknown coefficients, $M$ the number of terms of polynomial and $N_m$ the chosen basic functions.

Jun et al. [16] summarized the framework of reconstructing eigenstrain field and solving the inverse problem by finite element method (FEM). First, the FE model is set up according to the sample geometry, in which only the elastic material properties, such as Young’s modulus and Poisson’s ratio are needed. For regions where eigenstrains are possibly localized, fine mesh is recommended. Then, arbitrary eigenstrains are introduced in FE model by arbitrarily prescribing coefficients of thermal expansion. The eigenstrain is treated comparable to thermal strain since the modelling does not require explicit knowledge about its origin. After the completion of simulation, the elastic strains of the same experimental area are extracted for optimizing the coefficients. The simulations and optimizations are conducted until a good agreement is achieved. The final FE model provides extensive information such as residual stresses, strains, displacement, etc. Fig. 3 shows the residual strains predicted by this procedure [16].

![Fig. 3 Contour plots of (left) localised eigenstrain input at different positions and (right) the corresponding residual strain distributions [16]](image)

Besides FEM, boundary element method (BEM) can be used to calculate residual stresses as well. Qian et al. [22] approximated the eigenstrain field as a series of two-term polynomials. Based on the work by Cheng et al. [23], the corresponding domain integral is then transformed into boundary integral. Two examples are presented to demonstrate the effectiveness of this approach.

Instead of measuring and comparing strains directly, the deformation after welding can also be used to determine residual stresses. Wang et al. [24] estimated eigenstrains by analysing the deformation and the tendon force and applied them in an elastic FE analysis.

Similarly, a shell theory based scheme is developed by Song et al. [25, 26] to estimate the residual stress distribution. After analysing the distribution features of numerous residual stress solutions, Dong et al. [8] identified that the residual stress $\sigma$ at weld toe can be decomposed into three parts, through-thickness membrane $\sigma_m$, bending $\sigma_b$ and self-equilibrating $\sigma_{se}$ stresses:

$$\frac{\sigma(\xi)}{S_y} = \sigma_m + \sigma_b \xi + \sigma_{se}(\xi)$$

$$\sigma_{se}(\xi) = e \xi^2 + f \xi^3 + g \xi^4 + h \xi^5$$

$$\sigma_m = a_m \cdot \ln\left(\frac{r}{t}\right) + b_m$$

$$\sigma_b = a_b \cdot \ln\left(\frac{r}{t}\right) + b_b$$
where $\xi = 2 \frac{x}{t} - 1$ with $x$, $t$ and $r$ identically defined in Chapter 2 and $S_v$ is the material yield strength of base material. Coefficients $a_m$, $b_m$, $b_b$, $e$, $f$, $g$ and $h$ are determined using empirical equations of heat input and specimen geometry. With the knowledge of stresses at weld toe, the distribution of residual stresses in other area can be calculated analytically through classical shell theory. Yang et al. [27] adopted shell theory as well but considered the material as ideal plastic.

4 CONCLUSION

As far as concerned, the empirical method adopts a set of easy and practical equations to predict the residual stress profiles in welds. However, the case study [10] shows that it provides relatively conservative results in engineering applications. The treatment of heat input and component geometries can be improved to have a more realistic results. The reconstructive method is developed to treat the measurements in a more decent way. The framework mentioned in section 3 can be flexibly applied to any other complex structures. To achieve a stable solution of the stress/strain field, the choice of basic functions of eigenstrain and measurement points are of considerable importance.

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6 REFERENCES


