Fast evaluation of sound diffraction over complex and multiple rigid obstacles

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Summary
Accurate and efficient calculation of diffraction over rigid obstacles is needed for sound propagating over city canyons and for applications to urban noise mapping. Uniform diffraction theory offers very accurate solutions. However, these solutions depend on complicated input parameters and usually take more CPU time than engineering approximations. In this paper, simplified diffraction functions to approximate single, double and multiple diffraction are presented. Their formulation is based on a simplification of the Fresnel integral and their accuracies depend on the relative locations of the source, obstacle and receiver. Compared with the non-simplified diffraction model, most of the receiver positions can reach an acceptable accuracy except for positions close to the boundary line where a 3 dB error is expected.

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1. Introduction

Sound propagating over obstacles dominates the precision of sound pressure level predictions in the shielded areas. Methods based on the diffraction theories, such as those presented in [1, 2, 3], were shown to be very accurate [4, 5]. Relevant parameters are the diffraction path, diffraction angle and the wedge angle, often requiring complicated input parameters and long CPU time. Therefore, they are not easily implemented in urban noise mapping methods. The insertion loss in some engineering models, such as ISO9613-2 [6] and CNOSSOS-EU [7] models, are only relevant to the barrier width and difference between the direct path and diffracted path. These models are easier to implement, however, the accuracies are not always satisfied. In this article, a simple but still accurate method to calculate the sound diffraction over a thin wedge, a thick barrier and even complicated shapes will be introduced and validated. This method not only can be used to calculate the insertion loss of an obstacle but also the sound pressure behind the obstacle.

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The current model is based on the approximation of the Fresnel integrals. The Fresnel integrals can be considerably simplified if the input arguments are sufficiently large (>1). However, in urban configurations, these inputs are frequently close to zero [5], which leads to strong singularities. In this article, a different simplification of the Fresnel integral will be proposed, which is suitable for propagation in an urban area.

2. Single diffraction over a rigid wedge

According to [1, 8], the diffracted sound pressure is a product of a source term, a term related to propagation distance and a diffraction term. For a point source diffracted by a rigid wedge, as shown in figure 1(a), the diffracted sound pressure reads:

$$p_{diffr} = S_0 \frac{e^{ikL}}{L} D_1, \tag{1}$$

with

$$D_1 = \frac{e^{i\pi/4}}{\sqrt{2}} \left[A_D(X_+) + A_D(X_-)\right] \tag{2}$$

D_1 is the diffraction function, where $A_D = sign(X)[f(|X|) - ig(|X|)]$. $f(|X|)$ and $g(|X|)$ are functions of the Fresnel integral $C(X)$ and $S(X)$.

$$C(X) = \int_0^X \cos\left(\frac{1}{2}\pi t^2\right) dt,$$

$$S(X) = \int_0^X \sin\left(\frac{1}{2}\pi t^2\right) dt,$$

The $C(X)$ and $S(X)$ are approximated as the following simple form:

$$C(X) \approx 0.5 + \frac{0.37}{0.37+X} \sin\left(\frac{\pi}{2}X^2\right) \tag{3}$$

$$S(X) \approx 0.5 - \frac{0.37}{0.37+X} \cos\left(\frac{\pi}{2}X^2\right) \tag{4}$$

Substituting equations 3 and 4 into equation 2, the diffraction function for a rigid wedge is then obtained:

$$D_1 = \frac{e^{i\pi/4}}{\sqrt{2}} \left(\frac{0.37}{0.37+X_+} + \frac{0.37}{0.37+X_-}\right) \tag{5}$$

D_1 is determined by $r_s, \theta_s, r_r, \theta_r$ as shown in figure 1(a). The input argument X_+ and X_- are the same as in Ref. [1]. The corresponding insertion loss of a single wedge is:

$$IL_1 = -10\log_{10}\Big[\frac{R^2}{L^2}\Big(\frac{0.37}{0.37+X_+} + \frac{0.37}{0.37+X_-}\Big)^2\Big] \tag{6}$$

with R is length from the source to the receiver and L the length of the diffraction path.

3. Double diffraction over a wide rigid barrier

A double diffraction, as shown in figure 1(b), can be approximated as a single diffraction wave produced from edge 1 and then subsequently diffracted by edge 2 to reach the receiver. Therefore, the double diffraction function is the product of two single diffraction:

$$\begin{aligned} D &= D_1 D_2 \\ &= i/4 \left(\frac{0.37}{0.37+BX_{S+}} + \frac{0.37}{0.37+BX_{S-}}\right) \\ &\quad \left(\frac{0.37}{0.37+X_{R+}} + \frac{0.37}{0.37+X_{R-}}\right) \end{aligned} \tag{7}$$

For a double diffraction over a rectangular barrier, it could be easily proved that $X_{S+} = X_{S-}$ and $X_{R+} = X_{R-}$. Therefore, equation 7 can be simplified as:

$$D = i\left(\frac{0.37}{0.37+BX_{S+}}\frac{0.37}{0.37+X_{R+}}\right) \tag{8}$$

The input arguments X_{S+}, X_{R+} and B are the same as in Ref. [1].

The corresponding insertion loss of a rigid rectangular barrier is then:

$$IL_2 = -10\log_{10}\Big[\frac{R^2}{L^2}\Big(\frac{0.37}{0.37+BX_{S+}}\Big)^2 \Big(\frac{0.37}{0.37+X_{R+}}\Big)^2\Big] \tag{9}$$

4. Multiple diffraction over complex obstacles

Similar to the generalization from single diffraction to double diffraction, the multiple diffraction term can be considered as a relay from the previous diffraction edges as shown in figure 1 (c,d). The diffracted sound pressure is recursively calculated by the previous diffraction. Therefore, the $(n-1)^{th}$ diffracted sound pressure by path $S1\cdots n$ is:

$$p_{n-1}^{S1\cdots n} = p_1^{S12}\frac{L_1}{L_{n-1}}D_2 \ldots D_{n-1}e^{ik(L_n-L_1)} \tag{10}$$

Substituting equation (1) to equation (10), form for the sound pressure at the n^{th} diffraction point or a receiver point after n-1 diffraction then reads:

$$p_{n-1}^{S1\cdots n} = \left(\frac{1}{2}\right)^C S_0 \frac{e^{ikL_{n-1}}}{L_{n-1}} \prod_{l=1}^{n-1} D_l \quad n = 2, 3, \cdots \tag{11}$$

Equation 11 is similar as equation 1 and it equals the product of the divergence from the source and its diffraction function, where the diffraction function D_l is:

$$D_l = \frac{e^{i\pi/4}}{\sqrt{2}} \left(\frac{0.37}{0.37+B_l X_{l+}} + \frac{0.37}{0.37+B_l X_{l-}}\right) \tag{12}$$

where, $X_{l+} = \gamma_l M_\nu(\theta_{l+})$. $X_{l-} = \gamma_l M_\nu(\theta_{l-})$. $\theta_{l+} = \theta_{s,l} + \theta_{r,l}$. $\theta_{r,l}$ is the angle from the right diffraction edge and the connecting line between the diffraction edge to the "receiver" and $\theta_{s,l}$ is the angle from the right diffraction edge to the connecting line between the diffraction point to the "source". For the demonstration of these above inputs, figure 1 (c) and (d) show some examples. The parameters B_l, γ_l and $M_{\nu l}$ are:

$$B_l = \sqrt{\frac{W_{l,l+1}(r_s + \sum_{j=1}^{n-1} W_{j,j+1} + r_r)}{(r_s + \sum_{j=1}^{l} W_{j,j+1})(r_r + \sum_{j=l}^{n-1} W_{j,j+1})}} \tag{13}$$

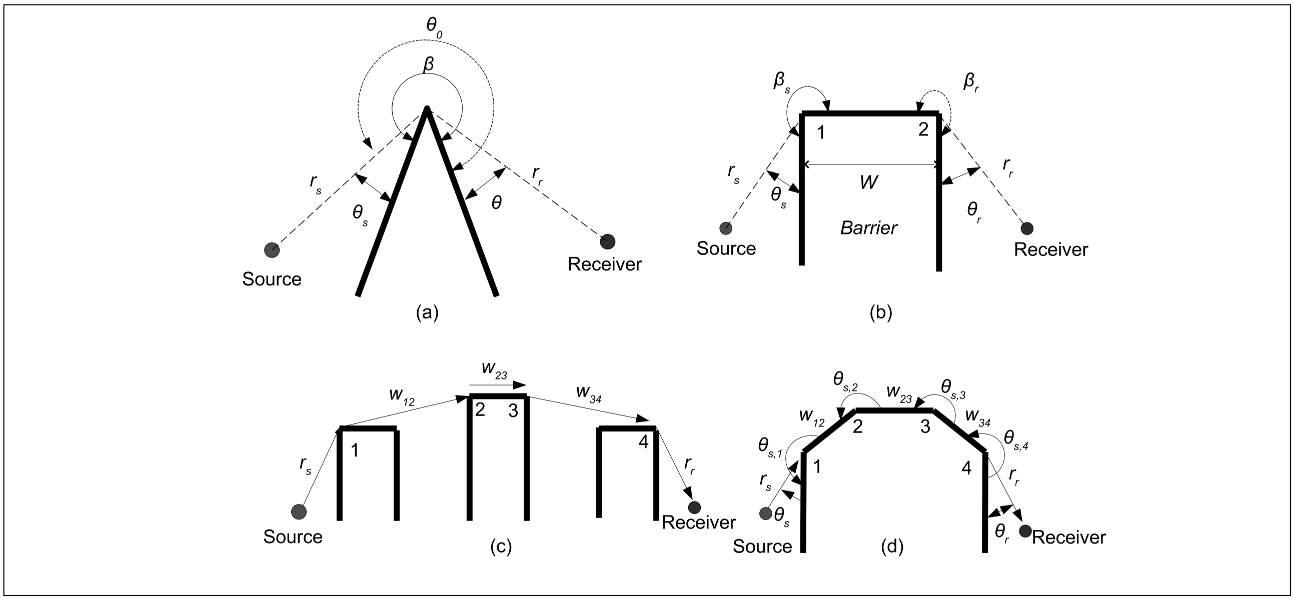

Figure 1. Geometry of single-wedge (a), double-edge (b) diffraction and multiple diffraction (c) and (d).

$$\gamma_l = \sqrt{\frac{2\left(r_s + \sum_{j=1}^{l-1} W_{j,j+1}\right)\left(r_r + \sum_{j=l}^{n-1} W_{j,j+1}\right)}{\lambda\left(r_s + \sum_{j=1}^{n-1} W_{j,j+1} + r_r\right)}} \quad (14)$$

$$M_{\nu l}(\theta_l) = (\cos \nu_l \pi - \cos \nu_l \theta_l)/(\nu_l \sin \nu_l \pi), \quad (15)$$

where r_s is the distance from the source to the first diffraction edge; $W_{l,l+1}$ is the distance between edge l and edge $l+1$; r_r is the distance from the receiver to the last diffraction edge.

The corresponding insertion loss for multiple diffraction is:

$$IL_n = -10\log_{10}\left[\frac{R^2}{L_{n-1}^2}\left(\frac{1}{2}\right)^{2C}\prod_{l=1}^{n-1} D_l^2\right] \quad (16)$$

5. Validation

For single and double diffraction, strict and relative complex methods have been published [1] [2]. To validate the here proposed model, the same validation cases are used as mentioned in Pierce's publication. Figure 2 and figure 3 show the validation results for different relative propagation paths. Our simplified method coincides with Pierce's method very well except for positions near the boundary line.

Multiple diffraction may occur in considerably different configurations as shown in figure 1(c) and (d). Here the configuration of (c) is chosen as the validation case whose dimensions are shown in figure 4 and the results are shown in figure 5. In the contour plots, the predicted errors in most of the positions are less than 2 dB.

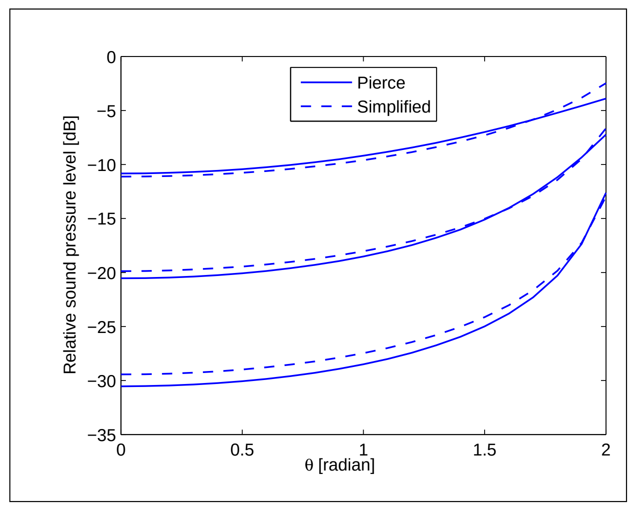

Figure 2. Validation for the single diffraction case. In the legend, "Pierce" is the method presented by Pierce; "Simplified" is the set of equations introduced in this paper. $r_s = r_r = 1\lambda, 10\lambda$ and 100λ, up to down, respectively, $\beta = 11/6\pi$, $\theta_s = \pi/6$.

6. Conclusions

A simplified model to calculate the diffraction over rigid obstacles is presented and validated. The simplified model results in less than 2 dB prediction errors in most of the tested locations. This model could be used to urban noise mapping.

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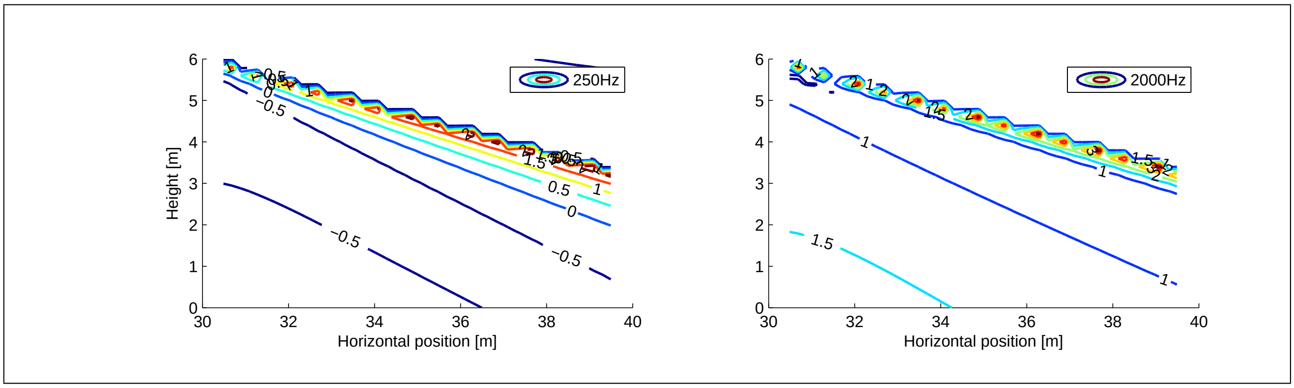

Figure 5. Contour plots showing the sound pressure level difference ($L_{simplified} - L_{Pierce}$) in the receiver zone as defined in figure 4. $L_{simplified}$ is the sound pressure level calculated by our simplified method and L_{Pierce} is the sound pressure level calculated by Pierce's [1] and Kawai's [2] method.

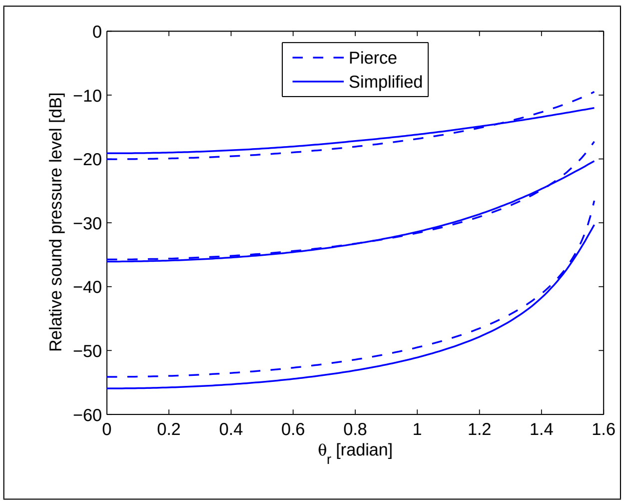

Figure 3. Validation for the double diffraction case. The legend is the same as single diffraction. $r_s = r_r = W = 1\lambda, 10\lambda$ and 100λ up to down respectively, $\beta_s = \beta_r = 1.5\pi$, $\theta_s = \pi/4$

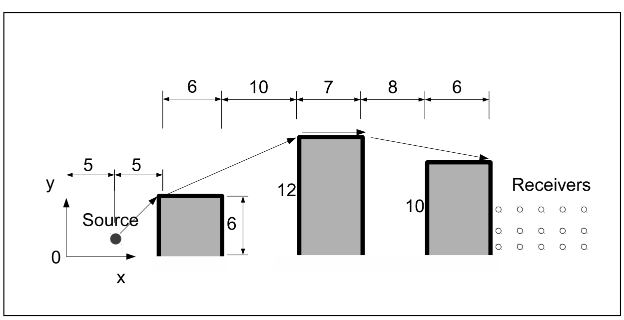

Figure 4. Configuration for the validation of multiple diffraction case.

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