Identification of a memory kernel in a nonlinear parabolic integrodifferential problem

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Abstract

The reconstruction of a solely time-dependent convolution kernel $K$ in the following nonlinear parabolic equation with unknowns $(K,u)$ is studied:

\begin{equation}
\partial_t u(x,t) - \nabla \cdot (\nabla \beta(u(x,t))) + \int_0^t K(t-s)u(x,s) \, ds = \ldots, \quad (1)
\end{equation}

with $x \in \Omega \subset \mathbb{R}^d$ and $t \in [0,T]$. The right-hand side (RHS) of (1) is not specified yet. The missing kernel is recovered from a global measurement over the domain, i.e.

\begin{equation}
\int_{\Omega} u(x,t) \, dx = m(t), \quad t \in [0,T]. \quad (2)
\end{equation}

Note that in [1], the reconstruction of $K$ based on the same measurement is studied in the semilinear equation

\begin{equation}
\partial_t u(x,t) - \Delta u(x,t) + K(t)h(x,t) + \int_0^t K(t-s)u(x,s) \, ds = f(u(x,t),\nabla u(x,t)).
\end{equation}

The main differences are that equation (1) is nonlinear and does not contain the term $Kh$. This term was crucial in the analysis made in [1]. The implications of this removal on the analysis of problem (1)-(2) and in particular on the choice of the RHS of (1) are discussed. After a specific choice for the RHS of (1), using observation (2), the inverse problem (1)-(2) can be reformulated in a direct setting. Afterwards, using Rothe’s method [2], the existence and uniqueness of a weak solution is shown and a numerical algorithm based on this method is illustrated by numerical experiments.
References

parabolic problem based on a global measurement. *Nonlinear Analysis: Theory, Methods