Transient Analysis and Stability Limits for Synchronous Reluctance Motors Considering Saturation Effects

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This paper investigates the influence of the magnetic saturation on the transient performance of Synchronous Reluctance Motors (SynRMs). In addition, the stability limits for the SynRM are studied using a more accurate model. The saturation, cross-saturation and position effects on both direct (d) and quadrature (q) axis flux linkages are considered. A Finite Element Method (FEM) is used to obtain an accurate representation for the dq -axis flux linkages relations. In order to reduce the calculation time of the finite element analysis, a look-up table (LUT) for the dq -axis flux linkages is generated based on the FEM to be used for simulating the SynRM characteristics. It is found that the magnetic saturation in the adopted motor results in an enlarged region of stable operation of the SynRM. The torque increases by about 200 % compared with the unsaturated case, which proves that it makes no sense to model this type of motor without saturation. The results show the importance of including the saturation factors on the performance of the SynRM and its stability limits. Hence, the magnetic saturation effect will not only affect the stability of the motor but also on the whole drive system.

Index Terms— FEM, Magnetic Saturation , Stability Limits , Synchronous Reluctance Motor.

I. NOMENCLATURE

i_d, i_q	Instantaneous direct and quadrature axis stator current respectively, A
L_d, L_q	Direct and quadrature axis stator inductance of SynRM respectively, H
P	Number of pole pairs
p	Differential operator (d/dt)
R_s	Stator resistance of SynRM, Ω
T_e	Electromagnetic torque of the motor, N.m
v_d, v_q	Instantaneous direct and quadrature component of stator voltage respectively, V
V_m	Maximum input voltage of the motor, V
I_m	Maximum input current of the motor, A
δ, α	Load angle and current angle, rad
ω_r, ω_s	Motor and synchronous speed, rad/s
θ_r	Rotor position, rad
λ_d, λ_q	Direct and quadrature axis flux linkages of SynRM as a function of i_d, i_q and θ_r respectively, V.s
ψ_d, ψ_q	Direct and quadrature axis flux linkages of SynRM as a function of i_d and i_q with averaging to θ_r respectively, V.s
PF	Motor power factor

II. INTRODUCTION

Synchronous Reluctance Motors (SynRMs) have many attractive features compared with other types of motors. This is due to the rugged construction and the absence of rare-earth magnets [1]. In addition, there are no cage, windings and magnets on the rotor. The torque per ampere is acceptable. This type of machines can withstand high temperatures as well

as high centrifugal forces: they are suitable for high speed applications in machine tools and molecular pumps[2, 3]. Moreover, the control system is similar to that of induction motor drives.

The SynRM (with a flux-barrier rotor type) behavior depends mainly on the saliency difference which is the difference between the direct and quadrature axis inductances (L_d, L_q). This difference is affected by the rotor geometry design and the magnetic material steel grade type of the motor core. The magnetic material steel grade has an effect on both the dq -axis inductances and the core loss of the SynRM due to different permeability [4]. In addition, the rotor position with respect to the stator has an effect on the dq -axis inductances. Moreover, the dq -axis inductances do not only depend on the self-currents but also on the mutual ones due to sharing the same flux path on the rotor core. This dependency is called the cross-saturation effect. Therefore, a model considering the saturation, cross-saturation and the position effects is necessary for an accurate representation for the SynRM control and efficiency optimization [4, 5, 6].

Several papers have been focused on the saturation and cross-saturation effects on the SynRM performance and control. Different models have been proposed to take into account the effect of the magnetic saturation for electrical machines [7, 8, 9, 10].

In [7], a saturation modelling in D-Q axis models of salient pole synchronous machines was proposed based on generalized forms tacking into account a single saturation factor with the same saturation level in both dq -axis inductances. In [8], the magnetic saturation effects on the control of a SynRM was studied based on a single saturation factor with measurable results assuming that the dq -axis inductances saturate to the same level at all the operating

conditions. The stability limits of interior permanent magnet (IPM) motors have been investigated in [9]. Different effects of magnetic saturation has been studied. The importance of including the magnetic saturation have been prove in studying the performance of IPM motors. In [10], the impact of cross saturation in SynRM of transverse-laminated type is investigated with a mixed theoretical and experimental approach considering assumptions in the measuring of the dq -axis flux linkages relations.

This paper analyses deeply the influence of the magnetic saturation and rotor position effects on the transient conditions for the SynRM performance and its stability limits. A Finite Element Method (FEM) is used to obtain an accurate representation for the dq -axis flux linkages relations. To reduce the calculation time, a Look-up table (LUT) based on the FEM is generated to be used in the simulation of the motor behaviour at steady-state and transient conditions.

This paper is organized as follows. Section III presents the modelling of the SynRM at transient condition considering the magnetic saturation and the position effects. Afterwards, section IV deals with the transient analysis for the SynRM performance. Finally, section V presents the conclusion for the studied work.

III. SYNRM MODELLING

A. Nonlinear dq model of SynRM

In order to eliminate the time-varying inductances in the voltage equations, the SynRM was represented in the dq -axis reference frame. This frame is fixed on the rotor, which rotates at ω_r [4, 11, 12].

The dq -axis voltage equations can be written as:

$$v_d = R_s i_d + p \lambda_d(i_d, i_q, \theta_r) - \omega_r p \lambda_q(i_d, i_q, \theta_r) \quad (1)$$

$$v_q = R_s i_q + p \lambda_q(i_d, i_q, \theta_r) + \omega_r p \lambda_d(i_d, i_q, \theta_r) \quad (2)$$

The dq -axis flux linkage relations are expressed as:

$$\lambda_d(i_d, i_q, \theta_r) = L_d(i_d, i_q, \theta_r) i_d \quad (3)$$

$$\lambda_q(i_d, i_q, \theta_r) = L_q(i_d, i_q, \theta_r) i_q \quad (4)$$

The electromagnetic torque can be obtained as:

$$T_e = \frac{3}{2} P (\lambda_d(i_d, i_q, \theta_r) i_q - \lambda_q(i_d, i_q, \theta_r) i_d) \quad (5)$$

The dq -axis currents and voltages can be expressed as a function of current angle (α).

$$i_d = I_m \cos(\alpha), \quad i_q = I_m \sin(\alpha) \quad (6)$$

$$v_d = -V_m \sin(\delta), \quad v_q = V_m \cos(\delta) \quad (7)$$

where α is the angle of the stator current space vector with respect to the d -axis of the motor as shown in Fig. 1.

The SynRM load angle (δ) can be calculated as:

$$\delta = \int (\omega_r - \omega_s) dt \quad (8)$$

Neglecting the stator resistance R_s , the power factor (PF) of the SynRM can be expressed as a function of the saliency ratio $K=L_d/L_q$:

$$PF = \frac{K-1}{\sqrt{K^2 \frac{1}{\sin^2(\alpha)} + \frac{1}{\cos^2(\alpha)}}} \quad (9)$$

Fig. 1. SynRM vector diagram

B. Finite Element Method (FEM) for the SynRM

The dq -axis flux linkages ($\lambda_d(i_d, i_q, \theta_r)$, $\lambda_q(i_d, i_q, \theta_r)$) of the SynRM have been calculated using FEM. The SynRM has four poles with symmetrical geometry so that it is sufficient to model only a quarter of the motor geometry. The parameters of the adopted motor are specified on table I (Appendix).

Figure 2 shows the flux paths in a quarter of the adopted motor geometry using FEM.

Fig. 2. Flux paths for the adopted SynRM using FEM for a quarter geometry with different positions

To obtain the dq -axis flux linkages relations for the SynRM, the FEM is supplied with different i_d , i_q and θ_r combinations for variable stator currents from 0 to the rated value (I_m). On the one hand, Fig. 3 shows the d -axis flux linkage of the SynRM for a half mechanical revolution based on different i_d and i_q . The variation of the d -axis flux linkage with respect to the position (θ_r) is shown. This is due to the variation of the magnetic reluctance between the slot and the teeth positions. It is clear from Fig. 3 that the d -axis flux linkage increases with increasing the i_d . In addition, the d -axis flux linkage decreases with increasing the i_q by about 15% (Fig. 3-a). On the other hand, Fig. 4 shows the q -axis flux linkage of the SynRM for a half mechanical revolution and for different i_d and i_q . It is obvious from Fig. 4 that the q -axis flux linkage increases with increasing the i_q . In addition, the effect of increasing i_d on the q -axis flux linkage is notable and leads to a decrease of about 50% (Fig. 4-a). Hence, the effect of i_d on the q -axis flux linkage is greater than the effect of i_q on the d -axis flux linkage (about 35%). This is due to the higher value of d -axis flux linkage compared to the q -axis (saliency factor equals about 5 at the rated stator current).

Figure 5 shows the average dq -axis flux linkages of the SynRM with respect to the position (θ_r). In addition, the nonlinearity of the dq -axis flux linkages with the currents can

be noticed. This leads to nonlinearity characteristics of dq -axis inductances (L_d , L_q); hence it has a strong effect on the SynRM performance.

Fig. 3. d-axis flux linkage ($\lambda_d(i_d, i_q, \theta_r)$) for the SynRM using FEM for half mechanical revolution.

Fig. 4. q-axis flux linkage ($\lambda_q(i_d, i_q, \theta_r)$) for the SynRM using FEM for half mechanical revolution.

Fig. 5. (a) d-axis flux linkage ($\psi_d(i_d, i_q)$) and (b) q-axis flux linkage ($\psi_q(i_d, i_q)$) for the SynRM using FEM with averaging for the position (θ_r).

III. TRANSIENT-STATE ANALYSIS FOR THE SYNRM

In order to study the effect of the magnetic saturation and the position on the transient operation of the SynRM, a dynamic model for the SynRM based on the FEM has been solved for different conditions. Three different cases are considered; (1) the general and more accurate modeling for the SynRM where the effect of the magnetic saturation and the position are taken into account, (2) the magnetic saturation effect is considered only without the position effect, and (3) the unsaturated case where the L_d and L_q are considered to be constant values. From Fig. 5, it can be deduced that there are

different values for L_d and L_q based on the currents. So the values of L_d and L_q set to be $L_d=0.0185$ H and $L_q=0.003$ H to represent an average values for the constant inductance case with neglecting the magnetic saturation and the position effects.

Figure 6 shows the run-up response of the SynRM for different loads. On the one hand, the oscillation on the torque waveform are mainly due to the position dependence of λ_d and λ_q . This can be understood by comparing with the torque wave form where the position effect is removed, i.e where λ_d and λ_q are averaged over θ_r .

Fig. 6. Run-up response of the SynRM with considering and neglecting saturation for different loads

Fig. 7. The variation of the motor torque with the load angle for considering and neglecting saturation

On the other hand, it is clear that the motor torque capability increases when considering the saturation factors. The motor torque stability region can be increased from about 21 N.m with neglecting the saturation effect to about 42 N.m with including the saturation factors. This is obvious also in Figs. 7 and 8. Figure 7 shows the transient variation of the motor torque with the load angle for considering and neglecting the saturation effect. The minor loops on Fig. 7 are due to the step change of the load. Figure 8 shows the load angle and the maximum stator current for different loads in considering and neglecting the saturation effect. The variation of the power factor for considering and neglecting the saturation effect can be seen from Fig. 9. It can be noted that the power factor of the saturated case is higher than the unsaturated one. This is due to the dependency of the power factor on saliency ratio (L_d/L_q) of the motor.

Fig. 8. The variation of the load angle and the maximum stator current of the SynRM with considering and neglecting saturation for different loads

Fig. 9. The variation of the power factor of the SynRM with considering and neglecting saturation for different loads

V. CONCLUSION

This article has presented the influence of the magnetic saturation on the SynRM performance at transient conditions. Moreover, the stability limits of operation for the adopted motor have been investigated. The dq -axis flux linkages of the motor considering the magnetic saturation and the position effects have been calculated using FEM. In order to reduce the calculation time of the FEM, a look-up table (LUT) is generated based on the FEM calculation to be used on the simulation for the SynRM characteristics at dynamic conditions. On the one hand, it's noted that the d -axis flux linkage varies linearly with i_d at lower values for i_d about half of the rated value and then becomes saturated with increasing i_d . On the other hand, q -axis flux linkage varies linearly with i_q . The effect of i_q on the d -axis flux linkage is very small compared to the effect of i_d on the d -axis. The variation on q -axis flux linkage has a great impact on the SynRM performance and its stability limits.

The magnetic saturation increases the SynRM stability operation region and the torque capability by about 200 % compared with the unsaturated case.

Finally, the results show the importance of including the magnetic saturation on the modeling of the SynRM performance and its stability limits.

APPENDIX

TABLE I
SYNRM PARAMETERS

Parameter	Value
Number of pole pairs	4
Number of rotor flux barriers per pole	3
Number of stator slots	36
Number of phases	3
Stator outer diameter	180 mm
Stator inner diameter	110 mm
Rotor outer diameter	109.4 mm
Rotor shaft diameter	35 mm
Axial length	140 mm
Air gap length	0.3 mm
Rated speed	6000 RPM
Rated frequency	200 Hz
Rated current	22 A
Rated voltage	380 V
Stator resistance	0.15 Ω
Moment of inertia	0.0025 Kg.m ²

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