Shape Correction Factor for Drying Shrinkage in a Concrete Cross-Section

N. Reybrouck¹; P. Criel¹; R. Caspee¹; and L. Taerwe¹

¹Magnel Laboratory for Concrete Research, Department of Structural Engineering, Ghent University, Technologiepark-Zwijnaarde 904, Ghent. E-mail: Nicky.Reybrouck@UGent.be

Abstract

A concrete member is subjected to loads for a long period of time, during which creep and shrinkage of concrete develop gradually. The prediction of this time-dependent behaviour is important as it may cause serious serviceability problems in concrete structures. A time-dependent analysis is commonly based on empirical equations according to design codes where the function describing the time dependent increment of shrinkage and creep is commonly, among others, defined based on the notional size of the element. In case of imbedded steel or insulated boundaries the moisture transport can be partially affected or prevented. Also, the geometry and size of the cross-section have an important effect on the shrinkage behaviour of a concrete member. Hence, the performance of commonly used empirical formulas may be improved by applying a correction factor on the notional size. In order to investigate the impact of these various factors on the net macroscopic shrinkage used in analysis and design, a discretized 2D physical model was developed. The model was used to simulate drying of a concrete cross-section by determining the moisture distribution in the cross-section as function of time.

INTRODUCTION

Creep and shrinkage of concrete result in long-term deformations that may cause serviceability problems and it is therefore important to predict this effect properly. The mechanisms causing creep and shrinkage of concrete show significant correlation with the moisture distribution of the concrete cross-section. This study is focussing on the determination of the influence of the geometry of a concrete member on the drying shrinkage response.

The effect of moisture transportation has shown considerable effect on shortening of concrete members in case of imbedded steel profiles (Seol 2008). The differential moisture distribution should therefore be taken into account in order to enhance the accuracy of the prediction of strain development. The deformation of concrete members depends highly on the possible moisture transport routes affected by for example a different shape geometry (T-beams), hollow areas (hollow core elements), obstructions (imbedded steel) and boundary conditions (isolations and exposed surfaces).

The moisture diffusion and differential drying shrinkage was determined using finite element analysis. This behaviour is determined by analysing the progress of moisture at each point of the concrete section in time.

NUMERICAL ANALYSIS OF DRYING CREEP AND SHRINKAGE

The creep and shrinkage behaviour is modelled using a finite element method combined with Fick's second law of diffusion, which describes the transient phenomena within the concrete section during each time-step of the analysis. The drying of a concrete cross-section with time is given by (Sue-Tae 2011):

$$\frac{\partial H}{\partial T} = \nabla \cdot (D_s(H, T) \nabla H)$$

where H is the relative pore humidity and t is time. The progress of moisture diffusion through these specimens was calculated using the non-linear moisture diffusion equation (Bazant 1972). For isothermal conditions the non-linear diffusion coefficient D can be expressed as a function of the relative pore humidity H as defined in Model Code 2010 (Fib 2010) as:

$$D(H) = D_1 \left(\alpha + \frac{1 - \alpha}{1 + \left(\frac{1 - H}{1 - H_c} \right)^n} \right)$$

where D_1 is the maximum of $D(H)$; D_0 is the minimum of $D(H)$; $\alpha = D_0/D_1$ (assumed 0.05); H_c = the relative pore humidity at $D(H) = 0.5D_1$ (assumed 0.80) and n is an exponent (assumed 15). The free shrinkage at each point of the section is determined based on local drying and calculated as suggested by (Kim 2012; Bazant 1994):

$$\Delta \varepsilon_{sh}(t_i, t_{i-1}) = \varepsilon_s^0 g_s(t_i)(1 - h(t_i, x, y)^3) - \varepsilon_s^0 g_s(t_{i-1})(1 - h(t_{i-1}, x, y)^3)$$

where $\Delta \varepsilon_{sh}(t_i, t_{i-1})$ is the shrinkage strain during time period (t_i, t_{i-1}) ; ε_s^0 is the magnitude of the final shrinkage; $g_s(t_i) = E_c(t_0)/E_c(t_i)$; $E_c(t)$ is the Young modulus at time t and $h(t, x, y)$ is the local moisture content at time t .

The moisture distribution at each time step and each material point within the concrete section is determined using the initial strains and induce non-uniform restraining stresses. An example of the moisture distribution of a T-shaped cross-section is shown in Figure 1. This analysis of shrinkage and creep strains was performed by using a numerical step-by-step procedure in time in combination with transformed section properties adjusted in time.

Figure 1. Geometry of the studied T cross-section and moisture content example.

The local shrinkage behaviour in a specific point of the cross-section is determined based on the local moisture content history. This local strain response throughout the structural element in space/time is then transformed to a section-wide temporal shrinkage response by applying the principle of Navier-Bernoulli. The fictitious force ΔN and bending moment ΔM_x corresponding to restrain the local free shrinkage increment $\Delta \varepsilon_{sh}(x, y, t_i, t_{i-1})$ at time interval (t_i, t_{i-1}) is determined by (Ghali 2011):

$$\Delta N = \int_A E_c(t) \Delta \varepsilon_{sh}(x, y, t_i, t_{i-1}) dA$$

$$\Delta M_x = \int_A E_c(t) \Delta \varepsilon_{sh}(x, y, t_i, t_{i-1}) y dA$$

in which y is the y -coordinate of the observed material point and A is the area of the cross-section. The strain increment $\Delta \varepsilon_{sh}(t_i, t_{i-1})$ and curvature increment $\Delta \kappa_{sh}(t_i, t_{i-1})$ are then calculated by:

$$\begin{Bmatrix} \Delta \varepsilon_{sh}(t_i, t_{i-1}) \\ \Delta \kappa_{sh}(t_i, t_{i-1}) \end{Bmatrix} = \frac{1}{E_c(t)} \begin{bmatrix} EA & ES \\ ES & EI \end{bmatrix}^{-1} \begin{Bmatrix} -\Delta N \\ -\Delta M_x \end{Bmatrix}$$

in which EA , ES and EI are respectively the area, the first moment of area and the second moment of area of the transformed cross-section multiplied with the modulus of elasticity $E_c(t)$. The final shrinkage $\varepsilon_{sh}(t)$ is given by:

$$\varepsilon_{sh}(t) = \sum_{i=1}^n \Delta \varepsilon_{sh}(t_i, t_{i-1})$$

RESULTS OF THE NUMERICAL SIMULATION

The size of each element used in the numerical analysis was chosen equal to 5 mm and the simulation was performed for a period of 10 years using time step increments of 5 hours. An example of the drying shrinkage development obtained by the numerical analysis for different geometries such as rectangular, circular and T-shaped cross-sections are given in Figure 2.

Figure 2. Drying shrinkage development for different cross-sections (a) Rectangular (b) Circular (c) T-beam with $z = 300$ mm, variable flange thickness a (d) T-beam with $z = 500$ mm, variable flange thickness a.

REMARK

Drying shrinkage results in a shortening of the concrete element. Cross-sections who do not have two axis of symmetry will also produce a curvature (inclined plane) due to the non-uniform shrinkage (see Figure 3). The curvatures as function of time obtained from the numerical analysis of a T-beam with a width of 500 mm are shown in Figure 4.

Figure 3. Curvature due to the non-uniform development of shrinkage.

**Figure 4. Curvature due to the development of shrinkage in case of T-beams.
(example $z = 500$ mm)**

SHAPE CORRECTION FACTOR

An averaged cross-sectional approach in which the behaviour of the structural member is equal for the whole cross-section is a simple, fast and efficient method for the analysis of creep and shrinkage in concrete structures and is widely used in engineering practice. The formulations given in Model Codes and guidelines such as CEB-FIP (fib 2010), ACI (ACI 1998), B3 (Bazant 1995), B4 (RILEM 2015) are generally used to calculate the shortening of concrete members. These methods provide fairly good results as an average for plain concrete cross-sections. It considers mix properties, size and exposed ambient conditions. The progress of drying shrinkage in Eurocode 2 is expressed as a function of the notional size of the element. Non-uniform behaviour of the cross-section cannot be captured correctly using these methods but can be accounted for using correcting parameters. The shrinkage and creep equations found in Eurocode have been modified to predict the shortening of the concrete element more accurately for different shapes. In this paper a modified shrinkage model is suggested to determine shape correction factor α_T to improve the accuracy of the formulations for drying creep of concrete by modifying the notional size h_0 in Eurocode 2. It is assumed that the differences between the different shapes are caused only by the moisture diffusion process. Consequently, the proposed method was used to perform a numerical analysis on a wide range of different shapes and dimensions for T-shaped beams to improve the local drying shrinkage prediction. The function $\beta_{ds}(t, t_s)$ (EN 1992-1-1) describes the time-development of the drying shrinkage $\varepsilon_{cds}(t, t_s)$ and is given as (with an additional proposed correction parameter α_T) :

$$\beta_{ds}(t, t_s) = \frac{t - t_s}{(t - t_s) + 0.04\sqrt{(\alpha_T h_0)^3}}$$

where $h_0 = 2 A_c / u$; A_c is the cross-section and u is the perimeter of the member in contact with the atmosphere.; α_T is a proposed shape correction factor for T-beams. Some examples for h_0 are shown in Table 1.

Table 1. Expected value for notional size h_0 according to Eurocode 2 applied to a square, circle and T-beam.

<i>Shape geometry of cross-section</i>	<i>Shape description</i>	$h_0 = \frac{2A_c}{u}$ (EN1992-1-1)
	Square	$\frac{z}{2}$
	Circle	$\frac{D}{2}$
	T-beam (see figure 1)	$\frac{2az - a^2}{2z}$

The parameter $\alpha_T h_0$ was determined based on the equation for $\beta_{ds}(t, t_s)$ using a least squares approximation of the drying shrinkage behaviour obtained by the numerical simulations. A comparison of the estimated notional size with correction factor $\alpha_T h_0$ obtained from simulations and the notional size parameter h_0 as defined by Eurocode 2 ($h_0 = 2 A_c/u$) is shown in Table 2 and Figure 5.

Table 2. Comparison of estimated values for notional size h_0 with the expected value according to Eurocode 2.

	$\alpha_T h_0$	h_0	α_T		$\alpha_T h_0$	h_0	α_T
R200	76.8	100	0.77	T400A200	120.9	150.0	0.81
R250	100.0	125	0.80	T400A250	152.2	171.9	0.89
R300	125.1	150	0.83	T400A300	165.2	187.5	0.88
R350	152.1	175	0.87	T400A350	181.1	196.9	0.92
R400	181.0	200	0.90	T500A50	25.1	47.5	0.53
R450	211.7	225	0.94	T500A100	53.7	90.0	0.60
R500	244.3	250	0.98	T500A150	86.5	127.5	0.68
C200	82.5	100	0.82	T500A200	122.7	160.0	0.77
C250	97.8	125	0.78	T500A250	160.5	187.5	0.86
C300	121.4	150	0.81	T500A300	184.4	210.0	0.88
C350	144.9	175	0.83	T500A350	223.9	227.5	0.98
C400	170.8	200	0.85	T500A400	240.1	240.0	1.00
C450	199.3	225	0.89	T500A450	245.0	247.5	0.99
C500	231.2	250	0.92	T800A800	478.6	400.0	1.20
T300A50	25.3	45.8	0.55	T800A600	455.9	375.0	1.22
T300A100	53.8	83.3	0.65	T800A400	303.5	300.0	1.01
T300A150	76.2	112.5	0.68	T800A200	123.7	175.0	0.71
T300A200	111.8	133.3	0.84	T1000A1000	661.7	500.0	1.32
T300A250	124.4	145.8	0.85	T1000A800	649.8	480.0	1.35
T400A50	25.2	46.9	0.54	T1000A600	521.5	420.0	1.24
T400A100	53.7	87.5	0.61	T1000A400	309.6	320.0	0.97
T400A150	86.3	121.9	0.71	T1000A200	124.3	180.0	0.69

Figure 5. Comparison $\alpha_T h_0$ obtained by simulations and h_0 according to Eurocode 2 for rectangular and circular cross-sections.

The goal of this paper is to define a general parameter α_T to denote the influence of the shape and size of T-beams on the drying shrinkage development. The following equation is proposed (with R^2 equal to 0.93):

$$\alpha_T = 1.3 \cdot a^{0.33}$$

in which α_T is the shape correction factor for T-beams and a is the thickness of the flange in meters (see Figure 1). A comparison of the proposed correction factor α_T given by the proposed equation and the value of α_T obtained by numerical results is shown in Figure 6.

Figure 6. Comparison proposed equation for $\alpha_T h_0$ and numerical results.

CONCLUSION

- The time-development of the drying shrinkage behaviour obtained by the numerical analysis for prisms and cylinders is comparable with the function $\beta_{ds}(t, t_s)$ given by EN 1992-1-1 based on the notional size h_0 .

- The presented results are based on simplifications and give agreeable results even in the case in which the effect of creep is neglected. Further refinements can be included to enhance the modelling of drying shrinkage.
- There is a large influence of the geometry of T-shaped beams on the drying shrinkage behaviour caused by the different flow paths for moisture transport and a simple modification parameter α_T was proposed in order to account for this effect.

REFERENCES

- ACI 209R. (1998). Prediction of creep, shrinkage, and temperature effects in concrete structures. *Amer concr inst Detroit*.
- Bazant, Z. P., & Baweja, S. (1995). Justification and refinement of Model B3 for concrete creep and shrinkage. 1. Statistics and sensitivity. *Materials and structures*, Vol.28.
- Bazant, Z. P., & Najjar, L. J. (1972). Nonlinear water diffusion In nonsaturated concrete. *Materiaux et Constructions*, Vol.5(No.25).
- Bazant, Z. P., & Xi, Y. (1994). Drying creep of concrete: constitutive model and new experiments separating its mechanisms. *Materials and structures*, Vol.27, pp.3-14.
- Fib. (2010). CEB-FIP Model Code 2010 In Comité Euro-Int Du Béton (Ed.), *Design code*.
- Ghali, A., Favre, R., & Elbadry, M. (2011). *Concrete Structures: Stresses and Deformations: Analysis and Design for Sustainability (4th edition)*: CRC Press.
- Jefferson, A. D., & Lyons, P. (2010). *Comparison of approaches for simulating moisture content changes in concrete*. Paper presented at the EURO-C 2010.
- Kim, J. K., & An, G. H. (2012). *An Analytical Study of Long-term Deformation in SRC Columns Considering Differential Moisture Distribution*. Paper presented at the Proceedings of the 3rd JCI-KCI-TCI Symposium on Recent Advancement of Technologies in Concrete Engineering Japan Concrete Institute.
- Reybrouck, N., Maes, M., Caspeele, R., Criel, P., & Taerwe, L. (2014). *Modeling the effect of random environmental influences on stochastic creep in concrete members*. Paper presented at the 12th international probabilistic workshop.
- RILEM. (2015). Model B4 for creep, drying shrinkage and autogenous shrinkage of normal and high-strength concretes with multi-decade applicability. In TC-242-MDC (Ed.), (Vol. Vol.48). *Materials and Structures*.
- Seol, H. C., Kwon, S.-H., Yang, J.-K., Kim, H.-S., & Kim, J.-K. (2008). *Effect of differential moisture distribution on the shortening of steel-reinforced concrete columns*. Paper presented at the Magazine of Concrete Research.
- Su-Tae, K., Jeong-Su, K., Yun, L., Yon-Dong, P., & Jin-Keun, K. (2011). Moisture Diffusivity of Early Age Concrete Considering Temperature and Porosity. *KSCE Journal of Civil Engineering*.