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Horn length:	0.5 m
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Thermal effects on transverse domain wall dynamics in magnetic nanowires

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Magnetic domain walls are proposed as data carriers in future spintronic devices, whose reliability depends on a complete understanding of the domain wall motion. Applications based on an accurate positioning of domain walls are inevitably influenced by thermal fluctuations. In this letter, we present a micromagnetic study of the thermal effects on this motion. As spin-polarized currents are the most used driving mechanism for domain walls, we have included this in our analysis. Our results show that at finite temperatures, the domain wall velocity has a drift and diffusion component, which are in excellent agreement with the theoretical values obtained from a generalized 1D model. The drift and diffusion component are independent of each other in perfect nanowires, and the mean square displacement scales linearly with time and temperature. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4921421>]

Several future spintronic devices^{1,2} rely on the ability to accurately manipulate the position of magnetic domain walls in nanowires. For instance, in the racetrack memory,¹ data are stored in magnetic domains separated by domain walls moved by spin polarized currents.^{3,4} Recently, new schemes have been proposed in which the domain walls themselves represent the data bits.⁵ However, domain walls are subject to thermal fluctuations which can compromise the position of the walls, and thus the integrity of the data. Consequently, thermal effects should be taken into account in the design and should first be understood. In micromagnetic simulations, thermal fluctuations are either included by a jump-noise process^{6,7} or a stochastic field⁸ as determined by Brown.⁹ Using this approach, the influence of thermal fluctuations at high driving forces has been investigated.¹⁰ It was found that the effect of thermal fluctuations is negligible in the flow regime. However, most experiments are performed at low velocities, because of the high current densities (of the order of $1\text{ A}/\mu\text{m}^2$) required to move domain walls.

In this letter, we investigate the effects of finite temperatures on transverse domain wall motion in the absence of magnetic defects such as grain boundaries. We find that thermal fluctuations result in a diffusive motion of the domain wall independent of the excitation current. These results are understood within the framework of a generalized 1D model¹¹ where we incorporated temperature and get an analytical expression for the diffusion, in excellent agreement with numerical simulations. This easy inclusion of thermal effects is important for future work where this approach can be applied to the more realistic case of nanowires with imperfections.

On the microscopic level, magnetization dynamics is described by the Landau-Lifshitz-Gilbert equation¹² extended

with two spin transfer torque terms¹³ due to the current density $\mathbf{J}$,

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} - [b\mathbf{J} \cdot \nabla] \mathbf{m} + \beta \mathbf{m} \times [b\mathbf{J} \cdot \nabla] \mathbf{m}. \quad (1)$$

In this equation, γ depicts the gyromagnetic ratio, α the Gilbert damping constant, $b = P\mu_B/eM_s(1 + \beta^2)$ with P the polarization of the spin-polarized current, e the electron charge, μ_B the Bohr magneton, β the degree of non-adiabaticity,^{14,15} and M_s the saturation magnetization. $\mathbf{m}$, the normalized magnetization (with unit length), and $\mathbf{H}_{\text{eff}}$, the effective field, are both space and time varying vector fields. The effective field consists of different terms:¹⁶ the external field, the demagnetizing field, and fields due to the exchange interaction and the anisotropy of the material. Thermal fluctuations are taken into account by a stochastic thermal field,⁹ $\mathbf{H}_{\text{th}}$, contributing to the effective field. This thermal field [Eq. (2)] is uncorrelated in space and time, and has a magnitude determined by the fluctuation dissipation theorem,^{9,10}

$$\begin{aligned} \langle \mathbf{H}_{\text{th}} \rangle_{\text{stat}} &= 0 \\ \langle \mathbf{H}_{\text{th},i}(t) \mathbf{H}_{\text{th},j}(t') \rangle_{\text{stat}} &= q \delta_d(t - t') \delta_{d,ij} \\ q &= \frac{2k_B T \alpha}{M_s \gamma \mu_0 V}. \end{aligned} \quad (2)$$

The operator $\langle \rangle_{\text{stat}}$ indicates a statistical average over different realizations, indices i, j , and k represent the axes in a cartesian system, δ_d is the Dirac delta function, k_B the Boltzmann constant, T the temperature, μ_0 the vacuum permeability, and V the volume on which the thermal field is calculated, which in our simulations is the size of the finite difference cells. We performed our simulations using the software package MuMax3.¹⁷ To simulate domain wall motion in an infinite wire, we restrict

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the computational region to an 800 nm wide window centered around the moving domain wall.⁴ Magnetic charges at the window edges are compensated. The cross-section S of the wire is $100 \times 10 \text{ nm}^2$.

To analyze the motion of the domain wall, we employ a generalization¹¹ of the 1D model.^{18–20} We have extended this model to include the effects of externally applied fields in any direction, combined with a spin-polarized current. The resulting formulas are shown in Eqs. (3) and (4), where $\langle \rangle$ represents a spatial average over the computational region R . In the derivation of the generalized 1D model, all terms which are asymmetric in the magnetization are collected in one contribution $\mathcal{O}(\text{asymmetric})$, which is equal to 0 for symmetric domain walls¹¹

$$\dot{x}(H_{\text{ext}}, J_x) = \frac{L_x \gamma \langle \delta \rangle}{2\alpha} H_{\text{ext},x} - \frac{\beta}{\alpha} b J_x - \frac{L_x}{2\alpha} \left\langle \delta \frac{\partial \phi}{\partial t} \right\rangle + \mathcal{O}(\text{asymmetric}), \quad (3)$$

$$\begin{aligned} \left\langle \delta \frac{\partial \phi}{\partial t} \right\rangle (H_{\text{ext}}, J_x) &= \frac{\gamma}{1 + \alpha^2} H_{\text{ext},x} \langle \delta \rangle - \frac{2}{L_x} \frac{\beta - \alpha}{1 + \alpha^2} b J_x \\ &+ \frac{\alpha \gamma}{1 + \alpha^2} (H_{\text{ext},z} \langle m_y \rangle - H_{\text{ext},y} \langle m_z \rangle) \\ &- \frac{\alpha \gamma}{1 + \alpha^2} \frac{M_s}{\langle \sqrt{\delta} \rangle^2} (N_z - N_y) \langle m_y \rangle \langle m_z \rangle \\ &+ \mathcal{O}(\text{asymmetric}). \end{aligned} \quad (4)$$

Here, L_x is the width of the computational window, N_y and N_z are effective demagnetizing factors, while the angle ϕ is the out-of-plane tilting of the domain wall. Furthermore, $\langle \delta \rangle$ is the fraction of R which is not magnetized along the nanowire

$$\langle \delta \rangle = \frac{\langle m_y^2 + m_z^2 \rangle}{\langle m_x^2 + m_y^2 + m_z^2 \rangle} = \langle m_y^2 + m_z^2 \rangle. \quad (5)$$

Hence, $L_x \langle \delta \rangle$ can be interpreted as a measure of the domain wall width. Equation (3) shows that in general, the domain wall velocity is not only determined by the direct action of a driving field or current but is also affected by the time variation of the magnetization tilting $\partial \phi / \partial t$ and the asymmetry of the domain wall. Below the Walker breakdown, the domain wall tilting is fixed, resulting in a vanishing third term of Eq. (3).

To isolate the effects of temperature, we performed a first set of simulations, applying only a spin-polarized current and assuming perfect adiabaticity ($\beta = 0$). Typical material parameters for Permalloy²¹ were used. In the absence of temperature, no net motion of the domain wall is expected under the Walker breakdown^{3,15} as is clear from Eq. (3). Hence, all steady state domain wall dynamics can be attributed to thermal effects. On the transverse domain wall, shown in Fig. 1(a), we have applied a current density $J = 1 \text{ A}/\mu\text{m}^2$ at 300 K for 100 ns. In Fig. 1(b), 1000 paths of the domain wall simulated with different realizations of the temperature are shown. The red line highlights one typical path. On the right side of Fig. 1(b), the distribution of the final domain wall positions after 100 ns is shown. Remarkably, the combined action of

FIG. 1. (a) The transverse domain wall positioned in the center of the computational region. The colors and arrows represent the direction of the magnetization. Note the absence of closure domains at the outer edges due to charge compensation. The lengths of the wire in the direction indicated by the axes in the coordinate system are 800, 100, and 10 nm, respectively. (b) The left side of the figure shows the positions as function of time for 1000 domain walls in a magnetic nanowire at 300 K, and $J = 1 \text{ A}/\mu\text{m}^2$ ($\beta = 0$). One randomly chosen path is highlighted in red. The right side shows the distribution of their final positions, with a fitted Gaussian curve centered at 0, proving that the motion is diffusion with no average displacement.

thermal fluctuations and current does not give rise to an average domain wall motion: the data are described by a Gaussian with zero average. Consequently, the motion can be interpreted as a random walk resulting in diffusion without net motion and characterized by a mean square displacement (MSD). To investigate the influence of current on the diffusion, we performed similar simulations varying J from 0 to $10 \text{ A}/\mu\text{m}^2$ (close to the Walker breakdown current density of $\pm 14 \text{ A}/\mu\text{m}^2$). For each current density, 500 realizations of the temperature are simulated. The results are shown in Fig. 2(a), where the red bullets represent the MSD divided by time, as this quantity is independent of the simulation time. As a function of current density, almost constant values of MSD/t are found, indicating that—apart from the indirect effect due to a decrease in $L_x \langle \delta \rangle$ for higher current densities (as shown in Fig. 3(e))—the current density does not influence the diffusion characteristics. In the nanowire, the domain wall tilts out of plane until it reaches an offset tilting determined by the current density. Here, at zero temperature, all torques cancel out and the wall does not move.³ Thermal fluctuations give rise to an additional field torque responsible for the motion of the domain wall. These thermal fluctuations are independent of the current density and thus give rise to similar diffusion. Repeating these simulations at different temperatures shows that MSD/t scales linearly with temperature [see Fig. 2(c)] as expected in a system described by normal diffusion.²² We aim to quantitatively describe this motion. To this end, we interpret our results within the framework of a generalized 1D model. Similar to the micromagnetic approach, we add temperature to the 1D model as a fluctuating thermal field with properties given by Eq. (2). Now, V is the volume of the domain wall.

Figure 3 clarifies the combined action of a constant field along the nanowire and a current on the domain wall at 0 K and with $\beta = 0$. The velocity of the domain wall increases

FIG. 2. (a) The domain walls' mean square displacement over time averaged each time over 500 realizations as function of current density at 3 different temperatures. The coloured dots with error bars show the simulated data with $\beta = 0$, while the full lines show the theoretical curves expected by Eq. (7). The black triangles show the mean square displacement corrected for the drift velocity for simulations with $\beta = 2\alpha$. The error bars on these values are comparable to their counterparts for $\beta = 0$ but are not shown for clarity. (b) The drift velocity of current driven domain walls with $\beta = 2\alpha$ at different temperatures. The datapoints (overlapping each other for all temperatures) are in almost perfect agreement with the theoretically expected velocities from Eq. (3), represented by the full line. (c) The mean square displacement over time as function of temperature at $J = 0$ A/ μ m². The full line is a fit to the data and shows that there is a linear temperature dependence.

with the increase in field strengths up to the Walker breakdown, see Fig. 3(a). Based on Eq. (3), we expect for a rigid, symmetric domain wall a linear dependence of the domain wall velocity on the external field. However, the domain wall width $L_x \langle \delta \rangle$ decreases with larger fields as shown in Fig. 3(b). Introducing this field dependence in Eq. (3) leads to the dotted lines in Fig. 3(a), showing that the reduction in domain wall width alone cannot explain the reduction in velocity. Indeed, the asymmetry of the wall significantly influences the dynamics. In Fig. 3(c), $L_x \langle m_x \delta \rangle$ versus applied field is shown. This quantity represents the net magnetization component along the nanowire axis within the domain wall, and is a measure for the domain wall asymmetry as it is zero for a symmetric domain wall and large for an asymmetric one. Panel(c) clearly shows that the wall gets increasingly asymmetric for larger fields. To take this asymmetry into account, we assume a linear dependence on the field [see Fig. 3(c)] with a slope $\sim \chi$, which can be interpreted as a susceptibility along the x -axis. This allows us to include the asymmetry in Eq. (3) by defining an effective $\langle \delta \rangle$,

$$\langle \delta_{\text{eff}} \rangle \equiv \langle \delta \rangle (1 - \chi) \approx 0.74 \langle \delta \rangle. \quad (6)$$

This value accommodates for the difference between the dotted lines and simulation data in Fig. 3(a). In other models,^{18–20,23} a similar rescaling is done by adopting different definitions of the domain wall width or by using the width as

a fitting parameter, implicitly taking the asymmetry into account. In the following, we remove $\mathcal{O}(\text{asymmetric})$ from the equations as these effects are now included in $\langle \delta_{\text{eff}} \rangle$.

Figures 3(d)–3(f) show similar simulation results, now with varying J ($\beta = 0$). In panel (d), the offset in velocity at $J = 0$ A/ μ m² is determined by the applied field and $\langle \delta_{\text{eff}} \rangle$ at this field. With the increase in current density, the domain wall velocity gradually goes down. This is explained by the reduction in domain wall width for increasing current, as shown in panel (e). Panel (f) shows that currents have no influence on the domain wall asymmetry. Remarkably, the curves in panel (d) show similar behaviour as the simulation results in Fig. 2(a), which suggests that the latter can indeed be described by the combined action of a thermal field and the driving current. However, unlike the static field, the thermal field does not give rise to a drift velocity but only to diffusion.

Now, we will introduce thermal effects in the 1D model to explain our observations. Contrary to the fields used in Fig. 3, the thermal field acting on the domain wall fluctuates in time, and has no preferential direction. The spread on the domain wall positions²² is described by the MSD = $\langle (\int_0^t \dot{x} dt')^2 \rangle_{\text{stat}}$, which can be quantified using Eq. (3),

$$\begin{aligned} \left\langle \left(\int_0^t \dot{x} dt' \right)^2 \right\rangle_{\text{stat}} &= \left\langle \left(\int_0^t \frac{L_x \gamma \langle \delta_{\text{eff}} \rangle}{2\alpha} H_{\text{ext},x} dt' \right)^2 \right\rangle_{\text{stat}} \\ &= \left\langle \frac{L_x^2 \gamma^2 \langle \delta_{\text{eff}} \rangle^2}{4\alpha^2} \left(\int_0^t H_{\text{th},x} dt' \right)^2 \right\rangle_{\text{stat}} \\ &= \frac{L_x^2 \gamma^2 \langle \delta_{\text{eff}} \rangle^2}{4\alpha^2} q t = \frac{\gamma k_B L_x \langle \delta_{\text{eff}} \rangle}{2\alpha M_s \mu_0 S} T t. \end{aligned} \quad (7)$$

Here, we assumed that the domain wall tilting follows the thermal fluctuations sufficiently fast to neglect $\langle \delta_{\text{eff}} \frac{\partial \phi}{\partial t} \rangle_{\text{stat}}$. Furthermore, we made use of the properties in Eq. (2) and $V = \langle \delta_{\text{eff}} \rangle L_x S$, the volume of the domain wall. Note that this volume is dependent on the current density [cf. Fig. 2(e)]. As expected, the MSD grows linearly in time. The full lines in Fig. 2(a) show the MSD/ t resulting from Eq. (7). For experimentally relevant (low) current densities, there is an almost perfect agreement between theory and simulation. Furthermore, Eq. (7) predicts that the MSD is linearly dependent on temperature, which is also confirmed by our simulations. Moreover, the model takes the small domain wall deformations into account via $\langle \delta_{\text{eff}} \rangle$. These result in the non-linear deviations for large current densities also found in the simulations shown in Fig. 3. The remaining slight difference between the full lines and the data points is explained by the fact that we assumed a linear scaling of the asymmetry with the externally applied field, while Fig. 3(c) shows that this approximation is only valid for small fields. Also, $\langle \delta_{\text{eff}} \frac{\partial \phi}{\partial t} \rangle_{\text{stat}}$ might have a small contribution in Eq. (7).

In a last set of simulations, we applied a spin polarized current, assuming non-adiabaticity ($\beta = 2\alpha$), and investigated the domain wall motion at different temperatures, again considering 500 realizations per datapoint. When $\beta \neq 0$, we expect a net velocity of the wall for any $J > 0$ A/ μ m². In Fig. 2(b), the full line shows the domain wall velocity at 0 K described by

FIG. 3. Field and current driven domain wall motion at zero temperature. Both driving forces are applied along the nanowire. (a) The velocity as function of field strength for different current densities. The datapoints show the micromagnetic results, while the lines show the theoretically expected values from Eq. (3) with (full lines) and without (dotted lines) taking the asymmetry into account. (d) presents the data from (a) in a different way, to clarify that the velocity of the nanowire slightly goes down as function of current density at a fixed field strength. (b) and (e) $L_x \langle \delta \rangle$, a measure for the width of the domain wall. Increasing fields and current densities decrease the size of the domain wall (note the difference in scales). Current driven domain walls remain symmetric up to the Walker breakdown while field driven domain walls become more asymmetric with increasing field strengths. The full black line illustrates that the asymmetry scales linear with field at low field strengths with a slope of 0.352 nm/mT.

Eq. (3), while the datapoints depict the average domain wall velocity at $T = 100, 200$, and 300 K. The fact that all data coincide confirms that the drift velocity of the domain wall is unaffected by temperature.¹⁰

Additionally, at nonzero temperatures, the domain wall motion has a diffusion component. The MSD/t (corrected for the drift velocity) is shown as black triangles in Fig. 2(a), indicating that the adiabatic and non-adiabatic systems exhibit identical diffusion properties. Hence, we can conclude that the diffusion is solely determined by the domain wall shape and the temperature [cf. Eq. (7)] and is not affected by the drift velocity.

To summarize, we have investigated the influence of temperature on transverse domain wall dynamics in magnetic nanowires. Temperature is included in the micromagnetic simulations and the generalized 1D model as a randomly fluctuating field acting on the discretization cell and domain wall volume, respectively. In general, the domain wall motion contains a drift and a diffusion component. We verified that the drift velocity of the domain wall is unaffected by temperature and found that the domain wall diffusion gives rise to a mean square domain wall displacement which grows linearly with time. The diffusion is solely determined by temperature and the domain wall shape, which changes at higher currents. The domain wall drift and

diffusion do not influence each other and can be quantitatively predicted by a generalized 1D model. Further research may focus on thermally activated domain wall motion in disordered media, i.e., the creep regime.²⁴ At low driving forces, material defects or grain boundaries⁴ are potential wells²⁵ which act as pinning centers for the domain wall. The escape rate from these centers due to thermal fluctuations will determine the creep velocity of the domain wall through the nanowires.²⁶

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