Transmission Line Modeling of Coupled Lines in a Semiconducting Substrate

Daniël De Zutter and Thomas Demeester

Abstract—This paper briefly describes some aspects of transmission line modeling. In particular some issues related to the most general case of multiconductor lines on a semiconducting substrate are addressed, and a number of examples illustrate the possibilities of the developed model.

Index Terms—Transmission line modeling, semiconducting substrate, surface admittance, RLGC-parameters.

I. MULTICONDUCTOR LINES AND TRANSMISSION LINE ANALYSIS

The general problem of multiconductor lines embedded in (planar) layered substrates has been abundantly treated in literature and in a large number of books. The analysis starts from the transversal cross-section, usually taken to be the \((x, y)\)-plane, and the structure is supposed to remain invariant along the longitudinal signal propagation direction (the \(z\)-axis). Solving the source-free Maxwell’s equations under the above assumptions leads to an eigenmode analysis. In the sinusoidal regime the typical signal propagation dependency of each mode becomes \(\cos (\omega t - \beta z + \phi)\) for a signal propagating along the positive \(z\)-axis, with \(\omega = 2\pi f\) and \(f\) the frequency, and with \(\beta(\omega)\) the (in general complex) frequency-dependent wavenumber of one of the modes that can be sustained by the multiconductor lines. As noted before many different configurations have been studied in literature with layered, isotropic or (bi)anisotropic substrate materials, lossless and lossy dielectrics, semi-conductors etc. For additional information we refer the reader e.g. to [1]. Finding eigenmodes and wavenumbers (eigenvalues) is only one aspect of the problem. As the multiconductor lines are part of a complete interconnection (in a cable, on a board, in a package, on chip, ...) it is important to obtain an \(R, L, G, C\) description. These familiar resistance, inductance, conductance and capacitance matrices allow to describe the behaviour of the multiconductor system by the so-called Telegrapher Equations:

\[
\frac{\partial \mathbf{v}(\omega, z)}{\partial z} = -(j\omega \mathbf{C} + \mathbf{G}) \mathbf{v}(\omega, z) \quad (1)
\]

\[
\frac{\partial \mathbf{i}(\omega, z)}{\partial z} = -(j\omega \mathbf{L} + \mathbf{R}) \mathbf{i}(\omega, z) \quad (2)
\]

Here \(\mathbf{v}(\omega, z)\) and \(\mathbf{i}(\omega, z)\) are \(N \times 1\) column vectors formed by the voltages and total currents associated with each conductor (the complete system consist of \(N + 1\) conductors, one of which has been taken as the reference conductor). The whole discussion now boils down to the way in which physical (and/or mathematical) meaning can be given to these voltages and currents and to the circuit parameters and how to express them in terms of the fields. In literature several ways to deal with this problem have been explored and discussed. At low frequencies voltages and currents can be defined in a completely consistent way. When frequencies increase, however, this is no longer the case. Definitions can then be based on, e.g., (partial) power conservation, reciprocity conservation or causality conservation. In all those cases one can either choose the current as the quantity to which a clear physical meaning is given (as is most often the case for conductors), or the voltage (e.g. for slotline analysis). In general it is no longer possible to give a clear physical meaning to both voltage and current. Moreover, the circuit parameters, i.e. the transmission line equivalent, will differ when adopting the different techniques. In our presentation we will give some more details on this aspect of the problem.

II. MULTICONDUCTOR LINES IN SEMICONDUCTING SUBSTRATES

This particular problem is very challenging due to the large influence of the semi-conducting substrate on the circuit parameters when the frequency varies from DC to about 100 GHz, when considering the still relevant harmonics in the transmitted bits. Of special importance is a possible slow-wave phenomenon meaning that in a particular frequency band signals propagate (much) slower, leading to an important bit distortion. A thorough analysis of a single conductor line embedded in a semiconducting substrate and an overview of related work are presented in [2] revealing the presence of dielectric, slow-wave and skin-effect modes. In this presentation we will discuss how this analysis can be extended to the multiconductor case. The problem is basically tackled by replacing the considered configuration by an equivalent one consisting of well-chosen surface charges and surface currents. These charges and currents depend, respectively, on the electric potential and on the longitudinal electric field. These relationships are expressed using the surface admittance concepts first put forward in [3]. We will also give some details on this in the oral presentation with special attention devoted to the meaning of voltages, current and the RLGC-parameters. Because mode orthogonality plays a crucial role in the extension from the single conductor to the multiple conductor case, it turns out to be “natural” to use the reciprocity conservation principle to derive a transmission line equivalent. The surface admittance concepts also allow to extend the analysis to other configurations such as coated conductors (which are of importance for on chip interconnects), still retaining a very high precision when predicting the skin-effect and current crowding over a very large frequency band.

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Based on the above principles a code was implemented capable of handling a large variety of configurations, some examples of which are discussed below.

III. EXAMPLES

The following paragraphs with examples are merely excerpts from [4] and [5], where a more extensive discussion and additional examples can be found.

A. MIS Microstrip Line (Excerpt from [4])

As a first example, consider the open MIS microstrip line, shown on the inset of Fig. 1. The line consists of a \( h_1 = 500 \mu m \) thick lossy substrate with conductivity \( \sigma_1 \), separated from a thin signal conductor by a lossless dielectric layer with thickness \( h_2 = 135 \mu m \). All materials are non-magnetic (as will be the case in all the examples). Simulations were performed at 1 GHz, for increasing values of \( \sigma_1 \) such that the fundamental mode evolves from a dielectric mode, over the slow-wave range, to a skin effect mode. In [2], an infinitely thin and perfect electric conducting (PEC) signal line was used. Our simulation of the PEC line (full line in Fig. 1) yields identical results for the attenuation constant \( \alpha \). Yet, a small difference in the effective relative permittivity \( \varepsilon_{eff} \) is noticeable because the results shown here were obtained by leaving away the top side of the box surrounding the structure in [2], resulting in a small shift of the inductance and hence the observed difference in \( \varepsilon_{eff} \). The difference is very small though, and the large box described in [2] allows a good approximation of the open line structure. Simulating the signal line as a copper conductor (\( \sigma_{Cu} = 58 \ MS/m \)) of the same width and with finite thickness (\( w = 600 \mu m, t = 20 \mu m \)) only slightly affects the results, as shown in Fig. 1 (dash-dot lines).

As an illustration of the frequency-dependency, the same configuration was simulated both for the PEC and the copper signal line, at the frequencies 10 MHz and 1 GHz. Fig. 2(a) shows that the resulting \( \varepsilon_{eff} \)-values are very similar, be it that at 10 MHz the skin effect mode is never reached (current crowding would only occur for a loss tangent, higher than \( 10^5 \)). In both cases, there is virtually no difference in \( \varepsilon_{eff} \) between the copper and the PEC conductor. However, now consider an analogous configuration, but with a smaller conductor, closer to the substrate (\( h_2 = 13.5 \mu m, w = 60 \mu m, \) and \( t = 2 \mu m \) for the copper strip). The thickness of the lower substrate and the material parameters remain unchanged. Fig. 2(b) shows that at 1 GHz, the finite conductivity of the line still has no influence on the propagation constant. The main difference between both configurations at 1 GHz, is the different value of \( \varepsilon_{eff} \) in the slow-wave range of the fundamental mode. In Fig. 2(b), \( \varepsilon_{eff} \) at 1 GHz is higher than in Fig. 2(a), because the lower substrate has a higher internal inductance (as it was not scaled together with the line), whereas the capacitance remains unchanged as soon as \( \sigma_1 > \omega \varepsilon_1 \). At 10 MHz, \( \varepsilon_{eff} \) is much larger in Fig. 2(b) than in Fig. 2(a), although only for the copper conductor. By decreasing the dimensions of the line, the resistance \( R \) increases with respect to the inductance \( L \) and the point where \( R \approx \omega L \) shifts towards higher frequencies. At 10 MHz the line in Fig. 2(b) operates in the so-called RC-range (\( R > \omega L \)).

B. Coated Conductor (Excerpt from [5])

As a second example, the coated conductor shown in Fig. 3 is modeled. The circuit inductance and resistance p.u.l. presented in Fig. 4 show that the RL-behavior of the coated conductor is quite different from that of a homogeneous copper conductor with the same dimensions. For skin-effect frequencies, the resistance increases more rapidly for the
coated conductor, given that a gradually larger part of the current flows within the less conducting coating. A better approximation for the actual resistance is obtained by a homogeneous conductor with a resistivity $\rho_{\text{av}} = 2.06 \ \mu\Omega\text{cm}$, found as the weighted average of $\rho_{\text{Cu}}$ and $\rho_{\text{Cr}}$ such that the same DC-resistance as the coated conductor is obtained. With the developed techniques it is now possible to actually calculate $\rho_{\text{eff}}(f)$. Fig. 5 shows this effective value as a function of frequency normalized on $\rho_{\text{Cu}}$. For low frequencies $\rho_{\text{eff}}$ coincides with $\rho_{\text{av}}$ as defined above. The difference between $\rho_{\text{eff}}$ and $\rho_{\text{av}}$ at higher frequencies clearly illustrates the effect of the coating. The DC-value $\rho_{\text{av}}$ is a good approximation for $\rho_{\text{eff}}(f)$, up to the frequencies at which the skindepth $\sqrt{\rho_{\text{av}}/\pi\mu_0 f}$ becomes comparable with the coating thickness, which is far beyond the relevant frequencies in current chip technologies.

C. Multi-Conductor Line Structure (Excerpt from [4])

In the final example of Fig. 6, a transmission line system of 8 coupled lines is analyzed. The dimensions are based on a currently used semiconductor technology. Four identical pairs of conductors ($c_1$ to $c_8$) and a reference conductor ($c_R$), all with conductivity $\sigma_{\text{sig}} = 40 \ MS/m$, are embedded in a dielectric layer above a thick semiconducting substrate, on top of a PEC plane. The substrate conductivity $\sigma_{\text{sub}} = 2 \ S/m$, unless indicated differently (as for Fig. 8 and 9). Locally (underneath $c_1$ and $c_2$), the substrate has been heavily doped ($\sigma_{\text{dep}} = 0.03 \ MS/m$). Permittivities are $\epsilon_{\text{sil}} = 4\epsilon_0$ and $\epsilon_{\text{sub}} = 12\epsilon_0$. The dimensions, in micrometers, are indicated in the cross-section (not shown in proportion). The structure is enclosed between two PEC ‘mirror’ walls at the left and right side, in order to imitate a wide slab (as was done in [2] as well).

In Fig. 7, the modal voltages on each signal conductor are presented for the 8 fundamental modes, at a frequency of 10 GHz and for a substrate conductivity $\sigma_{\text{sub}} = 2 \ S/m$. For a clear graphical presentation of the modes, each normalized modal voltage $V$ is presented with a modified amplitude $V_0 = |V| \cdot \text{sign}(\text{Re}(V))$ and a phase $\phi$, such that $V = V_0 e^{-j\phi}$. The modes fall apart into two groups: the modes ($m_1$-$m_4$), in which both conductors of each pair have more or less the same excitation, and those ($m_5$-$m_8$), with an opposite excitation of both conductors of each pair. In the next paragraphs, they are resp. called the even and the odd modes.

The behavior of the SWF and attenuation as a function of the substrate loss factor $\sigma_{\text{sub}}/\omega \epsilon_{\text{sub}}$ is shown in Fig. 8. There is a large difference in the behavior of the even and the odd modes, which can be explained as follows. Each pair can roughly be approximated as a symmetric line pair in its own respect. For such a line, the two fundamental modes $\beta_{\text{even}}$ and
in which the indices $V$ to $m$ conductor pairs (mode bar. The modes can be split up into a quasi ‘even’ excitation of each of the elements of the complex matrices $\tilde{C}$ and $\tilde{L}$, associated with the conductor pair $c_3$-$c_4$. The elements $C_{33}$ and $C_{44}$ correspond to $C_s$ from (3) (the small difference between them due to

$$\beta_{\text{even}}^2 = - \left( j\omega (C_s - |C_m|) + (G_s + G_m) \right) \cdot \left( j\omega (L_s + L_m) + (R_s + R_m) \right)$$

$$\beta_{\text{odd}}^2 = - \left( j\omega (C_s + |C_m|) + (G_s - G_m) \right) \cdot \left( j\omega (L_s - L_m) + (R_s - R_m) \right)$$

(3)

in which the indices $s$ and $m$ denote the diagonal, resp. the off-diagonal elements from the $(2 \times 2)$ circuit matrices associated with each line pair. As an illustration, Fig. 9 shows the relevant elements of the complex matrices $C$ and $L$, associated with the conductor pair $c_3$-$c_4$. The elements $C_{33}$ and $C_{44}$ correspond to $C_s$ from (3) (the small difference between them due to

the actual non-symmetry of the configuration), whereas $C_{34}$ corresponds to $C_m$. From Fig. 9(a), it becomes clear that the factor $(j\omega C_s + G_s)$ in (3) is dominated by the capacitance term and the numerical results confirm this is also the case for the other line pairs. For all line pairs, both $C_s$ (positive) and $C_m$ (negative) are influenced in the same way by the charge on the substrate’s surface. As soon as the loss tangent becomes larger than about 1, an increasing $\sigma_{\text{sub}}$ increases $C_s$, whereas $|C_m|$ decreases. For the odd mode, depending on $C_s + |C_m|$, the influence of $\sigma_{\text{sub}}$ is cancelled out, whereas it is reinforced for the even mode. This explains the flat behavior of the SWF in Fig. 8(a) around $\sigma_{\text{sub}}/\omega \epsilon_{\text{sub}} \approx 1$ for the odd modes, and the increase for the even modes. This effect is not very pronounced for mode $m_{14}$, although an ‘even’ mode. This is due to the reference conductor, shielding $c_5$ and $c_6$ from the substrate. The reason why $m_3$ has the highest and $m_1$ the lowest SWF from the even modes, is readily explained as well. The influence of the reference conductor and the doped part ($\sigma_{\text{doped}}$) in the substrate lead to higher capacitance elements associated with the conductors on the left ($c_1$, $c_2$, $c_5$, and $c_6$, strongly excited in mode $m_3$) than for those on the right (excited in mode $m_{14}$). At higher values of $\sigma_{\text{sub}}$, when the magnetic field can no longer fully penetrate the substrate, an analogue argumentation based on the inductance and resistance coefficients, explains the different $\sigma_{\text{sub}}$-dependence of the odd and the even modes.

Fig. 8 also shows that the overall odd mode SWF is higher than for the even modes. As both conductors of each pair are close to one another, $|C_m|$ is of the same order of magnitude as $C_s$ and $(C_s + |C_m|)$ is hence considerably larger than

Fig. 7. Modal voltages for the configuration of Fig. 6, at 10 GHz, and with $\sigma_{\text{sub}} = 2 \times 10^{-3}$. The complex modal voltages $V$ are found as $V = V_0 e^{-j\phi}$, with $V_0$ shown as bars and $\phi$ (expressed in degrees) at the corresponding bar. The modes can be split up into a quasi ‘even’ excitation of each of the conductor pairs (mode $m_2$ to $m_4$), and a quasi ‘odd’ excitation (mode $m_5$ to $m_8$). For each mode, the SWF is indicated as well.

\[ \beta_{\text{odd}} = \frac{V_{\text{odd}}}{V_{\text{even}}} \]

\[ \beta_{\text{even}} = \frac{V_{\text{even}}}{V_{\text{odd}}} \]

\[ \beta_{\text{odd}}^2 = - j\omega \left( C_s - |C_m| \right) \left( L_s + L_m \right) + \left( R_s + R_m \right) \]

\[ \beta_{\text{even}}^2 = - j\omega \left( C_s - |C_m| \right) \left( L_s + L_m \right) + \left( R_s + R_m \right) \]

\[ \beta_{\text{odd}} = \frac{V_{\text{odd}}}{V_{\text{even}}} \]

\[ \beta_{\text{even}} = \frac{V_{\text{even}}}{V_{\text{odd}}} \]

\[ \beta_{\text{odd}}^2 = - j\omega \left( C_s - |C_m| \right) \left( L_s + L_m \right) + \left( R_s + R_m \right) \]

\[ \beta_{\text{even}}^2 = - j\omega \left( C_s + |C_m| \right) \left( L_s + L_m \right) + \left( R_s + R_m \right) \]

\[ \beta_{\text{odd}} = \frac{V_{\text{odd}}}{V_{\text{even}}} \]

\[ \beta_{\text{even}} = \frac{V_{\text{even}}}{V_{\text{odd}}} \]

\[ \beta_{\text{odd}}^2 = - j\omega \left( C_s - |C_m| \right) \left( L_s + L_m \right) + \left( R_s + R_m \right) \]

\[ \beta_{\text{even}}^2 = - j\omega \left( C_s + |C_m| \right) \left( L_s + L_m \right) + \left( R_s + R_m \right) \]
Re than currents, opposite in both conductors of each pair, tend to repel.

The main effect is here that the odd modes yield a difference in the SWF of the even and odd modes. The dotted line denotes (a) Entries of $\text{Re}(\tilde{C}) = C$, compared to $-\text{Im}(\tilde{C}) = G/j\omega$ (scaled by a factor 10 for clarity), and (b) elements of $\text{Re}(\tilde{L}) = L$, compared to $-\text{Im}(\tilde{L}) = R/j\omega$.

The conductance coefficients $G_{33}$, $G_{34}$ and $G_{44}$ shown on Fig. 9(a), clearly demonstrate the semiconductor’s behavior. For a very low conductivity $\sigma_{\text{sub}}$, $\varepsilon_t$ must be taken into account but the transverse currents $\sigma_{\text{sub}}\varepsilon_t$ are still negligible. When $\sigma_{\text{sub}}$ increases, the transverse currents and the $G$-values also increase. However, as soon as the loss tangent becomes considerably higher than 1, charge relaxation reduces $\varepsilon_t$. This effect outweighs the increase in $\sigma_{\text{sub}}$ and hence the $G$-values decrease again and in the end become negligible as soon as the substrate’s surface potential has become constant.

Finally, the frequency-dependency of the SWF is shown in Fig. 10, for a substrate conductivity $\sigma_{\text{sub}} = 2 S/m$. The transition from the $RC$- to the $LC$-range takes place around 20 GHz. This frequency is quite high because the conductors’ cross-sections are small, resulting in large $R$-values.

**REFERENCES**


**Daniël De Zutter** was born in 1953. He received his M.Sc. Degree in electrical engineering, Ph.D. degree, and a thesis leading to a degree equivalent to the French Aggrégation or the German Habilitation from Ghent University, Ghent, Belgium, in 1976, 1981, and 1984, respectively. From 1984 to 1996 he was with the National Fund for Scientific Research of Belgium. He is currently a Full Professor of electromagnetics with Ghent University. Most of his earlier scientific work concerned the electrodynamics of moving media. His research currently focuses on all aspects of circuit and EM modeling of high-speed and high-frequency interconnections and packaging, EM compatibility, and numerical solutions of Maxwell’s equations. He has authored or coauthored over 140 international journal papers and 150 papers in conference proceedings.

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PROGRAM VI EIEC

INVITED TALKS

As in the previous issue, Aiguablava (Girona), the "VI Iberian Meeting on Computational Electromagnetics" will include a number of invited talks given by prestigious international experts. At the moment, the following invited speakers have confirmed their contributions:

- **Prof. Dr. Daniël De Zutter**, Electromagnetics Group, Faculty of Engineering, Universiteit Gent (Belgium). Dean of the Faculty of Engineering
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