1. Introduction

Quantitative microwave imaging is an example of an electromagnetic inverse scattering problem where one tries to reconstruct the complex permittivity of an inhomogeneous scatterer from a multi-view and/or a multi-frequency scattering experiment with microwaves. Since microwaves easily penetrate the human body and are less harmful than the ionizing X-rays, it is tempting to consider their application to breast cancer imaging, for the difference between the permittivities of the various tissues appears to be large. Moreover, new imaging techniques which are complementary to the existing modalities might help to reduce the large number of false positive and false negative diagnoses that still plague breast cancer screening. However, microwaves have a much larger wavelength than X-rays and the need to obtain sub-wavelength resolution renders the inverse scattering problem extremely ill-posed. On top of this, the problem is non-linear. Due to these difficulties microwave breast imaging is a challenging problem. In this contribution we discuss the implementation of a single-frequency quantitative microwave imaging algorithm and a preliminary study of its application to breast imaging.

2. A Microwave Imaging Algorithm

For simplicity we consider only inhomogeneous dielectric objects in a homogeneous background that extends to infinity. Consider a cuboidal investigation domain $D$ that contains the scatterers. We want to reconstruct the complex permittivity $\varepsilon(r)$ for points $r$ inside $D$ by illuminating $D$ with a number of time-harmonic electric dipole fields $E_i^0$ (in $r$, with orientation $u_i$) and by measuring for each such illumination the $u_r$-component of the scattered electric field $E_i^s(r_r)$ in the points $r_r$ (see Figure 1). To do this, a model $E_i^s(r,\varepsilon)$ (non-linear in $\varepsilon$) of the scattered electric field is fitted against the measured data, where the model parameters $\varepsilon_n$ are the values of the complex permittivity in the cells $n$ of a grid that covers $D$. This model can be evaluated for a given permittivity profile $\varepsilon$ by solving the contrast source volume integral equation in the domain $D$. A fast FFT-based solver was developed to this end [1].
To determine the model parameters, i.e. the permittivity in every cell of the discretization grid, the normalized least squares data fit function could be minimized
\[ F^{LS}(\varepsilon) = \frac{\| e^{sim}(\varepsilon) - e^{meas} \|^2}{\| e^{meas} \|^2}, \]
where the vectors \( e^{meas} \) and \( e^{sim}(\varepsilon) \) contain the measured and simulated data respectively. However, the minimum of this cost function is not well defined. Technical limitations often restrict the number of antennas that can be used to sample the scattered fields. This often results in an under-determined system, since a fine permittivity grid is needed to obtain a reasonable spatial resolution in the image. Apart from non-uniqueness that may result from this, there is a stability issue: noise on the data typically results in amplified noise on the reconstructions.

To reduce the degrees of freedom in the model and to mitigate noise amplification, a regularization strategy has to be applied. Therefore, we added a regularization term to the cost function yielding
\[ F(\varepsilon) = F^{LS}(\varepsilon)[1 + \alpha F^{S}(\varepsilon)], \]
where \( F^{S} \) is a smoothing function that penalizes non-smooth permittivity profiles. Note that the smoothing term has a weight in the cost function that is proportional to the least squares data fit. This allows for a controlled minimization of \( F^{LS} \), where the regularization starts as a severe constraint and is gradually relaxed, thereby allowing for more and more detail to be added in the reconstruction, while avoiding the heavily oscillating behavior that results from noise amplification.

The minimization itself is performed by means of a modified Gauss-Newton method [1], which converges rapidly.

3. Application to Breast Imaging

As a first feasibility study for breast imaging, we considered the numerical breast phantom of Figure 2. It is a coarse grid approximation to an MRI-based numerical breast phantom which we adopted from the recently founded online repository of the University of Wisconsin. We added an artificial tumor to this phantom and immersed it in a homogeneous background with permittivity \( \varepsilon = 10 - 2i \). The dipole sources and receiver locations are distributed around the breast as in Figure 1. The arrows indicate the dipole directions \( \mathbf{u} \) and \( \mathbf{u}^r \). 48 of these dipole positions are used for the transmitting dipoles and for every illumination the scattered field is measured in all 84 depicted positions along both dipole orientations. The data are simulated at an operating frequency of 2 GHz and afterwards corrupted by gaussian noise (SNR = 30 dB). Since
the number of data points is 8064 and the number of optimization variables is 15750, the problem is heavily underdetermined and the regularization is really necessary to make the problem solvable.

The optimization, which started from an empty domain $D$ (only background medium) and hence used no a priori information, ended after 8 Gauss-Newton iterations and the result is shown in Figure 1. It is clear that the reconstruction is a smoothed version of the real profile, but the tumor and the other inhomogeneities in the breast can be identified. However, the high permittivity values are not very well reconstructed. Simulations at higher frequencies presently consume too much memory to be run on our computer infrastructure.

4. Conclusion

We presented a promising preliminary result for microwave breast cancer imaging using a regularized Gauss-Newton inversion scheme. We expect improvements from tailoring the regularization to this specific application. Challenges are the inclusion of a more realistic scenario for body and antennas into the model.

![Image](image.png)

Figure 2: The permittivity in a cross section of the breast phantom. Top row: real part (left) and imaginary part (right) of the actual permittivity. Bottom row: real part (left) and imaginary part (right) of the reconstructed permittivity.

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URSI FORUM 2008
CROSS-BORDER RADIO SCIENCE

Brussels, 30 May 2008

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