Compressive Membrane Action in FRP-strengthened Concrete Beams

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Summary

A comparison of compressive membrane action (CMA) between standard reinforced concrete (RC) members and RC members strengthened with Fibre Reinforced Polymer (FRP) is performed. The comparison is based on an extension of Park and Gamble's model and quantified by defining an enhancement factor. Results indicate that the CMA effect decreases with increasing steel reinforcement for RC beams whereas this conclusion only holds for the effect of top steel reinforcement in case of FRP-strengthened RC beams. Results also show that the CMA effect increases with increasing concrete strength and is not significantly affected by span-to-depth ratio for both beam systems.

Keywords: Compressive membrane action; reinforced concrete; fibre reinforced polymers.

1. Introduction

Since compressive membrane action (CMA) has been recognized for laterally restrained members, research results have shown that CMA is beneficial in strength enhancement. With regard to the investigation of CMA in concrete members, a commonly applied proposed by Park and Gamble [1], is used here using plastic theory to consider CMA by performing analysis of strain compatibility and force equilibrium. With the increased application of Fibre Reinforced Polymer (FRP) to strengthen concrete structures, it is desirable to conduct a comparison of CMA between standard concrete members and FRP-strengthened concrete members.

The aim of this study is to investigate the differences of CMA between standard reinforced concrete (RC) members and FRP-strengthened RC members

2. Extension of Park and Gamble's model

2.1 Assumptions

According to Park and Gamble's method, a model with four idealized plastic hinges formed symmetrically along the beam is chosen to represent a standard RC beam or a FRP-strengthened RC beam. A perfectly rigid plastic mechanism is basically assumed. A complete symmetry along the span is assumed with respect to geometry, reinforcement, loading, boundary conditions and deformations. The lateral restraints are idealized to be equivalent axial springs

Fig. 1: Schematic view of a laterally restrained

with stiffness K_a . Fig. 1 shows the schematic view of the model, where β is the ratio of the span length l_n from the plastic hinge at the beam end to the nearest hinge in the span to the beam span l . The widely-adopted assumptions such as the assumptions of plane-section, a full composite action of FRP and an equivalent rectangular stress block for compressive concrete are considered to perform the analysis of compatibility and force equilibrium [2]. Note that the full composite action of FRP means that a failure occurs when the tensile steel reinforcement has yielded followed by concrete crushing whereas the FRP remains intact. Further, the axial force at the beam end (N_{u1}) is assumed to be equal to the axial force in the span (N_{u0}), as show in Eq. (1)

$$N = N_{u0} = N_{u1} \quad (1)$$

Besides, the compressive strain of the beam is assumed to be uniformly distributed along the beam length with a value of $\varepsilon = N/(E_c A_c)$, where N is the axial force, E_c is elastic modulus of concrete and A_c is the cross-sectional area.

2.2 Model prediction and verification

Fig. 2 shows one-half of the deformed shape. The axial force along the beam causes a horizontal displacement at each support, $t = N/K_a$. Due to the uniform compression strain along the beam, the contraction of the portion in Fig. 2 is $\varepsilon\beta l$ and the contraction of the middle portion is $(1-2\beta)\varepsilon l$. For a given beam deflection δ , we have $\tan\theta \approx \delta/\beta l$ assuming that the rotation at the beam end θ is so small that $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ hold. Recalling the expressions of the beam strain ε and lateral restraint deformation t , the compatibility requirement can be expressed as in Eq. (2)

$$h - x_0 - x_1 = \frac{\delta}{2} + \frac{N\beta l^2}{2\delta} \left(\frac{1}{E_c A_c} + \frac{2}{K_a l} \right) \quad (2)$$

where x_0 and x_1 are the neutral-axis depth in the span and at the beam ends, respectively; h is the beam depth. According to Fig. 2, Eq. (1) can be rewritten as

$$C_{c0} + C_{s0} - T_0 - F_0 = C_{c1} + C_{s1} - T_1 - F_1 \quad (3)$$

where C_{c0} and C_{c1} are the concrete compressive forces, C_{s0} and C_{s1} the steel compressive forces, T_0 and T_1 the steel tensile forces, and F_0 and F_1 the FRP tensile forces, acting on sections at the mid-span and the beam end, respectively.

Considering that all terms in Eq. (3) can be expressed by given geometric and material properties and the unknowns x_1 and x_0 based on a sectional force equilibrium analysis, x_0 and x_1 are obtained by solving the Eqs. (2) and (3) simultaneously for a given δ . Further, the moments at the midspan and the beam ends can be calculated and the load bearing capacity can be calculated. When repeating such procedures for different values of δ , a series of the load bearing capacities are obtained, and the maximum value can be seen as the ultimate load bearing capacity.

Note that, according to the assumed failure mode, the concrete compressive strain reaches to its

Fig. 2 Idealized deformation of one-half of the restrained beam

ultimate strain, the steel tensile strain exceeds its yield strain, and the strain of FRP is less than its ultimate strain. In this paper, the ultimate concrete strain is 3.5‰ [2], the steel yield strain is calculated by its yield stress, and the ultimate strain of FRP is based test result. In addition, the lateral stiffness is calculated based on the adjacent structural elements.

To verify the extensive model, a standard RC beam (A1) from [3] and a FRP-strengthened RC beam (FR-1) from [4] were selected, each of which can be idealized as the model shown in Fig. 1. The results show that the load bearing capacity is 181.8 kN and 146.6 kN for A1 and FR-1, respectively. This yields an acceptable 8% and 1% overestimation compared to the tested results of 168.0 kN and 145.5 kN, for A1 and FR-1, respectively, which validates the extended model for FRP-strengthened RC beams.

3. Comparison of CMA between standard RC beams and FRP-strengthened RC beams

Considering that CMA is affected by many influencing factors, it is necessary to select a dimensionless quantity to carry out the comparison of CMA between standard RC beams and FRP-strengthened RC beams. Therefore, a strength enhancement factor α_p is adopted and defined as $\alpha_{p,CMA,RC} = P_{CMA,RC}/P_0$ and $\alpha_{p,CMA,FRP} = P_{CMA,FRP}/P_{0,FRP}$, where $\alpha_{p,CMA,RC}$ and $\alpha_{p,CMA,FRP}$ are enhancement factors quantifying the CMA effect, $P_{CMA,RC}$ and $P_{CMA,FRP}$ are the beam peak resistances considering CMA calculated by the extensive method, P_0 and $P_{0,FRP}$ are the beam peak resistances calculated by *fib* bulletin 14 [2], each time for standard RC beams and FRP-strengthened RC beams, respectively.

A beam similar to the four-point loaded two-span beam in [5] is adopted as the benchmark beam in the parametric study. This 10 m beam is assumed to be laterally restrained ($K_a = 10^6$ kN/m) at both ends with a 200×400 mm section and a value of 0.20 for β is selected. C30/37 concrete is used and three steel bars are initially placed along the beam length at both the beam top ($2\phi 12 + 1\phi 18$, $A_{top} = 480.4$ mm²) and the beam bottom ($2\phi 12 + 1\phi 20$, $A_{bottom} = 540.1$ mm²). The elastic modulus and the yield stress of the steel bars are 200 GPa and 500 MPa, respectively. In cases of FRP strengthening, a CFRP layer with a section area of 120 mm² is applied along the beam at tension zones and the elastic modulus and the ultimate strain of the FRP are considered 190 GPa and 1.5%, respectively. All these values apply if no further information is indicated.

3.1 Effect of steel reinforcement

Considering the steel reinforcements at both top and bottom of the beam may have different effects on CMA, the effects of the steel reinforcement at the top and at the bottom are investigated separately, which is performed by consecutively fixing the area of steel bars at the bottom or that at the top. Fig. 3 shows the results of the effect of steel reinforcement on CMA for RC beams and FRP-strengthened RC beams.

A first conclusion from Fig. 3 is that that CMA is very significant in improving the beam strength. Taking the standard RC beam for example, a very significant strength increase of 40% can be obtained, compared to the benchmark beam, when CMA is taken into account. Fig. 3 also shows both the increase of the top steel bars and that of the bottom steel bars contribute to a linear decrease of the enhancement factor for standard RC beams, which implies that, in the general cases of CMA for standard RC beams, the more steel reinforcement is placed, the less the load bearing capacity is affected by CMA. However, for FRP-strengthened RC beams the increase of bottom steel reinforcement contributes to an increase of the enhancement factor. Note that, when the steel reinforcement increases, the decrease of the enhancement factor does not mean that the beam resistance decreases but means the increase rate of the beam resistance due to CMA is smaller than the increase rate of the beam resistance due to the increasing steel reinforcement.

As shown in Fig. 3, another difference of CMA between these two beam systems can be seen: the

influence of steel reinforcement on the beam resistance capacity under CMA in FRP-strengthened RC beams is less significant than that in standard RC beams. For instance, in cases where the top steel reinforcement is 0.4 times the A_{top} , the enhancement factor are 1.59 and 1.14 for the standard RC beam and the FRP-strengthened RC beam, respectively, which implies a 39% more enhancement due to CMA is obtained for the standard RC beam than the FRP-strengthened RC beam.

Fig. 3 Effect of steel reinforcement

Also, the decrease of the enhancement factor due to the increase of top steel reinforcement in standard RC beams is more significant than in FRP-strengthened RC beams given that the slope of $\alpha_{p,CMA,RC}$ is smaller than that of $\alpha_{p,CMA,FRP}$.

3.2 Effect of concrete strength and span-to-depth ratio

The comparison of the CMA effect between conventional RC beams and FRP-strengthened RC beams was further conducted by investigating the effects of concrete strength and span-to-depth ratio on CMA, as shown in Fig. 4 and Fig. 5, respectively. In Fig. 4, it can be seen that the enhancement factor increases with increasing concrete strength ($f_{ck} \leq 50$ MPa) for both standard RC beams and FRP-strengthened RC beams. Also, both the effect of CMA on the enhancement of the beam strength and the effectiveness of the effect of the increasing concrete strength in enhancing the beam strength in FRP-strengthened RC beams is less significant than that in standard RC beams. Fig. 5 shows that the span-to-depth ratio does not influence the enhancement factor for both standard RC beams and FRP-strengthened RC beams. This means the load bearing capacity considering CMA decreases to the same relative amount as the decrease of the load bearing capacity without the consideration of CMA when the span-to-depth ratio increases.

Fig. 4 Effect of concrete strength

Fig. 5 Effect of span-to-depth ratio

4. Conclusion

Park and Gamble's method to investigate the compressive membrane action (CMA) was first extended and verified to enable the analysis of CMA in FRP-strengthened RC members. Comparisons of the CMA effect on load bearing capacity between standard RC beams and FRP-strengthened RC beams were then conducted by defining a dimensionless enhancement factor as the ratio of the beam resistance considering CMA to the beam resistance without the consideration of CMA. Results indicate that the CMA effect decreases with increasing steel reinforcement for RC beams whereas this conclusion in case of FRP-strengthened RC beams only holds for the effect of top steel reinforcement on CMA. An interesting finding is that, the decrease of the enhancement factor due to the decrease of top steel reinforcement in standard RC beams is more significant than in FRP-strengthened RC beams. Results also show that the CMA effect increases with increasing concrete strength and is not significantly affected by span-to-depth ratio for both the RC and FRP-strengthened RC beam systems.

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