Charge shielding in magnetized plasmas

Shaojie Wang,¹ Ulrich Stroth,² and Guido Van Oost³

¹Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China and Center for Magnetic Fusion Theory, Chinese Academy of Sciences, Hefei, Anhui 230031, China

²Institut für Plasmaforschung, Universität Stuttgart, D-70569 Stuttgart, Germany

³Department of Applied Physics, Ghent University, B-9000 Gent, Belgium

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The shielding of a charge sheet in a magnetized plasma is investigated by taking account of the diamagnetic drift start-up current in addition to the polarization current. For a charge sheet with an infinitesimal width, the shielding is the same as the conventional Debye shielding if the charge sheet is perpendicular to the magnetic field; the shielding length is $\sqrt{2}$ times larger than the conventional one if the charge sheet is parallel to the magnetic field. When the scale length of the charge sheet is comparable or smaller than the ion Larmor radius, the electric field is significantly enhanced within the charge sheet, while far away from the charge sheet, the electric field is shielded to the usual $1/\epsilon_r$ level (where ϵ_r is the diamagnetic coefficient of the magnetized plasma). © 2010 American Institute of Physics. [doi:10.1063/1.3497274]

I. INTRODUCTION

Shielding of the electric fields generated by free charge in plasmas is of fundamental importance in plasma physics. When a free charge is introduced into a plasma, charged particles respond to the electric field generated by the free charge, and this leads to the result that the electric field is determined by the dynamic response of the plasma to the electric field applied externally. For a nonmagnetized plasma, the plasma equilibrium is simply the balance of the force due to the electric field and the force due to the plasma pressure gradient; this results to the well-known Debye shielding.¹ However, in a magnetized plasma, due to the Lorentz force applied by the magnetic field, the perpendicular dynamics is different from the nonmagnetized plasma, while the parallel dynamics is the same as the nonmagnetized plasmas. Therefore, it is of interest to investigate the shielding of the perpendicular (with respect to the magnetic field) electric field generated by the free charge.

Recently, it was found that the polarization drift is significant in affecting the shielding of a point charge in a magnetized dusty plasma;² various effects on the Debye shielding problem have been actively investigated.^{3–7} In Ref. 3, Debye shielding in strongly coupled plasmas is investigated; in Refs. 4 and 5, dynamic shielding effects are investigated; and in Refs. 6 and 7, plasma sheaths in magnetized plasmas are investigated.

It is also of interest to understand the perpendicular electric field generated by the free charge in magnetic confinement plasmas, especially for the bias experiment where the free charge was introduced to the magnetized plasma by an electrode inserted into the plasma.⁸ Although various theories and modelings have been carried out for understanding the perpendicular electric fields generated in magnetic confinement plasmas,^{9–12} it seems that these researches are still not conclusive. In addition to the well-known polarization current, various currents, including ion orbit loss current,⁹ turbulent Reynolds stress effects,¹⁰ and the collisional vis-

cosity effects,¹¹ have been proposed. However, it appears that these currents do not result to perpendicular electric field consistent with the ion radial force balance equation,¹² which has been confirmed by experiments.

In this paper, we will investigate the shielding of a charge sheet in a magnetized plasma by taking into account of the polarization current and the diamagnetic-drift start-up (DS) current. The remaining part of this paper is organized as follows. In Sec. II, we present the theoretic model; in Sec. III, we discuss the electric field generated by a charge sheet with an infinitesimal width; in Sec. IV, we discuss the electric field generated by a charge sheet in Gaussian distribution with different width; and in Sec. V, we present the concluding remarks.

II. THEORETIC MODEL

We begin with the time-derivative of Poisson's equation

$$\nabla \cdot \partial_t \mathbf{E} = \frac{1}{\epsilon_0} \partial_t \rho_{\text{ex}} + \frac{1}{\epsilon_0} q_i \partial_t n_i, \quad (1)$$

and the ion density continuity equation

$$\partial_t n_i + \nabla \cdot n_i \frac{m_i}{q_i B^2} \partial_t \left(\mathbf{E} + \frac{-\nabla p_i}{n_i q_i} \right) = 0. \quad (2)$$

Here, ρ_{ex} is the free charge generated by externally applied charge; q_i , n_i , and m_i are the ion charge, density, and mass, respectively; and $p_i = n_i T_i$ is the ion pressure, with T_i being the ion temperature. Note that the time change of the electron density is ignored in Eq. (1) due to the relatively small electron mass. In Eq. (2), the contribution from $\mathbf{E}$ is the well-known polarization current; the contribution from the ion pressure gradient is the DS current. The DS current is easily understood by recognizing the fact that the ion pressure gradient divided by the ion charge density exerts a force on the ions in a way similar to the electric field.

For a typical magnetized plasma, the free charge density is smaller than the ion charge density. Therefore, we can write $n_i = n_0 + \delta n_i$ and find the linearized basic equations,

$$\nabla \cdot \partial_t \mathbf{E} = \frac{1}{\varepsilon_0} \partial_t \rho_{\text{ex}} + \frac{1}{\varepsilon_0} q_i \partial_t \delta n_i, \quad (3)$$

$$\begin{aligned} \partial_t \delta n_i + \nabla \cdot \left(\frac{n_0}{\Omega_i B} \partial_t \mathbf{E} - \nabla^2 \left(\frac{T_i}{m_i \Omega_i^2} \partial_t \delta n_i \right) \right) \\ = - \nabla \cdot \left(\frac{\nabla n_0 T_i}{n_0} \frac{1}{m_i \Omega_i^2} \partial_t \delta n_i \right), \end{aligned} \quad (4)$$

where Ω_i is the ion gyrofrequency.

For mathematical simplicity, we consider a homogeneous plasma embedded in a homogeneous magnetic field. Equations (3) and (4) can be simply reduced to

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \rho_{\text{ex}} + \frac{1}{\varepsilon_0} q_i \delta n_i, \quad (5)$$

$$\varepsilon_r \delta n_i - \frac{1}{2} \rho_i^2 \nabla^2 \delta n_i = -(\varepsilon_r - 1) \frac{1}{q_i} \rho_{\text{ex}}, \quad (6)$$

where $\varepsilon_r = 1 + n_i m_i / \varepsilon_0 B^2$ is the usual dielectric coefficient of the magnetized plasma and $\rho_i = \sqrt{2 m_i T_i / q_i B}$ is the ion Larmor radius. Equations (5) and (6) will be used in the following to discuss the charge shielding problems.

III. ELECTRIC FIELD GENERATED BY A CHARGE SHEET WITH INFINITESIMAL WIDTH

In this section, we take a slab model. We assume the magnetic field is in Z direction and the external charge is homogeneous in the Y and Z directions. The initial plasma is homogeneous and infinite. Consider a charge sheet with infinitesimal small width,

$$\rho_{\text{ex}} = \sigma \delta \left(\frac{X}{\rho_i} \right), \quad (7)$$

where σ is a constant denoting the surface charge density.

Normalization is introduced with $x = X / \rho_i$. Equations (5) and (6) are reduced to

$$\frac{d}{dx} E = \frac{\rho_i}{\varepsilon_0} \rho_{\text{ex}} + \frac{\rho_i}{\varepsilon_0} q_i \delta n_i, \quad (8)$$

$$\varepsilon_r \delta n_i - \frac{1}{2} \frac{d^2}{dx^2} \delta n_i = -(\varepsilon_r - 1) \frac{1}{q_i} \frac{\sigma}{\rho_i} \delta(x). \quad (9)$$

The solution to Eq. (9) is found as

$$\delta n_i = - \frac{\varepsilon_r - 1}{\sqrt{2 \varepsilon_r}} \frac{\sigma}{\rho_i q_i} \exp[-\sqrt{2 \varepsilon_r} x^2]. \quad (10)$$

Substituting it into Eq. (8), we found the electric field

$$E = E_p + E_{\text{DS}}, \quad (11a)$$

$$E_p = \text{sgn}(x) \frac{\sigma}{2 \varepsilon_0 \varepsilon_r}, \quad (11b)$$

$$\begin{aligned} E_{\text{DS}} &= \text{sgn}(x) \frac{\sigma}{2 \varepsilon_0} \frac{\varepsilon_r - 1}{\varepsilon_r} \exp[-\sqrt{2 \varepsilon_r} x^2] \\ &= \text{sgn}(X) \frac{\sigma}{2 \varepsilon_0} \exp[-\sqrt{X^2 / \lambda_D}]. \end{aligned} \quad (11c)$$

Clearly, Eq. (11b) is the result of polarization current without the DS current. Equation (11c) is the effect of the DS current.

E_{DS} decays from its maximal equal to the electric field in a free space; after a distance $\lambda_D = (\varepsilon_0 T_i / n_i q_i^2)^{1/2}$, it decays to $1/e$ of its peak, and after a distance $\lambda_e = \log(\varepsilon_r) \lambda_D$, it decays to $1/\varepsilon_r$ of its peak. $\lambda_e = 0.065 \rho_i$ for $\varepsilon_r = 10^4$.

Clearly, there are two scale lengths here; in $\sim \lambda_D$, the perpendicular electric field decays to $1/e$ of its peak, and in $\sim \lambda_e = \log(\varepsilon_r) \lambda_D \sim 10 \lambda_D$, it decays to $1/\varepsilon_r$ of its peak. Note that this is consistent with the results in Ref. 6, which pointed out that in a magnetized plasma, the perpendicular sheath length has two scales: one is $\sim \lambda_D$ and the other is $\sim 10 \lambda_D$.

For a charge sheet perpendicular to the magnetic field, the electric field is parallel to the magnetic field; the conventional Debye shielding is given by¹

$$E = \text{sgn}(Z) \frac{\sigma}{2 \varepsilon_0} \exp[-\sqrt{Z^2 / (\lambda_D / \sqrt{2})}],$$

with $T_i = T_e$ and $q_i = 1$.

Clearly, the perpendicular shielding length is $\sqrt{2}$ times the conventional parallel shielding length; this is consistent with the results in Ref. 7, which pointed out that the sheath length in a magnetized plasma is larger in the perpendicular direction than in the parallel direction.

To end this section, we briefly summarize the underlying physical mechanism of the Debye shielding in a magnetized plasma. The anisotropic shielding is due to the difference between the perpendicular dynamics and the parallel dynamics, as is discussed in Sec. I. The two scales of the perpendicular electric field are determined by the two different components of the perpendicular currents: the DS current due to the time varying of the diamagnetic drift determines the small scale $\sim \lambda_D$ [see Eq. (11c)], the usual polarization current due to the time varying of the $E \times B$ drift sets the asymptotic value of the perpendicular electric field [see Eq. (11b)] in a large scale $\lambda_e = \log(\varepsilon_r) \lambda_D$ determined by the DS current; it is not surprising that the asymptotic value of the perpendicular electric field introduces the usual perpendicular dielectric constant ε_r in a magnetized plasma.

IV. ELECTRIC FIELD GENERATED BY A CHARGE SHEET IN GAUSSIAN DISTRIBUTION WITH DIFFERENT WIDTH

For a more realistic model, we consider a charge sheet in Gaussian distribution,

$$\rho_{\text{ex}} = \sigma \frac{1}{\rho_i} \frac{\exp[-(x/\alpha)^2]}{\sqrt{\pi} \alpha}, \quad (12)$$

and $\alpha \rho_i$ is clearly the half-width of the Gaussian charge sheet. Note that in Eq. (12), the total charge is kept constant for different values of α .

From the last section, we identified the Green's function in the slab model,

$$G(x - x_s) = G_P(x - x_s) + G_{DS}(x - x_s), \quad (13a)$$

$$G_P(x - x_s) = \text{sgn}(x - x_s) \frac{1}{2\epsilon_0\epsilon_r}, \quad (13b)$$

$$G_{DS}(x - x_s) = \text{sgn}(x - x_s) \frac{1}{2\epsilon_0\epsilon_r} (\epsilon_r - 1) \times \exp[-\sqrt{2\epsilon_r}(x - x_s)^2]. \quad (13c)$$

For the Gaussian charge sheet case, the electric field can be found by the standard method of the Green's function with the help of Eq. (13),

$$E = E_P + E_{DS}, \quad (14a)$$

$$E_P = \frac{\sigma}{2\epsilon_0\epsilon_r} \text{erf}\left(\frac{x}{\alpha}\right), \quad (14b)$$

$$E_{DS} = \frac{\sigma}{2\epsilon_0\epsilon_r} \frac{1}{\sqrt{\pi}\alpha} \frac{2x}{\alpha^2} \exp\left[-\left(\frac{x}{\alpha}\right)^2\right], \quad (14c)$$

where in obtaining Eq. (14c) from integrating the product of the source term [Eq. (12)] and the Green's function [Eq. (13c)], we made use of the asymptotic method by noting that ϵ_r is typically a large number. An analytical expression can be found for a general distribution of charge sheet; it is shown in the Appendix.

Equation (14) is plotted in Fig. 1 for different values of α (the half-width of the charge sheet normalized by the ion Larmor radius). From the Appendix, it can be seen that the half-width here should be understood as the scale length of the distribution. It can be seen from Fig. 1 that when the half-width is larger than the ion Larmor radius, the DS effect is weak. When the half-width is comparable to the ion Larmor radius, the DS effect significantly enhances the electric field with the charge sheet. When the half-width is 25% of the ion Larmor radius, the electric field is enhanced by an order of magnitude. When the half-width is equal to the ion Larmor radius, the electric field is enhanced by 50%. When the half-width is larger than 4 Larmor radii, the DS effect disappears.

V. CONCLUDING REMARKS

In this paper, we have discussed the shielding of a charge sheet in a magnetized plasma by taking account of the DS current in addition to the polarization current. For an infinitesimal width of charge sheet parallel to the magnetic field, it is found that the perpendicular electric field generated by the free charge is shielded within a range of λ_D , and that out of the range of $\lambda_e = \log(\epsilon_r)\lambda_D$, the electric field decays to $1/\epsilon_r$ of the corresponding electric field in free space. For an infinitesimal width of charge sheet perpendicular to the magnetic field, the parallel electric field generated by the free charge is shielded within a range of $\lambda_D/\sqrt{2}$ through the

FIG. 1. Electric field [normalized by $\sigma/(2\epsilon_0\epsilon_r)$] generated by charge sheet with Gaussian distribution for different half-widths (αp_i). Dashed line: $E_P(x)$, effects of polarization drift; dotted line: $E_{DS}(x)$, effects of diamagnetic drift start-up drift; and solid line: $E(x)$, the total electric field.

mechanism of the conventional Debye shielding. The perpendicular shielding length is $\sqrt{2}$ times the conventional parallel shielding length.

For a charge sheet in Gaussian distribution, it is found that the half-width (the scale length of the distribution) is an important factor. When the half-width is much larger than the ion Larmor radius, the effect of DS current is weak compared to the effect of polarization. When the half-width is comparable or smaller than the ion Larmor radius, the DS current significantly enhances the electric field within the charge sheet. In the present work, we ignored the effects of nonlinearity and the effects of background plasma inhomogeneity; these effects will be addressed in the future.

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APPENDIX: ELECTRIC FIELD FOR A GENERAL CHARGE SHEET PARALLEL TO THE MAGNETIC FIELD

For a charge sheet with a general distribution

$$\rho_{\text{ex}} = \sigma \frac{1}{\rho_i} g(x), \quad (\text{A1})$$

the electric field is found in the following way:

$$E = \int_{-\infty}^{\infty} dx_s \rho_i G(x - x_s) \rho_{\text{ex}}(x_s) = E_P + E_{\text{DS}}. \quad (\text{A2})$$

$$E_P = \int dx_s \text{sgn}(x - x_s) \frac{\sigma}{2\epsilon_0 \epsilon_r} g(x_s), \quad (\text{A3})$$

$$\begin{aligned} E_{\text{DS}} &= \int_{-\infty}^{\infty} dx_s \text{sgn}(x - x_s) \frac{\sigma}{2\epsilon_0 \epsilon_r} (\epsilon_r - 1) \\ &\quad \times \exp[-\sqrt{2\epsilon_r}(x - x_s)^2] g(x_s) \\ &= \frac{\sigma}{2\epsilon_0 \epsilon_r} (\epsilon_r - 1) \int_{-\infty}^{\infty} dz \text{sgn}(z) \exp[-\sqrt{2\epsilon_r}z^2] g(x - z). \end{aligned} \quad (\text{A4})$$

To evaluate the above integral, we make use of the fact that ϵ_r is typically a large number and note that $\text{sgn}(z)$ is an odd function. Therefore, the integral is asymptotically evaluated,

$$\begin{aligned} E_{\text{DS}} &= \frac{\sigma}{2\epsilon_0 \epsilon_r} (\epsilon_r - 1) \int_{-\infty}^{\infty} dz \text{sgn}(z) \exp[-\sqrt{2\epsilon_r}z^2] \\ &\quad \times \left[\frac{dg(x - z)}{dz} \right]_{z=0} = \frac{\sigma}{2\epsilon_0 \epsilon_r} (\epsilon_r - 1) \frac{-dg(x)}{dx} 2 \\ &\quad \times \int_0^{\infty} dz \exp[-\sqrt{2\epsilon_r}z^2] z = \frac{\sigma}{2\epsilon_0 \epsilon_r} \frac{\epsilon_r - 1}{\epsilon_r} \frac{-dg(x)}{dx}. \end{aligned} \quad (\text{A5})$$

¹R. Goldston and P. Rutherford, *Introduction to Plasma Physics* (IOP, Bristol, 1995).

²P. K. Shukla, M. Nambu, and M. Salimullah, *Phys. Lett. A* **291**, 413 (2001).

³D. Sarmah, M. Tassarotto, and M. Salimullah, *Phys. Plasmas* **13**, 032102 (2006).

⁴A. B. Reimann and J. Fajans, *Phys. Plasmas* **9**, 1091 (2002).

⁵C. Hansen and J. Fajans, *Phys. Rev. Lett.* **74**, 4209 (1995).

⁶D. D. Tskhakaya, P. K. Shukla, B. Eliasson, and S. Kuhn, *Phys. Plasmas* **12**, 103503 (2005).

⁷B. P. Pandey, A. Samarian, and S. V. Vladimirov, *Plasma Phys. Controlled Fusion* **50**, 055003 (2008).

⁸R. J. Taylor, M. L. Brown, B. D. Fried, H. Grote, J. R. Liberati, G. J. Morales, P. Pribyl, D. Darrow, and M. Ono, *Phys. Rev. Lett.* **63**, 2365 (1989).

⁹K. C. Shaing and E. C. Crume, Jr., *Phys. Rev. Lett.* **63**, 2369 (1989).

¹⁰P. H. Diamond, Y.-M. Liang, B. A. Carreras, and P. W. Terry, *Phys. Rev. Lett.* **72**, 2565 (1994).

¹¹T. E. Stringer, *Nucl. Fusion* **35**, 1008 (1995).

¹²F. L. Hinton and G. M. Staebler, *Phys. Fluids B* **5**, 1281 (1993).