Bayesian inference of the missing resonances in $K^+\Lambda$ photoproduction

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Abstract. A Regge-plus-resonance framework featuring consistent couplings for nucleon resonances up to spin $J = 5/2$ is adopted to perform a Bayesian analysis of the world’s $\gamma p \rightarrow K^+\Lambda$ data. It is concluded that the following nucleon resonances have the highest probability of decaying into the $K^+\Lambda$ channel: $S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{13}(1720)$, $D_{13}(1900)$, $P_{13}(1900)$, $P_{11}(1900)$, and $F_{15}(2000)$.

Keywords: Kaon photoproduction, Bayesian analysis

INTRODUCTION

The strange quark plays a peculiar role in Quantum Chromodynamics (QCD) as it is neither light nor heavy. A thorough knowledge of the nucleon-resonance ($N^*$) content of open-strangeness production $\gamma p \rightarrow K^+\Lambda$ reactions would improve our understanding of the energy spectrum and the strong decay properties of baryons. The $\gamma p \rightarrow K^+\Lambda$ cross sections are of the order of $\mu b$ and there are no outspoken structures in their measured energy dependence [1]. This points towards overlapping resonances and/or a dominant role for the background diagrams. Obviously, these observations complicate the extraction of the relevant $N^*$ content from the data. In Ref. [2] a single-channel Regge-plus-resonance (RPR) model which cleanly separates the resonant and non-resonant contributions to $\gamma p \rightarrow K^+\Lambda$, is proposed. The background is described in terms of Reggeized $t$-channel $K^+(494)$ and $K^{*+}(892)$ exchange and can be parametrized by three coupling strengths ($g_{K^+\Lambda p}$, $G_{K^{*+}\Lambda p}$, $G_{K^{*+}\Lambda p}$) and two phases (either 1 or $\exp[-i\pi\alpha_K(t)]$) [2, 3, 4, 5]. The background parameters are constrained against the 262 data points from Jefferson lab with photon energies $E_\gamma > 2.6$ GeV where no individual resonances are expected to contribute. The resonances are implemented with Feynman $s$-channel diagrams for $N^*$ with $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ and $M_{N^*} \leq 2$ GeV. We adopt consistent couplings for the resonances [6]. These couplings are paramount for the analysis presented here, and require one parameter for each $J = \frac{1}{2}$ resonance and two parameters for each $N^*$ with $J \geq \frac{3}{2}$. In addition, there is a cutoff parameter in the hadronic form factor, which we assume identical for all $N^*$. 
BAYESIAN MODEL SECTION AND THE RESONANT CONTENT OF $\gamma p \rightarrow K^+ \Lambda$

In order to determine the resonant content of the $\gamma p \rightarrow K^+ \Lambda$ reaction in a statistically sound way, we have evaluated the $N^*$ content of the RPR model against the world’s data in a Bayesian analysis [3, 4]. The data comprise 3455 differential cross sections, 2241 single polarization observables (beam, target, and recoil) and, 452 double polarization observables (only beam-recoil data is published). In the analysis, 11 resonance candidates with masses up to 2 GeV are included. The spin-isospin quantum numbers and masses of the included $N^*$ can be found in Fig. 1. We evaluate all possible combinations of the 11 candidate resonances. The best model is selected from the $2^{11} = 2048$ model variants by calculating the Bayesian evidence $Z$ for each model against the world’s $p(\gamma, K^+ \Lambda)$ data. A model’s probability $P(M|\{d_k\})$ is defined as the probability for a model $M$ given the data $\{d_k\}$. Bayes’ theorem allows one to connect this quantity to the evidence $Z = P(\{d_k\}|M)$.

The determination of $Z$ is computationally very expensive as it requires the likelihood for all possible values of the model parameters [3]. For the priors of the $N^*$ parameters we adopt a uniform distribution whereby the upper bound is determined by naturalness arguments: no resonance is expected to generate strength which exceeds 5 times the measured total cross section. The absolute $Z_i$ values have little meaning, and only relative quantities $\Delta \ln Z \equiv \ln Z_A/\ln Z_B$ can be interpreted with the aid of Jeffreys’ scale [5]. We find that a model (coined RPR-2011) with 14 $N^*$ parameters provides the best evidence of describing the data. The RPR-2011 features the $S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{13}(1720)$, $D_{13}(1900)$, $P_{13}(1900)$, $P_{11}(1900)$, and $F_{15}(2000)$ resonances. It is worth noticing that we find decisive evidence (measured according to Jeffreys’ scale [3, 5]) that model variants with a larger amount of $N^*$ parameters provide a worse description of the current data than RPR-2011. This proves that in a Bayesian analysis the most complex model does not necessarily emerge as the “best” model and that there is a cost for introducing additional model parameters.

The probability of a resonance $R$ given the $\gamma p \rightarrow K^+ \Lambda$ data can be obtained by marginalizing over all possible models $M_i$ which include a specific resonance $R$

$$P(R|\{d_k\}) \Rightarrow \sum_{M_i} P(M_i|\{d_k\}) = \sum_{M_i} P(M_i) \frac{P(\{d_k\}|M_i)}{Z_i} \frac{P(M_i)}{P(\{d_k\})}.$$  \hspace{1cm} (1)

Results are contained in Fig. 1 and confirm the resonance content of RPR-2011.

CONCLUSION

The $\gamma + p \rightarrow K^+ + \Lambda$ reaction is background dominated and has an overlapping $N^*$ content which makes conventional isobar approaches and analysis schemes less appropriate. Bayesian methodology is a great tool for model selection under the condition that a moderate number of tunable parameters is involved. This confines the Bayesian methodology to a single-channel analysis. A Bayesian analysis within a coupled-channel framework is computationally prohibitive with the current computational resources. The RPR approach is a single-channel model for $K^+ \Lambda$ photoproduction and provides an economical
FIGURE 1. The computed $P(R | \{d_k\})$ as they have been defined in Eq. 1. There is decisive evidence for three resonances predicted by constituent quark models (CQM) and a mass of about 1900 MeV.

description from threshold up to $E_{\gamma}^{lab} \leq 16$ GeV, the highest energy for which exclusive data are available. A Bayesian analysis of the world’s data within the RPR framework points to 8 $N^*$’s with a decay into $K^+\Lambda$. Thereby we find evidence for the $P_{13}(1900)$ with an $\ast \ast \ast$ overall status ($\ast \ast \ast$ status for seen in $K^+\Lambda$) in PDG [9], and for the $P_{11}(1900)$, $D_{13}(1900)$ which are “missing resonances”. We have evaluated the merits of the RPR model and have found that it has predictive power for $e p \rightarrow e' K^+ \Lambda$ [3], that it can predict the observables for kaon-hyperon production from the neutron [7], and that it provides a good elementary production operator for $d (\gamma, K^0) Y$ processes [8].

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