Topics on the Portfolio Management of Financial Investments
Doctoral Jury

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Korte voorstelling van het proefschrift

Het gebruik van financiële opties kent twee voorname doelstellingen: een hedgingmotief en een speculatief motief. In het eerste geval tracht men de waarde van een onderliggend actief te beschermen, terwijl men in het tweede geval hoopt op rendementswinsten op de optieposities, los van een onderliggende investering.

Na het inleidend hoofdstuk, wordt in hoofdstukken twee tot vier van het doctoraat het eerste motief beschouwd. We gaan ervan uit dat een investering in een obligatie wordt beschermd door het kopen van een putoptie. Deze optie geeft het recht om de obligatie te verkopen aan een vooraf bepaalde prijs (de uitoevenprijs). Dit recht wordt interessant indien door rentestijgingen, de waarde van de obligatie is gedaald. We veronderstellen dat de investeerder slechts een beperkt budget heeft om de putoptie te kopen en dat getracht wordt het risico van de investering te minimaliseren. Ten tweede gaan we ervan uit dat als risico maatstaven Value-at-Risk (VaR) en Tail Value-at-Risk (TVaR) beschouwd worden. Dit zijn twee populaire risicomaatstaven in de financiële wereld. VaR\(_{\alpha,T}\) geeft het verlies op de investering weer in een rampscenario, met een betrouwbaarheid van \(1 - \alpha\), gedurende de periode \([0,T]\). TVaR beschouwt de grootte van de mogelijke verliezen, eens de VaR drempel overschreden is. We leiden, rekening houdend met deze twee veronderstellingen, theoretisch de optimale uitoevenprijs af. Steeds vinden we dat de optimale uitoevenprijs onafhankelijk is van het beschikbare hedgingbudget. In het tweede hoofdstuk gebruiken we als rentemodel het model van Vasicek. Als onderliggend actief richten we ons op een nulcoupon obligatie. Hoofdstuk drie verbreedt de mogelijke rentemodellen door enkel te veronderstellen dat de verdeling van de toekomstige obligatieprijzen gekend is. Daarnaast leiden we de optimale uitoevenprijs ook af voor een coupondragende obligatie. In het vierde hoofdstuk richten we ons op de affiene rentemodellen, en gaan we uitgebreider in op het Hull-White één-factormodel. We implementeren onze strategie aan de hand van een Belgische overheidsobligatie en houden er rekening mee dat de marktprijzen van opties niet noodzakelijk gelijk zijn aan de theoretische optieprijzen. We tonen het (economisch) belang aan van de juiste keuze van de uitoevenprijs.
Het vijfde hoofdstuk gaat in op het speculatief motief van financiële opties. We analyseren daarvoor de transacties van online optiebeleggers van een vooraanstaande Nederlandse bank, gedurende de periode Januari 2006-December 2007. Er is duidelijk sprake van een home bias: meer dan 95% van de transacties hebben als onderliggende waarde een Nederlands aandeel. We beschrijven de belangrijkste beslissingvariabelen van een optietransactie. Er worden meer callopties dan putopties verhandeld. Dit is voornamelijk het geval voor de investeerders die niet zo frequent handelen. We berekenen de rendementen gerealiseerd op de optietransacties en observeren belangrijke rendementsverschillen tussen long- en short- posities, alsook tussen actief gesloten en afgelopen opties. Terwijl het gemiddelde rendement van long-posities positief is, is dat voor short posities negatief. De medianen tonen echter een ander beeld. Daar zien we dat de short posities beter presteren dan de long-posities. Het verschil wordt verklaard door het verschil in rendementen die mogelijkerwijs kunnen behaald worden door enerzijds long-en anderzijds short-posities. Terwijl long-rendementen niet lager kunnen gaan dan -100%, en langs de positieve kant (theoretisch) oneindig kunnen zijn, geldt het omgekeerde voor short-rendementen. Het verlies kan veel hoger oplopen dan 100%, maar de winst is beperkt tot 100%. Er wordt in onze analyses ook aangetoond dat rendementen afhankelijk zijn van de looptijd van de transacties. Hoe langer de looptijd van de transacties, hoe slechter de performantie. Transacties die gewoonweg aflopen op vervaldag, vertonen een veel slechter rendement dan transacties die actief gesloten worden. Als laatste punt stellen we vast dat het bij investeerders die frequent in opties handelen veel vaker voorkomt dat de optietransacties (in zijn geheel) tot winst leiden.

Het zesde hoofdstuk beschouwt een andere vorm van indirecte participatie in aandelenmarkten, namelijk beleggingsfondsen. Beleggingsfondsen worden geacht betere rendementen te behalen dan individuele beleggers, voornamelijk omdat de beheerders verondersteld worden meer kennis van en ervaring met aandelen te hebben. Echter, empirische evidentie wijst niet steeds in die zin. Aan de hand van een unieke dataset van transacties uitgevoerd door beleggingsfondsen, dragen we bij tot dit debat. Ten eerste gaan we na of beleggingsfondsen transacties uitvoeren die rendabel zijn. Over het geheel
van onze steekproef, vinden we inderdaad dat dit geval is, wat in het voordeel pleit van de beleggingsfondsen. Verdere tests wijzen echter uit dat de resultaten voornamelijk gedreven worden door UK fondsen, terwijl voor Europese en Amerikaanse fondsen geldt dat er geen verschil te vinden is tussen de aankoop- en verkooppportebeurte. Daarnaast onderzoeken we of beleggingsfondsen ook onderhevig zijn aan het dispositie-effect. Dit effect, vaak bevestigd in onderzoeken bij individuele beleggers, houdt enerzijds in dat men minder snel aandelen verkoopt waarvan de huidige prijs zich onder de aankoopprijs bevindt. Anderzijds worden aandelen die boven de aankoopprijs noteren gemakkelijker verkocht. Het geheel van onze steekproef vertoont geen gedrag dat hierop wijst. Echter, opnieuw stellen we regionale verschillen vast. UK fondsen blijken wel vatbaar voor het dispositie-effect, terwijl we geen evidentie vinden bij Europese en Amerikaanse fondsen. We stellen wel vast dat er een dispositie-effect optreedt indien we winst of verlies afteutsen met betrekking tot de hoogste koers behaald door een aandeel.
Chapter 1: Introduction
1. Outline of the dissertation

Financial options give the holder the right to buy (in case of a call option) or to sell (in case of a put option) a specific asset at a predetermined price. Options are mainly used for two reasons: hedging and speculation. In the first case, options are employed in order to protect the decrease of the value (and thus return) on a particular investment below a certain threshold. In the second case, an investor trades in options with the goal of achieving high returns on the invested money. Chapters two to four of this dissertation focus on the use of options as a hedging instrument. In the fifth chapter, we document the use of options as a speculative tool. We therefore make use of a data set containing stock option trades of online investors at a large Dutch bank, over the period January 2006-December 2007. This chapter thus investigates one particular form of indirect participation in the stock market, that is, by means of derivatives. However, there are other indirect ways of exposing your wealth to stock market fluctuations, without directly investing in shares. A very popular vehicle to this end is mutual funds. These funds pool the money of several investors, and take and implement investment decisions for these investors. These mutual funds claim that the pooling of money offers several benefits. Most notably is the claim that the managers of these mutual funds have much more knowledge and experience in making stock market investments than most individual investors. These individual investors are often assumed to behave in an irrational way. In the last chapter, we examine whether two of these irrational traits, commonly termed behavioural biases, also hold for institutional investors. These two behavioural biases are overconfidence and the disposition effect, which is the tendency to hang on to losing stocks too long. These biases are tested using a sample of daily mutual fund transactions, obtained from a leading global custodian, who is responsible for executing trades for mutual funds.
Chapter 1

2. Main findings of the different chapters

Chapters two to four focus on the use of options as a risk management tool. More specifically, we focus on the protection of an investment in a bond. In each chapter, we assume a put option will be used to protect the investment in the bond. The investor (we assume a company) has then yet to decide on two important features of an option: the time to maturity and the strike price. The time to maturity is set equal to the horizon of the period over which protection is required. We thus solely focus on determining the strike price. The strike price is, together with the volatility of the underlying asset and the time to maturity, the most important driver of the price of an option. Put options become more expensive as the strike price increases. This is intuitive, since the strike price determines at which price the holder of the investor can sell the asset. Therefore, the trade-off for the investor is clear: fixing a higher strike price lowers the loss in case of an unfavourable evolution of the underlying asset, but increases the premium (price of the option) that needs to be paid upfront. We assume that the amount the investor wants to pay is limited. Additionally, we assume that the investor uses some kind of risk criterion for assessing the hedging success. The risk criteria considered are Value-at-Risk (VaR) and Tail Value-at-Risk (TVaR). Especially VaR is very popular in the financial world, cfr. Basel II. VaR,\(\alpha\) is the loss of the worst case scenario on the investment at a \(1−\alpha\) confidence level during the period \([0,T]\). TVaR considers the size of the possible losses, once the VaR threshold is crossed. We answer the following question: what is the (optimal) strike price for the put option that minimizes the (TVaR) of the investment, taking into account our budget constraint. We thus deduce the optimal strike price. Our setup is similar in spirit as Ahn et al. (1999). They focus on a stock as underlying asset, while we consider a bond (both zero-coupon and coupon-bearing). Since the value of a bond heavily depends on the interest rate evolution, we carefully select a process for the evolution of the interest rate. There exists a whole literature concerning interest rate modelling (see e.g. Brigo and Mercurio (2007)). In the first contribution, we select the Vasicek (1977) model, in which it is assumed that the instantaneous interest rate follows a mean-reverting process. We perform the (TVaR) minimization for a zero-coupon bond. This results in an analytical
expression of which the solution is the optimal strike price. Absent from this expression is
the available hedging budget. This means that changing the hedging budget will not change
the optimal strike price. A change in the hedging budget will only change the proportion of
the put option that we will be able to buy. We also provide a numerical illustration that
shows the economic loss attached to diverging from the optimal strike price.

The second contribution generalizes the first in two ways. Firstly, we extend the analysis to a
coupon-bearing bond, which can also be understood as a portfolio of zero-coupon bonds.
Secondly, formulas are derived for any short rate model with a given distribution of future
bond prices. This comprises numerous popular interest rate models, e.g. one- and two-factor
Hull-White, two-factor additive Gaussian model G2++, two-factor Heath-Jarrow-Morton
with deterministic volatilities (where all future bond prices are lognormally distributed).

In the fourth chapter we consider interest rate models with an affine term structure.
Technically, this means that the zero-coupon bond price \( P(T, S) \) can be written in the form
\[
P(T, S) = A(T, S)e^{-\int_0^T r(t) \, dt},
\]
with the instantaneous interest rate \( r(T) \). The difference with the
third chapter is that here the results will be expressed in terms of the cumulative distribution
(cdf) of the instantaneous rate \( r(T) \), while in chapter three they were in terms of the cdf of the
zero-coupon bond itself.

We strongly focus on the Hull-White one-factor model in the fourth chapter, first discussed
in Hull and White (1990). This model has seen a widespread proliferation in the financial
world, for mainly two reasons. Firstly, it allows for closed form solutions for both bonds and
plain vanilla European bond options. In particular, for the option on a coupon-bearing bond
one can apply the Jamshidian decomposition (see Jamshidian (1989)). Secondly, it can be
fitted to the current term structure. After having derived the optimal strike price, we
implement our procedure by hedging a Belgian government bond. We take into account that
market option prices can diverge from theoretical option prices. We therefore perform the
analysis by using option prices provided to us by a financial institution. The difference
between the theoretical and market prices indeed introduces a small spread between the
optimal strike prices.
The next chapter considers the use of stock options by individual investors. We therefore make use of a data set containing the trades of a sample of online investors of a major Dutch bank, for the period January 2006-December 2007. As documented for common stock investments, a severe home bias exists in the trades. More than 95% of the trades concern a Dutch underlying stock. Our results reveal that investments in call options seem to be more popular than put options. This especially holds true for rather inexperienced/infrequent traders. We further provide descriptive statistics on two important decision variables of options, namely the moneyness and the time to maturity. Next, we document the (transactional) returns obtained on option trades. We report several return distribution characteristics for two return measures: a holding period transaction return (not taking into account the duration of the trades), and a daily return measure, taking into account the difference in duration between the different trades. We document return differences between long and short positions. Whereas the median return of the short position exceeds the median return of long transactions, the mean returns show a reverse picture: long positions are performing better than short positions. This difference can be explained by the difference in downside risk between the two positions. Whereas for short positions, the loss is (theoretically) unlimited, it amounts to a maximum of 100% for long positions. This downside risk is not to be neglected of course. We further show a difference in returns between trades of different durations. The longer the duration, the worse the performance gets. We also discuss the influence of trading frequency (which proxies for experience) on the wealth created by investing in options. More specifically, we show that the more trades, the higher the odds that option trading results in wealth creation instead of destruction. One possible interpretation is that there are learning effects in option trading.

The last chapter then also considers the stock market as underlying market, but focuses on the mutual fund industry. As stated earlier, these funds offer an alternative to directly investing in the stock market. We examine two behavioural biases that are often confirmed in research on individual investor behaviour: overconfidence and disposition behaviour. These biases are examined using a unique sample of daily transactions of mutual funds, obtained from a global custodian. The mutual fund transactions have an international spread,
in the sense that assets (most importantly equities and bonds) are traded on markets throughout the world. We start with examining the post transaction profitability of purchases and sales. We find performance differences between European, UK and US (oriented) funds. UK funds stand out, making profitable trades. For European and US funds, profitability of purchases and sales seems to be equal. Next, we focus on the disposition effect, which is explained as originating from the prospect theory of Kahneman and Tversky (1979). Due to loss aversion, investors will refrain from realizing losses. Odean (1998) provided compelling evidence of this behaviour for a sample of individual investors. The basic idea from our setup follows Odean (1998): we assess the difference in the propensity to realize gains versus losses. Equal propensities would imply no disposition effect. Using as reference point the average purchase price to assess gains and losses, we find no evidence of disposition for European and US funds. UK funds seem to display behaviour akin to the disposition effect. The choice of the reference point is non-trivial. This was already documented earlier, and is again confirmed in our analysis. Taking as a reference point the maximum price a stock attained over a particular time horizon, we confirm the disposition effect for all of our subsamples. This paper thus further extends the literature on the behaviour and performance of the mutual fund industry.

3. Relevance of the different chapters

The contributions of my PhD dissertation are situated in three domains: risk management (chapters two to four), options (chapters two to five), and mutual fund performance (chapter six). I believe the current state of both the economy and the financial markets easily legitimates the contributions. Indeed, the end phase of my PhD time coincides with a period of large turmoil on financial markets around the world, mainly initiated by the subprime crisis in the US. This subprime crisis knows several causes, but one of the fundamental causes is the way financial institutions granted mortgages (loans which are used for building or buying a home) to private customers. It is argued that banks and insurance companies were too careless when scrutinizing the potential of a borrower to make repayments on the
mortgage. Moreover, ingenious and attractive mortgage based products were created that allowed even the least solvent borrowers to contract a loan and start making repayments. This is the first phase where the risk measurement and management is fundamentally bad. Secondly, there is the issue of securitization. A substantial amount of the mortgages were securitized, meaning that these mortgages were repacked and sold in tranches to third parties. Therefore, these third parties, spread internationally and over different types of investors, also became involved. However, here again sound risk management practices could have prevented much of the losses incurred by third parties. If they had put more effort into examining what they were planning to buy, a lot of the investments would not have passed the tests. The two aforementioned roles for risk management would each time have prevented the purchase of a particular asset. Needless to say that, once the asset is in the portfolio of the bank or company, risk management remains imperative. That is what the first three chapters stress. Regardless of what kind of asset is in the portfolio (bond, stock, or a more complex tool), a thorough identification of the risks the asset is exposed to remains of high importance, along with possibilities to control these risks. Of course, the cost of implementing a hedge should not be neglected, and judged vis-à-vis the expected risk reduction.

In chapter five the use of stock options by individual investors is discussed. The contribution of this chapter is interesting in the light of two recent (and connected) developments. First of all, on the first of November 2007, MiFiD (Markets in Financial Instruments Directive) came into force. The most important goals of this directive are to create efficient financial markets, promote competition and provide EU-wide standards of investor protection. Especially this last part is of relevance for the fourth chapter. MiFiD provides a categorization of customers. Individual investors will be classified as ‘retail clients’ and enjoy the highest level of protection. Two important protection mechanisms are the suitability test and the appropriateness test. We focus on the latter test, which applies to the provision of execution-only services. These firms only provide the possibility to execute a particular trade, but do not provide investment advice. The firm must request information from the client regarding his knowledge and experience to enable the firm to assess whether the service or product is
appropriate for the client. Based on the answers of the investor, the firm communicates to the client whether it would be appropriate for the investor to engage in the trading of complex products. However, this advice is not binding. An investor can still pursue these trades.

This is where, in my view, the second development starts to play an important role. That is, the increasing popularity of online banking and investing. The Internet led to a spectacular increase in the possibilities for active investing. However, Barber and Odean (2002) already showed that for private investors, this increased trading flexibility is not always to the benefit of the investors, in terms of achieved profitability. An important reason is overconfidence. Overconfidence is, I think, even much more dangerous in an option context, due to the much higher attainable returns. Therefore, communications of the banks stating that option trading is not appropriate, will tend to be dismissed easily by overconfident people, even more when, as can be assumed, communication will be done by postal means.

The results of this chapter indeed clearly shows that returns of option trades can be spectacular. This makes option trading very attractive to investors with high levels of risk tolerance. Unfortunately, returns can be extreme in both a positive and negative way. The fourth chapter thus serves as an extra warning signal for investors, suggesting that indeed option trading should be done with care and a firm knowledge of the associated risk.

The last chapter discusses mutual fund behaviour and performance. The mutual fund industry promotes itself by pointing to the benefits. Three evident benefits are (1) the increased possibility for diversification, (2) the reduced transaction costs due to economies of scale and (3) the (assumed) ability of fund managers to select assets which generate excess returns. Ever since the seminal paper of Jensen (1968), this last benefit has been questioned. Several authors have pointed out that the net return on mutual funds does not exceed the return on a market index (see Malkiel (1995), Gruber (1996), Carhart (1997)). This would imply that mutual funds have no value for the private investor. However, more recently, researchers examined the portfolio composition of mutual funds and concluded that actively managed funds indeed hold stocks and bonds which make an abnormal profit (see amongst others Daniel et al. (1997), Chen et al. (2000) and Wermers (2000)). This finding is not necessarily inconsistent with the findings of Jensen (1968) and others. Transaction costs and
management fees probably drive down this abnormal return to the level of the market return. The final answer to the question whether mutual fund investments are profitable is yet to be given. Part of it is due to the fact that most data sets only contain monthly and quarterly holdings, and not transactions. Therefore, transactions within this time frame are ignored. Furthermore, most research is focused on the US market. The mutual fund data set at our disposal does not suffer from these shortcomings. Therefore, I am convinced the last chapter adds to the debate on the mutual fund industry.

4. Avenues for further research

The different topics and data sets used in this dissertation are worthwhile to be examined further. Concerning chapters two to four, extensions to other types of underlying assets (such as e.g. a basket of assets), other interest rate models (which do not allow the Jamshidian decomposition for an option written on a sum of multiple underlyings), other hedging instruments (e.g. swaptions) and a more general class of risk measures (including TVaR) are a natural point to start.

The fifth chapter provides numerous possibilities for further research. First of all, a portfolio approach to the performance measurement should give complementary insights into how profitable option trading really is. Secondly, next to the transaction data set, we also have a data set containing specific investor characteristics related to the investor psychology, trading behaviour and information use. These characteristics come from a survey conducted on the sample of the online investors and contains both hard (gender, age, education) and soft information. This soft information measures items such as (over)confidence, optimism, risk and loss aversion. These important behavioural concepts can be linked to the use of options, and the achieved performance. This allows us to check whether the biases influencing behaviour in the stock market also drive behaviour in the option market. Thirdly, the link between the option portfolio and share portfolio of the investors can be established. Indeed, in the fifth chapter we left aside the share positions of the investors. However, it is evident
that some of the option trades and positions will be linked to the share portfolio, to pursue particular strategies such as covered calls, or protective puts.

The data set underlying the last chapter also contains a host of research venues. More specifically, the transactional nature of the data allows for interesting event studies, making further evaluation of the capabilities of mutual fund managers possible. These capabilities can relate to both security selection and market timing. Concerning security selection, I envisage a study on the ability of mutual fund managers to invest in firms with positive earnings surprises. Since earnings are an important determinant of the stock price, earnings forecasts from analysts and earnings announcement from companies receive a lot of attention. Companies that are able to exceed the earnings forecasts of analysts, often enjoy a sizeable increase in their stock price. Therefore, a fund manager that invested in the stocks which surprised the market, are believed to have security selection skills. Concerning market timing, I would like to focus on the bond part of the data set. Common interest rate related factors account for the vast majority of the returns of bonds. These drivers are for example the short rate, the maturity spread, the curvature and the liquidity spread. Depending on the expectations a fund manager has concerning the evolution of these factors, market timing would imply that the investments are more focused on bonds that are sensitive to the factors which are believed to evolve positively. If ex-post the evolution was indeed beneficial, the fund manager is indeed perceived as a good market timer.

**References**

Chapter 1

Chapter 2:

Managing Value-at-Risk for a bond using bond put options
Managing Value-at-Risk for a bond using bond put options

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Abstract. This paper studies a strategy that minimizes the Value-at-Risk (VaR) of a position in a zero coupon bond by buying a percentage of a put option, subject to a fixed budget available for hedging. We elaborate a formula for determining the optimal strike price for this put option in case of a Vasicek stochastic interest rate model. We demonstrate the relevance of searching the optimal strike price, since moving away from the optimum implies a loss, either due to an increased VaR or due to an increased hedging expenditure. In this way, we extend the results of [Ahn et al., 1999], who minimize VaR for a position in a share. In addition, we look at the alternative risk measure Tail Value-at-Risk.

Keywords: Value-at-Risk, bond hedging, Vasicek interest rate model.

1 Introduction

Many financial institutions and non-financial firms nowadays publicly report Value-at-Risk (VaR), a risk measure for potential losses. Internal uses of VaR and other sophisticated risk measures are on the rise in many financial institutions, where, for example, a bank risk committee may set VaR limits, both amounts and probabilities, for trading operations and fund management. At the industrial level, supervisors use VaR as a standard summary of market risk exposure. An advantage of the VaR measure, following from extreme value theory, is that it can be computed without full knowledge of the return distribution. Semi-parametric or fully non-parametric estimation methods are available for downside risk estimation. Furthermore, at a sufficiently low confidence level the VaR measure explicitly focuses risk managers and regulators attention on infrequent but potentially catastrophic extreme losses.
Value-at-Risk (VaR) has become the standard criterion for assessing risk in the financial industry. Given the widespread use of VaR, it becomes increasingly important to study the effects of options on the VaR-based risk management.

The starting point of our analysis is the classical hedging example, where an institution has an exposure to the price risk of an underlying asset. This may be currency exchange rates in the case of a multinational corporation, oil prices in the case of an energy provider, gold prices in the case of a mining company, etc. The corporation chooses VaR as its measure of market risk. Faced with the unhedged VaR of the position, we assume that the institution chooses to use options and in particular put options to hedge a long position in the underlying.

[Ahn et al., 1999] consider the problem of hedging the Value-at-Risk of a position in a single share by investing a fixed amount $C$ in a put option. The principal purpose of our study is to extend these results to the situation of a bond. We consider the well-known continuous-time stochastic interest rate model of [Vasicek, 1977] to investigate the optimal speculative and hedging strategy based on this framework by minimizing the Value-at-Risk of the bond, subject to the fixed amount $C$ which is spent on put options. In addition, we consider an alternative risk measure Tail Value-at-Risk (TVaR), for which we solve the minimization problem and obtain the optimal hedging policy.

The discussion is divided as follows: Section 2 presents the general risk management model, introduces the Vasicek model and considers hedging with bond put options. Afterwards, Section 3 discusses the optimal hedging policy for VaR, considers the closely related risk measure TVaR and introduces comparative statics. Section 4 consists of a numerical illustration. Finally, Section 5 summarizes the paper, concludes and introduces further research possibilities.

2 The mathematical framework

Consider a portfolio with value $W_t$ at time $t$. The Value-at-Risk of this portfolio is defined as the $(1 - \alpha)$-quantile of the loss distribution depending on a time interval with length $T$. The usual holding periods are one day or one month, but institutions can also operate on longer holding periods (e.g. one quarter or even one year), see [Dowd, 1998]. A formal definition for the $\text{VaR}_{\alpha,T}$ is

$$\Pr(W_0 - W_{T}^{d} \geq \text{VaR}_{\alpha,T}) = \alpha,$$

with $W_{T}^{d}$ the value of the portfolio at time $T$, discounted back until time zero by means of a zero coupon with maturity $T$. In other words $\text{VaR}_{\alpha,T}$ is the loss of the worst case scenario on the investment at a $1 - \alpha$ confidence level during the period $[0, T]$. It is possible to define
the VaR\(_\alpha,T\) in a more general way

\[
\text{VaR}_{\alpha,T} = \inf \left\{ Y \mid \Pr(W_0 - W_T^Y > Y) \leq \alpha \right\}.
\]

In this study, we focus on the hedging problem of a zero-coupon bond. Therefore, we need to define a process that describes the evolution of the instantaneous interest rate, and enables us to value the zero-coupon bond. As term structure model, we consider the Vasicek model, which is a typical example of an affine term structure model.

### 2.1 The Vasicek model

[Vasicek, 1977] assumes that the instantaneous interest rate follows a mean reverting process also known as an Ornstein-Uhlenbeck process:

\[
dr(t) = \kappa(\theta - r(t))dt + \sigma dZ(t)
\]

(1)

for a standard Brownian motion \(Z(t)\) under the risk-neutral measure \(Q\), and with constants \(\kappa\), \(\theta\) and \(\sigma\). The parameter \(\kappa\) controls the mean-reversion speed, \(\theta\) is the long-term average level of the spot interest rate around which \(r(t)\) moves, and \(\sigma\) is the volatility measure. The reason of the Vasicek model’s popularity is its analytical and mathematical tractability. An often cited critique is that applying the model sometimes results in a negative interest rate.

It can be shown that the expectation and variance of the stochastic variable \(r(t)\) are:

\[
E^Q[r(t)] = m = \theta + (r(0) - \theta)e^{-\kappa t}
\]

(2)

\[
\text{Var}^Q[r(t)] = s^2 = \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa t}\right).
\]

(3)

Based on these results, Vasicek develops an analytical expression for the price of a zero-coupon bond which has value 1 on maturity date \(S\)

\[
Y(t, S) = \exp\left[A(t, S) - B(t, S)r(t)\right],
\]

(4)

where

\[
B(t, S) = \frac{1 - e^{-\kappa(S-t)}}{\kappa},
\]

(5)

\[
A(t, S) = (B(t, S) - (S-t))(\theta - \frac{\sigma^2}{2\kappa^2}) - \frac{\sigma^2}{4\kappa}B(t, S)^2.
\]

(6)

Since \(A(t, S)\) and \(B(t, S)\) are independent of \(r(t)\), the distribution of a bond price at any given time must be lognormal with parameters \(\Pi\) and \(\Sigma^2\):

\[
\Pi(t, S) = A(t, S) - B(t, S)m, \quad \Sigma(t, S)^2 = B(t, S)^2s^2,
\]

(7)

with \(m\) and \(s^2\) given by (2) and (3).
From the formulae (4)-(7), we can see that for \( S \geq T \) the present value (using a zero-coupon bond for discounting) of the loss of the (unhedged) portfolio can be expressed as function of \( z \)

\[
L_0 = W_0 - W_T^d = Y(0, S) - Y(0, T)e^{H(T,S)+\Sigma(T,S)z} := f(z)
\]  

(8)

where \( f \) is a strictly decreasing function, \( z \) is a stochastic variable with a standard normal distribution and \( d \) means equality in distributional sense. Therefore, the VaR\( _{\alpha,T} \) of such a portfolio is determined by the formula

\[
\text{VaR}_{\alpha,T}(L_0) = f(c(\alpha)),
\]

(9)

where \( c(\alpha) \) is the cut off point for the standard normal distribution at a certain percent level i.e. \( \Pr(z \leq c(\alpha)) = \alpha \).

Since the distribution of the unhedged position in the zero-coupon bond is lognormal in the Vasicek model, from the formulae (8)-(9) we observe that the Value-at-Risk measure for the zero-coupon bond can be expressed as

\[
\text{VaR}_{\alpha,T}(L_0) = Y(0, S) - Y(0, T)e^{H(T,S)+\Sigma(T,S)c(\alpha)},
\]

where \( c(\cdot) \) is the percentile of the standard normal distribution.

### 2.2 Put options and hedging

We recall from [Ahn et al., 1999] the classical hedging example, where an institution has an exposure to the price risk of an underlying asset \( S_T \). The hedged future value of this portfolio at time \( T \) is given by

\[
H_T = \max(hX + (1-h)S_T, S_T),
\]

(10)

where \( 0 \leq h \leq 1 \), represents the hedge ratio, that is, the percentage of put option \( P \) used in the hedge and \( X \) is the strike price of the option.

In our setup, the underlying security is a bond and the hedging tool is a bond put option, the price of which will be worked out hereafter. We recall that the price of a European call option with the zero-coupon bond which matures at time \( S \) as the underlying security and with strike price \( X \) and exercise date \( T \) (with \( T \leq S \)) is at date \( t \) given by:

\[
C(t, T, S, X) = Y(t, S)\Phi(d_1) - XY(t, T)\Phi(d_2),
\]

(11)

where

\[
d_1 = \frac{1}{\sigma_p} \log \left( \frac{Y(t, S)}{XY(t, T)} \right) + \frac{\sigma_p^2}{2T}, \quad d_2 = d_1 - \sigma_p,
\]

\[
\sigma_p = \frac{\sigma}{\kappa} \left( 1 - e^{-\kappa(S-T)} \right) \sqrt{\frac{1 - e^{-2\kappa(T-t)}}{2\kappa}},
\]

\[16\]
and $\Phi(\cdot)$ is the cumulative distribution function of a standard normal random variable. The Put-Call parity model is designed to determine the value of a put option from a corresponding call option and provides in this case the following European put option price corresponding to (11):

$$P(t, T, S, X) = -Y(t, S)\Phi(-d_1) + XY(t, T)\Phi(-d_2). \quad (12)$$

### 3 The bond hedging problem

#### 3.1 VaR minimization

Analogously to [Ahn et al., 1999], we assume that we have one bond and we use only a percentage of a put option on the bond to hedge. We will find the optimal strike price which minimizes VaR for a given hedging cost.

Indeed, let us assume that the institution has an exposure to a bond, $Y(0, S)$, which matures at time $S$, and that the company has decided to hedge the bond value by using a percentage of one put option $P(0, T, X)$ with strike price $X$ and exercise date $T$ (with $T \leq S$). Then we can look at the future value of the hedged portfolio (which is composed of the bond $Y$ and the put option $P(0, T, X)$) at time $T$ as a function, analogously to (10), of the form

$$H_T = \max(hX + (1 - h)Y(T, S), Y(T, S)).$$

If the put option finishes in-the-money (a case which is of interest to us), then the discounted value of the future value of the portfolio is

$$H^d_T = ((1 - h)Y(T, S) + hX)Y(0, T).$$

Taking into account the cost of setting up our hedged portfolio, which is given by the sum of the bond price $Y(0, S)$ and the cost $C$ of the position in the put option, we get for the present value of the loss

$$L_0 = Y(0, S) + C - ((1 - h)Y(T, S) + hX)Y(0, T),$$

and this under the assumption that the put option finishes in-the-money. We recall that $Y(T, S)$ has a lognormal distribution with parameters $\Pi$ and $\Sigma^2$, given by (7). Therefore the loss function equals in distributional sense

$$Y(0, S) + C - ((1 - h)e^{\Pi(T, S) + \Sigma(T, S)z} + hX)Y(0, T), \quad (13)$$

where $z$ again denotes a stochastic variable with a standard normal distribution. The Value-at-Risk at an $\alpha$ percent level of a position $H = \{Y, h, P\}$ consisting of a bond $Y$ and $h$ put options $P$ (which are assumed to be in-the-money) with a strike price $X$ and an expiry date $T$ is equal to

$$\text{VaR}_{\alpha, T}(L_0) = Y(0, S) + C - ((1 - h)e^{\Pi(T, S) + \Sigma(T, S)\epsilon(\alpha)} + hX)Y(0, T), \quad (14)$$
where we recall that \( c(\alpha) \) is the percentile of the standard normal distribution.

Similar to the Ahn et al. problem, we would like to minimize the risk of the future value of the hedged bond \( H_T \), given a maximum hedging expenditure \( C \). More precisely,

\[
\min_{X,h} Y(0, S) + C - ((1 - h) e^{\Pi(T,S) + \Sigma(T,S)c(\alpha)} + hX)Y(0, T)
\]

subject to the restrictions \( C = hP(0, T, S, X) \) and \( h \in (0, 1) \).

Solving this constrained optimization problem, we find that the optimal strike price \( X^* \) satisfies the following equation

\[
P(0, T, S, X^*) - (X^* - e^{\Pi(T,S) + \Sigma(T,S)c(\alpha)}) \frac{\partial P(0, T, S, X^*)}{\partial X} = 0,
\]

or equivalently, when taking (12) into account,

\[
e^{\Pi(T,S) + \Sigma(T,S)c(\alpha)} = \frac{Y(0, S)\Phi(-d_1(X^*))}{Y(0, T)\Phi(-d_2(X^*))}.
\]

We note that the optimal strike price is independent of the hedging cost. Also, the optimal strike price is higher than \( e^{\Pi(T,S) + \Sigma(T,S)c(\alpha)} \). This has to be the case since \( P(0, T, S, X) \) is always positive and the change in the price of a put option due to an increase in the strike is also positive. This result is also quite intuitive since there is no point in taking a strike price which is situated below the bond price you expect in a worst case scenario.

### 3.2 Tail VaR minimization

In this section, we introduce the concept of Tail Value-at-Risk, TVaR, also known as mean excess loss, mean shortfall or Conditional VaR. We further demonstrate the ease of extending our analysis to this alternative risk measure.

A drawback of the traditional Value-at-Risk measure is that it does not care about the tail behaviour of the losses. In other words, by focusing on the VaR at, let’s say a 5% level, we ignore the potential severity of the losses below that 5% threshold. In other words, we have no information on how bad things can become in a real stress situation. Therefore, the important question of ‘how bad is bad’ is left unanswered. TVaR is trying to capture this problem by considering the possible losses, once the VaR threshold is crossed.

Formally,

\[
TVaR_{\alpha,T} = \frac{1}{\alpha} \int_{1-\alpha}^{1} \text{VaR}_{1-\beta,T} d\beta.
\]

This formula boils down to taking the arithmetic average of the quantiles of our loss, from \( 1 - \alpha \) to 1 on, where we recall that \( \text{VaR}_{1-\beta,T} \) stands for the quantile at the level \( \beta \).
If the cumulative distribution function of the loss is continuous, which is the case in our problem, TVaR is equal to the Conditional Tail Expectation (CTE) which for the loss $L_0$ is calculated as:

$$\text{CTE}_{\alpha,T}(L_0) = E[L_0 \mid L_0 > \text{VaR}_{\alpha,T}(L_0)].$$

A closely related risk measure concerns Expected Shortfall (ESF). It is defined as:

$$\text{ESF}(L_0) = E[(L_0 - \text{VaR}_{\alpha,T}(L_0))_+].$$

In order to determine TVaR$_{\alpha,T}(L_0)$, we can make use of the following equality:

$$\text{TVaR}_{\alpha,T}(L_0) = \text{VaR}_{\alpha,T}(L_0) + \frac{1}{\alpha} \text{ESF}(L_0) = \text{VaR}_{\alpha,T}(L_0) + \frac{1}{\alpha} E[(L_0 - \text{VaR}_{\alpha,T}(L_0))_+].$$

This formula already makes clear that TVaR$_{\alpha,T}(L_0)$ will always be larger than VaR$_{\alpha,T}(L_0)$.

In our case, the loss has a lognormal distribution under the risk-neutral measure $Q$, because of the lognormality of our bond prices. This allows us, after noting that in view of (13)-(14) the ESF for the loss $L_0$ can be simplified to

$$\text{ESF}(L_0) = (1 - h) Y(0, T) e^{\Pi(T,S)} E^Q[(e^{\Sigma(T,S)c(\alpha)} - e^{\Sigma(T,S)z})_+]$$

to write the ESF as

$$\text{ESF}(L_0) = (1 - h) Y(0, T) e^{\Pi(T,S)} \left[\alpha e^{\Sigma(T,S)c(\alpha)} - e^{\Sigma(T,S)\Phi(c(\alpha) - \Sigma(T,S))}\right].$$

This reduces the TVaR$_{\alpha,T}(L_0)$ to:

$$\text{TVaR}_{\alpha,T}(L_0) = Y(0, S) + C - hXY(0, T) - \frac{1}{\alpha} (1 - h) e^{\Pi(T,S)} + \frac{1}{\alpha} e^{\Sigma(T,S)} \Phi(c(\alpha) - \Sigma(T,S)) Y(0, T).$$

We again seek to minimize this TVaR, in order to minimize potential losses. The procedure for minimizing this TVaR is analogue to the VaR minimization procedure. The resulting optimal strike price $X^*$ can thus be determined from the formula below:

$$\frac{1}{\alpha} e^{\Pi(T,S)} + \frac{1}{\alpha} e^{\Sigma(T,S)} \Phi(c(\alpha) - \Sigma(T,S)) = \frac{Y(0, S) \Phi(-d_1(X^*))}{Y(0, T) \Phi(-d_2(X^*))}.$$

### 3.3 Comparative statics

We examine how changes in the parameters of the Vasicek model influence the optimal strike price, by means of the derivatives of the optimal strike price with respect to these parameters.
For both \( \text{VaR}_{\alpha,T} \) and \( \text{TVaR}_{\alpha,T} \), the optimal strike price is implicitly defined by
\[
F(X, \beta) = \text{FAC} \cdot Y(0, T)\Phi(-d_2) - Y(0, S)\Phi(-d_1) = 0,
\]
with \( \beta \) the vector including the Vasicek parameters, that is \( \theta, \kappa \) and the volatility \( \sigma \), see Section 2.1, and with \( \text{FAC} \) representing \( e^{H(T,S)+\Sigma(T,S)c(\alpha)} \) in the case of \( \text{VaR}_{\alpha,T} \) and \( \frac{1}{\alpha}e^{H(T,S)+\frac{1}{2}\Sigma^2(T,S)\Phi(c(\alpha) - \Sigma(T,S))} \) in the case of \( \text{TVaR}_{\alpha,T} \).

Taking into account the implicit function theorem, we obtain the required derivatives as follows:
\[
\frac{\partial F}{\partial X} dX + \frac{\partial F}{\partial \beta} d\beta = 0 \iff \frac{dX}{d\beta} = -\frac{\frac{\partial F}{\partial \beta}}{\frac{\partial F}{\partial X}}.
\]

The denominator of (17) is equal for the different derivatives, and is given by
\[
\frac{\partial F}{\partial X} = \text{FAC} \cdot Y(0, T)\Phi(-d_2) - Y(0, S)\Phi(-d_1),
\]
with \( \Phi \) being the density function of a standard normal random variable, while the numerator of (17) can be obtained by applying the following formula,
\[
\frac{\partial F}{\partial X} = \frac{\partial \text{FAC}}{\partial \beta} Y(0, T)\Phi(-d_2) + \text{FAC} \cdot \frac{\partial Y(0, T)}{\partial \beta} - \Phi(-d_1) + Y(0, S)\varphi(d_1) \frac{\partial d_1}{\partial \beta}.
\]

These derivatives are rather involved and do not lead to a straightforward interpretation of their sign and magnitude. Therefore, we will describe the derivatives in the next paragraph using a numerical illustration.

Further relevant derivatives are \( \frac{dx}{ds} \) and \( \frac{dx}{dT} \) to study the response of the optimal strike price to a change in the maturity of both the underlying bond and the maturity of the bond option used to hedge the exposure. They follow from formulae (17)-(19), after having replaced \( \beta \) by \( S \) and \( T \) respectively, and taking into account the simplification due to the fact that \( Y(0, T) \) is independent of \( S \), and \( Y(0, S) \) is independent of \( T \). Again, we leave the interpretation of these derivatives to the next section.

A last derivative of interest is the one with respect to \( \alpha \), formally \( \frac{dx}{d\alpha} \):
\[
\frac{dx}{d\alpha} = -\frac{1}{\frac{\partial F}{\partial X}} \cdot \frac{\partial \text{FAC}}{\partial \alpha} Y(0, T)\Phi(-d_2),
\]
where \( \frac{\partial \text{FAC}}{\partial \alpha} \) is respectively given by
\[
\frac{e^{H(T,S)+\Sigma(T,S)c(\alpha)} \Sigma(T,S)}{\varphi(c(\alpha))} \left[ \frac{\alpha \varphi(c(\alpha) - \Sigma(T,S))}{\varphi(c(\alpha))} - \Phi(c(\alpha) - \Sigma(T,S)) \right] \quad \text{(TVaR)}.
\]
4 Numerical results

We illustrate the usefulness of the above results for the VaR case (TVaR case is ongoing research). In order to provide a credible numerical illustration, we take the parameter estimates for the Vasicek model from [Chan et al., 1992], who compare a variety of continuous-time models of the short term interest rate with respect to their ability to fit the U.S. Treasury bill yield. This results in the following parameter values: \( \sigma = 0.02, \theta = 0.0866, \kappa = 0.1779, r(0) = 0.06715 \). We assume that the market price of risk parameter equals zero such that the risk neutral probability coincides with the historical one. Next, we should consider the budget the financial institution is willing to spend on the hedging. Standardising the nominal value of the bond at issuance to 1, we start with a hedging budget of 0.05, so \( C = 0.05 \). We also assume the bank is considering the VaR at the five percent level, meaning that \( \alpha = 5\% \).

We considered two situations, one in which the bank wishes to hedge a bond with a maturity of one year \( (S = 1) \), and one for a bond with a maturity of ten year \( (S = 10) \). We observe that our strategy is successful in decreasing the risk, while, since we use options, still providing us with upward potential. In the one year bond case, the mean reduction in VaR (calculated as the difference between the VaR of the hedged position and the VaR of the unhedged position, divided by VaR of the unhedged position) over the holding period amounts to 6.25\%. The maximum reduction is 26.23\%, whereas the lowest reduction is 3.25\%. In the ten year bond case, the mean VaR reduction over the holding period is 5.36\%. The maximum reduction that can be achieved amounts to 26.15\%. The minimum reduction is 2.59\%.

As already mentioned above, we are also interested in the effect of changes in the parameter estimates of the Vasicek model on the optimal strike price. We examine these effects using the first example, in which the bond matures in one year. An increase in one of these parameters always leads to a lower optimal strike price. The influence of a 1\% increase in \( \kappa \) only marginally effects the strike price. Changes in \( \theta \) also have a moderate impact on the optimal strike. The most influential parameter of the Vasicek model undoubtedly is the volatility. Whereas for \( \kappa \) and \( \theta \) the impact constantly decreases as the holding period comes closer to the maturity of the bond, we find a non-monotonic relationship between the derivative (with respect to the volatility) and the difference between the holding period \( T \) and the maturity \( S \) of the bond.

Increasing the maturity of the bond decreases the strike price, while increasing the holding period (meaning that the holding period moves closer to the maturity of the bond) increases the strike price. Reducing the certainty with which a bank wishes to know the value it can lose, or in other words, increasing \( \alpha \) leads to an increased strike price. This increase again depends on the holding period in a non-monotonic way.
5 Conclusion

In this paper, we studied the optimal risk control for one bond using a percentage of a put option by means of Value-at-Risk and Tail Value-at-Risk, widespread concepts in the financial world. The interest model we use for valuation, is the Vasicek model. The optimal strategy corresponds to buying a put option with optimal strike price in order to have a minimal VaR or TVaR given a fixed hedging cost. We did not obtain an explicit result, but numerical methods can be easily implemented to solve for the optimal strategy. For the VaR case, we demonstrate the relevance of searching for this optimal strike price, since moving away from this optimum implies a loss, either because of an increased VaR, or an increased hedging expenditure. For TVaR, the numerical illustration is part of ongoing research.

Further analysis has been oriented towards more general interest rate models with an affine term structure such as the Hull-White model and towards coupon bonds, see [Heyman et al., 2006a] and [Heyman et al., 2006b].

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References

Chapter 3:

Minimizing the (conditional) Value-at-Risk for a coupon-bearing bond using a bond put option
MINIMIZING THE (CONDITIONAL) VALUE-AT-RISK FOR A COUPON-BEARING BOND USING A BOND PUT OPTION.

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Abstract

In this paper, we elaborate a formula for determining the optimal strike price for a bond put option, used to hedge a position in a bond. This strike price is optimal in the sense that it minimizes, for a given budget, either Value-at-Risk or Conditional Value-at-Risk. Formulas are derived for both zero-coupon and coupon bonds, which can also be understood as a portfolio of bonds. These formulas are valid for any short rate model with a given distribution of future bond prices.

1. INTRODUCTION

The importance of a sound risk management system can hardly be underestimated. The advent of new capital requirements for both the banking (Basel II) and insurance (Solvency II) industry, are two recent examples of the growing concern of regulators for the financial health of firms in the economy. This paper adds to this goal. In particular, we consider the problem of determining the optimal strike price for a bond put option, which is used to hedge the interest rate risk of an investment in a bond, zero-coupon or coupon-bearing. In order to measure risk, we focus on both Value-at-Risk and Conditional Value-at-Risk. Our optimization is constrained by a maximum hedging budget. Alternatively, our approach can also be used to determine the minimal budget a firm needs to spend in order to achieve a predetermined absolute risk level. This paper can be seen as an extension of Ahn et al. (1999), who consider the same problem for an investment in a share.
2. LOSS FUNCTION AND RISK MEASURES

Consider a portfolio with value $W_t$ at time $t$. $W_0$ is then the value or price at which we buy the portfolio at time zero. $W_T$ is the value of the portfolio at time $T$. The loss $L$ we make by buying at time zero and selling at time $T$ is then given by $L = W_0 - W_T$. The Value-at-Risk of this portfolio is defined as the $(1 - \alpha)$-quantile of the loss distribution depending on a time interval with length $T$. A formal definition for the VaR$_{\alpha,T}$ is

$$\text{Pr}[L \geq \text{VaR}_{\alpha,T}] = \alpha.$$  

In other words, VaR$_{\alpha,T}$ is the loss of the worst case scenario on the investment at a $(1 - \alpha)$ confidence level at time $T$. It is also possible to define the VaR$_{\alpha,T}$ in a more general way

$$\text{VaR}_{\alpha,T}(L) = \inf\{Y \mid \text{Pr}(L > Y) \leq \alpha\}.$$  

Although frequently used, VaR has attracted some criticisms. First of all, a drawback of the traditional Value-at-Risk measure is that it does not care about the tail behaviour of the losses. In other words, by focusing on the VaR at, let’s say a 5% level, we ignore the potential severity of the losses below that 5% threshold. This means that we have no information on how bad things can become in a real stress situation. Therefore, the important question of ‘how bad is bad’ is left unanswered. Secondly, it is not a coherent risk measure, as suggested by Artzner et al. (1999). More specifically, it fails to fulfil the subadditivity requirement which states that a risk measure should always reflect the advantages of diversifying, that is, a portfolio will risk an amount no more than, and in some cases less than, the sum of the risks of the constituent positions. It is possible to provide examples that show that VaR is sometimes in contradiction with this subadditivity requirement.

Artzner et al. (1999) suggested the use of Conditional VaR (CVaR) as risk measure, which they describe as a coherent risk measure. CVaR is also known as TVaR, or Tail Value-at-Risk and is defined as follows:

$$\text{CVaR}_{\alpha,T}(L) = \frac{1}{\alpha} \int_0^\alpha \text{VaR}_{\beta,T}(L) d\beta.$$  

This formula boils down to taking the arithmetic average of the quantiles of our loss, from 0 to $\alpha$ on, where we recall that VaR$_{\beta,T}$ stands for the quantile at the level $1 - \beta$, see (1). This formula already makes clear that CVaR$_{\alpha,T}(L)$ will always be larger than VaR$_{\alpha,T}(L)$.

If the cumulative distribution function of the loss is continuous, CVaR is also equal to the Conditional Tail Expectation (CTE) which for the loss $L$ is calculated as:

$$\text{CTE}_{\alpha,T}(L) = \mathbb{E}[L \mid L > \text{VaR}_{\alpha,T}(L)].$$

3. THE BOND HEDGING PROBLEM

Analogously to Ahn et al. (1999), we assume that we have, at time zero, one bond with maturity $S$ and we will sell this bond at time $T$, which is prior to $S$. In case of an increase in interest rates, not hedging can lead to severe losses. Therefore, the company decides to spend an amount $C$ on
hedging. This amount will be used to buy one or part of a bond put option, so that, in case of a substantial decrease in the bond price, the put option can be exercised in order to prevent large losses. The remaining question now is how to choose the strike price. We will find the optimal strike prices which minimize VaR and CVaR respectively for a given hedging cost. An alternative interpretation of our setup is that it can be used to calculate the minimal hedging budget the firm has to spend in order to achieve a specified VaR or CVaR level. The latter setup was followed in the paper by Miyazaki (2001).

3.1. Zero-coupon bond

Let us assume that the institution has an exposure to a bond, \( Y(0, S) \), with principal \( K = 1 \), which matures at time \( S \), and that the company has decided to hedge the bond value by using a percentage \( h \) (0 < \( h < 1 \)) of one put option \( P(0, T, S, X) \) with strike price \( X \) and exercise date \( T \) (with \( T \leq S \)). Further, we assume that the distribution of \( Y(T, S) \) is known and is continuous and strictly increasing. We will denote its cumulative distribution function (cdf) under the measure in which we measure the VaR or the CVaR by \( F_{Y(T,S)}(\cdot) \). For example when the short-rate model is one of the following commonly used interest rate models such as Vasicek, one- and two-factor Hull-White, two-factor additive Gaussian model G2++, two-factor Heath-Jarrow-Morton with deterministic volatilities, see e.g. Brigo and Mercurio (2001), then \( Y(T, S) \) has a lognormal distribution.

Analogously to Ahn et al. (1999), we can look at the future value of the hedged portfolio that is composed of the bond \( Y \) and the put option \( P(0, T, S, X) \) at time \( T \) as a function of the form

\[
H_T = \max(hX + (1 - h)Y(T, S), Y(T, S)).
\]

In a worst case scenario — a case which is of interest to us — the put option finishes in-the-money. Then the future value of the portfolio equals

\[
H_T = (1 - h)Y(T, S) + hX.
\]

Taking into account the cost of setting up our hedged portfolio, which is given by the sum of the bond price \( Y(0, S) \) and the cost \( C \) of the position in the put option, we get for the value of the loss:

\[
L = Y(0, S) + C - ((1 - h)Y(T, S) + hX),
\]

and this under the assumption that the put option finishes in-the-money.

Note that this loss function can be seen as a strictly decreasing function \( f \) in \( Y(T, S) \):

\[
f(Y(T, S)) := Y(0, S) + C - ((1 - h)Y(T, S) + hX).
\]

**VaR minimization**

We first look at the case of determining the optimal strike \( X \) when minimizing the VaR under a constraint on the hedging cost.

Recalling (1) and (4), the Value-at-Risk at an \( \alpha \) percent level of a position \( H = \{Y, h, P\} \) consisting of a bond \( Y \) and \( h \) put options \( P \) (which are assumed to be in-the-money at expiration)
with a strike price $X$ and an expiry date $T$ is equal to

$$\text{VaR}_{\alpha, T}(L) = Y(0, S) + C - ((1 - h)F^{-1}_{Y(T,S)}(\alpha) + hX),$$

(6)

where $F^{-1}_{Y(T,S)}(\alpha)$ is the percentile of the cdf $F_{Y(T,S)}$, i.e. $\Pr[Y(T, S) \leq F^{-1}_{Y(T,S)}(\alpha)] = \alpha$.

Similar to the Ahn et al. problem, we would like to minimize the risk of the future value of the hedged bond $H_T$, given a maximum hedging expenditure $C$. More precisely, we consider the minimization problem

$$\min_{X, h} Y(0, S) + C - ((1 - h)F^{-1}_{Y(T,S)}(\alpha) + hX)$$

subject to the restrictions $C = hP(0, T, S, X)$ and $h \in (0, 1)$.

This is a constrained optimization problem with Lagrange function

$$L(X, h, \lambda) = \text{VaR}_{\alpha, T}(L) - \lambda(C - hP(0, T, S, X)),$$

containing one multiplicator $\lambda$. Note that the multiplicators to include the inequalities $0 < h$ and $h < 1$ are zero since these constraints are not binding. Taking into account that the optimal strike $X^*$ will differ from zero, we find from the Kuhn-Tucker conditions

$$\begin{cases}
\frac{\partial L}{\partial X} = -h + h\lambda \frac{\partial P}{\partial X}(0, T, S, X) = 0 \\
\frac{\partial L}{\partial h} = -(X - F^{-1}_{Y(T,S)}(\alpha)) + \lambda P(0, T, S, X) = 0 \\
\frac{\partial L}{\partial \lambda} = C - hP(0, T, S, X) = 0 \\
0 < h < 1 \quad \text{and} \quad \lambda > 0
\end{cases}$$

that this optimal strike $X^*$ should satisfy the following equation

$$P(0, T, S, X) - (X - F^{-1}_{Y(T,S)}(\alpha)) \frac{\partial P}{\partial X}(0, T, S, X) = 0.$$ 

(7)

By a change of numeraire, it is well known that the put option price equals the discounted expectation under the $T$-forward measure of the the pay-off:

$$P(0, T, S, X) = Y(0, T)E^T[(X - Y(T, S))_+] = Y(0, T)F^T_{Y(T,S)}(X).$$

Its first order derivative with respect to the strike $X$ gives the cumulative distribution function $F^T_{Y(T,S)}$ of $Y(T, S)$ under this $T$-forward measure, see Breeden and Litzenberger (1978):

$$\frac{\partial P}{\partial X}(0, T, S, X) = Y(0, T)F^T_{Y(T,S)}(X).$$

(8)

Hence, (7) is equivalent to

$$P(0, T, S, X) - (X - F^{-1}_{Y(T,S)}(\alpha))Y(0, T)F^T_{Y(T,S)}(X) = 0.$$ 

\footnote{In case of an unhedged portfolio, take $C = h = 0$ in (4) and in (6) to obtain the loss function $L$ with corresponding $\text{VaR}_{\alpha, T}(L)$.}
Important remarks

1. We note that the optimal strike price is independent of the hedging cost $C$. This independence implies that for the optimal strike $X^*$, VaR in (6) is a linear function of $h$ (or $C$):

$$\text{VaR}_{\alpha,T}(L) = Y(0, S) - F_{Y(T,S)}^{-1}(\alpha) + h(P(0, T, S, X^*) + F_{Y(T,S)}^{-1}(\alpha) - X^*).$$

So, there is a linear trade-off between the hedging expenditure and the VaR level. It is a decreasing function since in view of (8) $\frac{\partial P}{\partial X}(0, T, S, X^*) < 1$ and thus according to (7) $X^* - F_{Y(T,S)}^{-1}(\alpha) > P(0, T, S, X^*)$.

Although the setup of the paper is determining the strike price which minimizes a certain risk criterion, given a predetermined hedging budget, this trade-off shows that the analysis and the resulting optimal strike price can evidently also be used in the case where a firm is fixing a nominal value for the risk criterion and seeks the minimal hedging expenditure needed to achieve this risk level. It is clear that, once the optimal strike price is known, we can determine, in both approaches, the remaining unknown variable (either VaR, either $C$).

2. We also note that the optimal strike price is higher than the bond VaR level $F_{Y(T,S)}^{-1}(\alpha)$. This has to be the case since $P(0, T, S, X)$ is always positive and the change in the price of a put option due to an increase in the strike is also positive. This result is also quite intuitive since there is no point in taking a strike price which is situated below the bond price you expect in a worst case scenario.

When moreover the optimal strike is smaller than the forward price of the bond, i.e.

$$X^* < \frac{Y(0, S)}{Y(0, T)},$$

then the price of put option to buy will be small.

3. The assumption of continuity and strictly monotonicity of the distribution of $Y(T, S)$ can be weakened. In that case we should work with the general definition (2) of VaR.

CVaR minimization

In this section, we demonstrate the ease of extending our analysis to the alternative risk measure CVaR (3) by integration of (6):

$$\text{CVaR}_{\alpha,T}(L) = Y(0, S) + C - hX - \frac{1}{\alpha}(1 - h) \int_0^\alpha F_{Y(T,S)}^{-1}(\beta)d\beta. \quad (9)$$

We again seek to minimize this risk measure, in order to minimize potential losses. The procedure for minimizing this CVaR is analogue to the VaR minimization procedure. The resulting optimal strike price $X^*$ can thus be determined from the implicit equation below:

$$P(0, T, S, X) - (X - \frac{1}{\alpha} \int_0^\alpha F_{Y(T,S)}^{-1}(\beta)d\beta)\frac{\partial P}{\partial X}(0, T, S, X) = 0, \quad (10)$$

or, equivalently by (8), from

$$P(0, T, S, X) - (X - \frac{1}{\alpha} \int_0^\alpha F_{Y(T,S)}^{-1}(\beta)d\beta)Y(0, T)F_{Y(T,S)}^T(X) = 0.$$
As for the VaR-case the optimal strike $X^*$ is independent of the hedging cost $C$ and CVaR can be plotted as a linear function of $C$ (or $h$) representing a trade-off between the cost and the level of protection.

For the same reason as in the VaR-case, the optimal strike $X^*$ has to be higher than the bond CVaR level $\frac{1}{\alpha} \int_0^\alpha F_{Y(T,S)}^{-1}(\beta)d\beta$.

3.2. Coupon-bearing bond

We consider now the case of a coupon-bearing bond paying cash flows $C = [c_1, \ldots, c_n]$ at maturities $S = [S_1, \ldots, S_n]$. Let $T \leq S_1$. The price of this coupon-bearing bond in $T$ is expressed as a linear combination (or a portfolio) of zero-coupon bonds:

$$CB(T, S, C) = \sum_{i=1}^n c_i Y(T, S_i).$$  \hspace{1cm} (11)

As in the previous section, the company wants to hedge its position in this bond by buying a percentage of a put option on this bond with strike $X$ and maturity $T$. In order to determine the strike $X$, the VaR or the CVaR of the hedged portfolio at time $T$ is minimized under a budget constraint. Comparing the results in the previous section for VaR and CVaR minimization for a hedged position in zero-coupon bond we note that both cases can in fact be treated together.

We first have a look at the value of a put option on a coupon-bearing bond as well as at the structure of the loss function.

Since the zero-coupon bonds $Y(T, S_i)$ all depend on the same short rate at $T$, the vector $(Y(T, S_1), \ldots, Y(T, S_n))$ is comonotonic, see Kaas et al. (2000). By the properties of comonotonic vectors, the coupon-bearing bond $CB(T, S, C)$ (11) is a comonotonic sum with cumulative distribution function $F_{CB}^T(\cdot)$ under the $T$-forward measure. This implies that a European option on a coupon-bearing bond decomposes into a portfolio of options on the individual zero-coupon bonds in the portfolio, which gives in case of a put with maturity $T$ and strike $X$:

$$CBP(0, T, S, C, X) = \sum_{i=1}^n c_i P(0, T, S_i, X_i), \quad \text{with} \quad \sum_{i=1}^n c_i X_i = X.$$  \hspace{1cm} (12)

This result, now well-known as the Jamshidian decomposition, was found in Jamshidian (1989) in case of a Vasicek interest rate model. Kaas et al. (2000) obtained this result in a more general framework of stop-loss premiums and gave an explicit expression for the $X_i$:

$$X_i = (F_{Y(T,S_i)}^T)^{-1}(F_{CB}^T(X)).$$  \hspace{1cm} (13)

Repeating the reasoning of Section 3.1 we may conclude that in a worst case scenario the loss of the hedged portfolio at time $T$ composed of the coupon-bearing bond (11) and the put option (12) equals a strictly decreasing function $f$ of the random variable $CB(T, S, C)$:

$$L = CB(0, S, C) + C - ((1 - h)CB(T, S, C) + hX) := f(CB(T, S, C)).$$  \hspace{1cm} (14)
VaR and CVaR minimization

The VaR of this loss that we want to minimize under the constraints $0 < h < 1$ and $C = hCBP(0, T, S, C, X)$, is given by

$$VaR_{\alpha,T}(L) = f(F_{\text{CB}}^{-1}(\alpha)) = CB(0, S, C) + C - (1 - h)F_{\text{CB}}^{-1}(\alpha) + hX,$$  \hspace{1cm} (15)

where $F_{\text{CB}}^{-1}$ stands for the inverse cdf of the coupon-bearing bond under the measure in which VaR (and CVaR) is measured.

By integrating this relation (15), after replacing $\alpha$ by $\beta$, with respect to $\beta$ between the integration bounds 0 and $\alpha$, we find for the CVaR of the loss:

$$CVaR_{\alpha,T}(L) = CB(0, S, C) + C - hX - \frac{1}{\alpha}(1 - h)\int_0^\alpha F_{\text{CB}}^{-1}(\beta)d\beta.$$  \hspace{1cm} (16)

Also here we note the similarity in the expressions for the risk measures (RM) VaR and CVaR which could be collected in one expression:

$$RM_{\alpha,T}(L) = CB(0, S, C) + C - hX - (1 - h)g(F_{\text{CB}}^{-1}(\alpha))$$  \hspace{1cm} (17)

with $g(F_{\text{CB}}^{-1}(\alpha)) = \begin{cases} F_{\text{CB}}^{-1}(\alpha) & \text{if } RM = \text{VaR} \\ \frac{1}{\alpha}\int_0^\alpha F_{\text{CB}}^{-1}(\beta)d\beta & \text{if } RM = \text{CVaR}. \end{cases}$  \hspace{1cm} (18)

Although the marginal distributions $F_{Y(T,S_i)}$ are known, the distribution $F_{\text{CB}}$ of the sum can in general not be obtained. However, in the case of a comonotonic sum we have, see again Kaas et al. (2000),

$$F_{\text{CB}}^{-1}(p) = \sum_{i=1}^n c_i F_{Y(T,S_i)}^{-1}(p) \text{ for all } p \in [0,1],$$  \hspace{1cm} (19)

and similarly for the inverse cdfs under the T-forward measure.

We now want to solve the constrained optimization problem

$$\min_{X,h} RM_{\alpha,T}(L) \text{ subjected to } C = hCBP(0, T, S, C, X), \quad 0 < h < 1.$$  \hspace{1cm}

From the Kuhn-Tucker conditions we find that the optimal strike price $X^*$ satisfies the following equation

$$CBP(0, T, S, C, X) - \frac{(X - g(F_{\text{CB}}^{-1}(\alpha)))}{CBP(0, T, S, C, X)} = 0.$$  \hspace{1cm} (20)

Rewriting this equation in terms of the put options on the individual zero-coupon bonds cfr. (12), invoking (19) and using the linearity of the function $g$ (18), leads to the following equivalent set of equations:

$$\sum_{i=1}^n c_i P(0, T, S_i, X_i) - (X - \sum_{i=1}^n c_i g(F_{Y(T,S_i)}^{-1}(\alpha))) \sum_{i=1}^n c_i \frac{\partial P}{\partial X_i}(0, T, S_i, X_i) \frac{\partial X_i}{\partial X} = 0 $$  \hspace{1cm} (21)

$$\sum_{i=1}^n c_i X_i = X$$  \hspace{1cm} (22)

$$\sum_{i=1}^n c_i \frac{\partial X_i}{\partial X} = 1,$$  \hspace{1cm} (23)
where \( X_i \) is defined by (13).

We can further simplify relation (21) by applying relation (8) to the strike \( X_i \) given by (13), i.e.

\[
\frac{\partial P}{\partial X_i}(0, T, S_i, X_i) = Y(0, T) F^T_{Y(T, S_i)}(\{(F^T_{Y(T, S_i)})^{-1}(F^T_{CB}(X))\}) = Y(0, T) F^T_{CB}(X).
\]

Hence, this derivative is independent of \( i \) which implies in view of (23) that

\[
\sum_{i=1}^{n} c_i \frac{\partial P}{\partial X_i}(0, T, S_i, X_i) \frac{\partial X_i}{\partial X} = Y(0, T) F^T_{CB}(X) \sum_{i=1}^{n} c_i \frac{\partial X_i}{\partial X} = Y(0, T) F^T_{CB}(X).
\]

We introduce the short hand notation

\[
A_X := F^T_{CB}(X).
\]

Substitution of (13), (22) and (24) in (21) leads to the following equation that we have to solve for \( A_X \):

\[
\sum_{i=1}^{n} c_i P(0, T, S_i, (F^T_{Y(T, S_i)})^{-1}(A_X)) - Y(0, T) A_X \sum_{i=1}^{n} c_i [(F^T_{Y(T, S_i)})^{-1}(A_X) - g(F^{-1}_{Y(T, S_i)}(\alpha))] = 0.
\]

Once, we know \( A_X \) we immediately have the optimal strike \( X^* \) from (22):

\[
X^* = \sum_{i=1}^{n} c_i (F^T_{Y(T, S_i)})^{-1}(A_X).
\]

**Remarks**

1. We note that also in the case of a coupon-bearing bond the optimal strike price is independent of the hedging cost and that one can look at the trade-off between the hedging expenditure and the RM level, cfr. Section 3.1.

2. Also here we may weaken the assumption of continuity and strictly monotonicity of the distribution functions \( F_{Y(T, S_i)} \). In that case we have to invoke Kaas et al. (2000) with a so-called \( \eta \)-inverse distribution of a random variable \( Y \) which is defined as the following convex combination:

\[
F^{-1}_Y(p) = \eta F^{-1}_Y(p) + (1 - \eta) F^{-1}_Y(p), \quad p \in (0, 1), \quad \eta \in [0, 1],
\]

\[
F^{-1}_Y(p) = \inf \{ y \in \mathbb{R} \mid F_Y(y) \geq p \}, \quad p \in [0, 1],
\]

\[
F^{-1}_Y(p) = \sup \{ y \in \mathbb{R} \mid F_Y(y) \leq p \}, \quad p \in [0, 1].
\]

Thus relation (12) holds with

\[
X_i = (F^T_{Y(T, S_i)})^{-1}(F^T_{CB}(X)), \quad \eta \in [0, 1] \text{ is determined from}
\]

\[
\sum_{i=1}^{n} c_i (F^T_{Y(T, S_i)})^{-1}(F^T_{CB}(X)) = X.
\]
4. APPLICATION: HULL-WHITE MODEL

As an application, we focus on the Hull-White one-factor model, first discussed by Hull and White in 1990 (see Hull and White (1990)). We choose this model because it is still an often used model in financial institutions for risk management purposes, (see Brigo and Mercurio (2001)).

Hull and White (1990) assume under the risk-neutral measure $Q$ that the instantaneous interest rate follows a mean reverting process also known as an Ornstein-Uhlenbeck process:

$$dr(t) = (\theta(t) - \gamma(t)r(t))dt + \sigma(t)dZ(t)$$

(28)

with $Z(t)$ a standard Brownian motion under $Q$, and with time dependent parameters $\theta(t)$, $\gamma(t)$, and $\sigma(t)$. The parameter $\theta(t)$ is the time dependent long-term average level of the spot interest rate around which $r(t)$ moves, $\gamma(t)$ controls the mean-reversion speed and $\sigma(t)$ is the volatility function. By making the mean reversion level $\theta(t)$ time dependent, a perfect fit with a given term structure can be achieved, and in this way arbitrage can be avoided. In our analysis, we will keep $\gamma$ and $\sigma$ constant, and thus time-independent. According to Brigo and Mercurio (2001), this is desirable when an exact calibration to an initial term structure is wanted. This perfect fit then occurs when $\theta(t)$ satisfies the following condition:

$$\theta(t) = F^M_t(0,t) + \gamma F^M_t(0,t) + \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma t})$$

where, $F^M_t(0,t)$ denotes the instantaneous forward rate observed in the market on time zero with maturity $t$.

It can be shown (see Hull and White (1990)) that the expectation and variance of the stochastic variable $r(t)$ are:

$$E[r(t)] = m(t) = r(0)e^{-\gamma t} + a(t) - a(0)e^{-\gamma t}, \quad \text{Var}[r(t)] = s^2(t) = \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma t})$$

(29)

with the expression $a(t)$ calculated as follows:

$$a(t) = F^M_t(0,t) + \frac{\sigma^2}{2} \left( \frac{1 - e^{-\gamma t}}{\gamma} \right)^2.$$

Based on these results, Hull and White developed an analytical expression for the price of a zero-coupon bond with maturity date $S$:

$$Y(t, S) = A(t, S)e^{-B(t,S)r(t)},$$

where

$$B(t, S) = \frac{1 - e^{-\gamma(S-t)}}{\gamma}, \quad A(t, S) = \frac{Y^M_t(0,S)}{Y^M_t(0,t)}e^{B(t,S)F^M_t(0,t)} - \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma t})B^2(t,S)$$

with $Y^M_t$ the bond price observed in the market. Since $A(t, S)$ and $B(t, S)$ are independent of $r(t)$, the distribution of a bond price at any given time must be lognormal with parameters $\Pi$ and $\Sigma^2$:

$$\Pi(t, S) = \ln A(t, S) - B(t, S)m(t), \quad \Sigma(t, S)^2 = B(t, S)^2s^2(t),$$
with \( m(t) \) and \( s^2(t) \) given by (29). Thus under the risk neutral measure the inverse cdf of \( Y(T, S) \) is given by
\[
F_{Y(T, S)}^{-1}(p) = e^{\Pi(T, S) + \Sigma(T, S)\Phi^{-1}(p)}, \quad p \in [0, 1],
\]
and we can compute the (standard) integral
\[
\int_0^\alpha F_{Y(T, S)}^{-1}(\beta) d\beta = e^{\Pi(T, S)} \int_0^\alpha e^{\Sigma(T, S)\Phi^{-1}(\beta)} d\beta = e^{\Pi(T, S) + \frac{1}{2}\Sigma(T, S)\Phi(\Phi^{-1}(\alpha) - \Sigma(T, S))}. \tag{31}
\]
By a change of numeraire it can be shown that \( Y(T, S) \) remains lognormally distributed under the \( T \)-forward measure but now with parameters \( \Pi^T \) and \( \Sigma^T \) given by:
\[
\Pi^T(T, S) = \ln \left( \frac{Y(0, S)}{Y(0, T)} \right) - \frac{1}{2} (\Sigma^T(T, S))^2, \quad \Sigma^T(T, S) = \Sigma(T, S). \tag{32}
\]
Hence, the inverse cdf of \( Y(T, S) \) under the \( T \)-forward measure is known explicitly:
\[
(F_{Y(T, S)}^T)^{-1}(p) = e^{\Pi^T(T, S) + \Sigma(T, S)\Phi^{-1}(p)}, \quad p \in [0, 1],
\]
as well as the put option price and its derivative with respect to the strike:
\[
P(0, T, S, X) = -Y(0, S)\Phi(-d_1(X)) + XY(0, T)\Phi(-d_2(X)),
\]
\[
\frac{\partial P}{\partial X}(0, T, S, X) = Y(0, T)\Phi(-d_2(X)),
\]
with, when taking (32) into account,
\[
d_1(X) = \frac{1}{\Sigma(T, S)} \left[ \ln \left( \frac{Y(0, S)}{Y(0, T)} \right) - \ln(X) \right] + \frac{1}{2} \Sigma(T, S) = \frac{\Pi^T(T, S) - \ln(X)}{\Sigma(T, S)} + \Sigma(T, S) \tag{34}
\]
\[
d_2(X) = d_1(X) - \Sigma(T, S) = \frac{\Pi^T(T, S) - \ln(X)}{\Sigma(T, S)}. \tag{35}
\]
For the zero-coupon case, substitution of the relations above in (7) and in (10) gives the following implicit relation for the optimal strike \( X^* \):
\[
G(\Phi^{-1}(\alpha)) = \frac{Y(0, S)\Phi(-d_1(X))}{Y(0, T)\Phi(-d_2(X))},
\]
with
\[
G(\Phi^{-1}(\alpha)) = \begin{cases} e^{\Pi(T, S) + \Sigma(T, S)\Phi^{-1}(\alpha)} & \text{if VaR} \\ \frac{1}{\alpha} e^{\Pi(T, S) + \frac{1}{2}\Sigma^2(T, S)\Phi(\Phi^{-1}(\alpha) - \Sigma(T, S))} & \text{if CVaR}. \end{cases} \tag{36}
\]
For the coupon-bearing bond case, the above relations for the distribution and the put option price hold but with \( S \) and \( X \) replaced by \( S_i \) and \( X_i \). The expressions (34) and (35) for \( d_1(X_i) \) and \( d_2(X_i) \) can further be simplified in view of (13),(25) and (31):
\[
d_1(X_i) = \Sigma(T, S_i) - \Phi^{-1}(A_X), \quad d_2(X_i) = -\Phi^{-1}(A_X).
In this way, the set of equations (26)-(27) to find the optimal strike $X^*$ is equivalent with:

$$\sum_{i=1}^{n} c_i \left[ -Y(0, S_i) \Phi(\Phi^{-1}(A_X) - \Sigma(T, S_i)) + Y(0, T) A_X e^{\Pi(T, S_i) + \Sigma(T, S_i) \Phi^{-1}(A_X)} \right]$$

$$= Y(0, T) A_X \sum_{i=1}^{n} c_i \left[ e^{\Pi(T, S_i) + \Sigma(T, S_i) \Phi^{-1}(A_X)} - G_i(\Phi^{-1}(\alpha)) \right]$$

$$X^* = \sum_{i=1}^{n} c_i e^{\Pi(T, S_i) + \Sigma(T, S_i) \Phi^{-1}(A_X)}$$

where $G_i(\Phi^{-1}(\alpha))$ is defined by (36) when replacing $S$ by $S_i$.

For a complete numerical example we refer to Deelstra et al. (2005) and Heyman et al. (2006).

5. CONCLUSIONS

We provided a method for minimizing the risk of a position in a bond (zero-coupon or coupon-bearing) by buying (a percentage of) a bond put option. Taking into account a budget constraint, we determine the optimal strike price, which minimizes a Value-at-Risk or Conditional Value-at-Risk criterion. Alternatively, our approach can be used when a nominal risk level is fixed, and the minimal hedging budget to fulfill this criterion is desired. From the class of short rate models which result in lognormally distributed future bond prices, we have selected the Hull-White one-factor model for an illustration of our optimization.

References


Chapter 4:

Risk management of a bond portfolio using options
Risk management of a bond portfolio using options

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Abstract

In this paper, we elaborate a formula for determining the optimal strike price for a bond put option, used to hedge a position in a bond. This strike price is optimal in the sense that it minimizes, for a given budget, either Value-at-Risk or Tail Value-at-Risk. Formulas are derived for both zero-coupon and coupon bonds, which can also be understood as a portfolio of bonds. These formulas are valid for any short rate model that implies an affine term structure model and in particular that implies a lognormal distribution of future zero-coupon bond prices. As an application, we focus on the Hull-White one-factor model, which is calibrated to a set of cap prices. We illustrate our procedure by hedging a Belgian government bond, and take into account the possibility of divergence between theoretical option prices and real option prices. This paper can be seen as an extension of the work of Ahn et al. (1999), who consider the same problem for an investment in a share.

\textit{JEL classification:} G11, C61

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\textit{Key words:} (Tail) Value-at-Risk, bond hedging, affine term structure model

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1 Introduction

Several studies document risk management practices in a corporate setting, see for example Bodnar et al. (1998), Bartram et al. (2004), Prevost et al. (2000). Survey techniques are often employed to get insights into why and how firms implement hedging strategies. In the vast majority of studies, the widespread usage of these hedging policies is confirmed. In each of the above mentioned surveys, at least 50% of the firms reported that they make use of some kind of derivatives. The most popular derivatives are forwards, options and swaps. These instruments can be used to hedge exposures due to currency, interest rate and other market risks. Swaps are most frequently used to tackle interest rate risks, followed by forwards and options. Using these kind of derivatives is surely a first step in successful risk management. However, a second step is formed by using these derivatives in an optimal way. Although tools like swaps and options are basic building blocks for all sorts of other, more complicated derivatives, they should be used prudently and a firm knowledge of their properties is needed. These derivatives have a multitude of decision parameters, which necessitates thoroughly investigating the influence of these parameters on the aims of the hedging policies and the possibility to achieve these goals.

The literature on risk management is much more silent on how to optimally decide on these parameters. The present study partly fills this gap. We consider the problem of determining the optimal strike price for a bond put option, which is used to hedge the interest rate risk of an investment in a bond. In order to measure risk, we focus on both Value-at-Risk (VaR) and Tail Value-at-Risk (TVaR). Our optimization is constrained by a maximum hedging budget. Alternatively, our approach can also be used to determine the minimal budget a firm needs to spend in order to achieve a predetermined absolute risk level.

The setup of our paper is similar in spirit as Ahn et al. (1999). However, we emphasize that our paper contributes in several aspects. First of all, our analysis is carried out for another asset class. Whereas Ahn et al. (1999) consider stocks, our focus lies on bonds. The importance of bonds as an investment tool can hardly be underestimated. As reported in the European institutional market place overview 2006 of Mercer Investment Consulting (see MercerAssetAloc (2006)), pension funds in continental Europe invest more than half of their resources in bonds. This makes fixed income securities an asset class that should not be neglected. Secondly, Ahn et al. (1999) assume that stock prices are driven by a geometric Brownian motion. Our analysis generalises their results since we only assume that the price of the asset we consider is driven by a one factor model with an affine structure. This encompasses the Brownian motion process which is often used for stocks, but also allows for mean reverting processes, which are crucial in interest rate modelling and the pricing of fixed income securities. Concrete examples of the term structure models that are captured by our approach are: Vasicek, one-factor Hull-White and one-factor Heath-Jarrow-Morton with deterministic volatility. Furthermore, we develop formulas for not only a zero-coupon bond, but also for a coupon-bearing bond. Finally, as risk measure, we consider both VaR and TVaR. As stated below, VaR is a very popular risk measure
but it is not free of criticism. An important drawback of VaR is that it is a risk measure which ignores what really happens in the tail. Furthermore, it is not a coherent measure, as precised by Artzner et al. (1999). These two problems are tackled when TVaR is used as risk measure.

Taking into account the advent of new capital regulations in both the bank (Basel II) and the insurance industry (Solvency II), our insights can play a role in implementing a sound risk management system.

In the next section we introduce the loss function as well as the risk measures that will be used. In Section 3 we formulate the bond hedging problem, first for a zero-coupon bond and next for a coupon-bearing bond. We assume a short rate model for the instantaneous interest rate with an affine term structure. Not only the VaR of the loss function but also its TVaR is minimized under the budget constraint. We pay special attention to the case that the zero-coupon bond price is lognormally distributed. In Heyman et al. (2006) we treat this problem theoretically in a more general framework by only assuming that the cumulative distribution function of the zero-coupon bond price at a later time instance before maturity is known. In Section 4 we illustrate the procedure by hedging a Belgian government bond, and take into account the possibility of divergence between theoretical option prices and real option prices. Section 5 concludes the paper.

2 Loss function and risk measures

Consider a portfolio with value $W_t$ at time $t$. $W_0$ is then the value or price at which we buy the portfolio at time zero. $W_T$ is the value of the portfolio at time $T$. The loss $L$ we make by buying at time zero and selling at time $T$ is then given by $L = W_0 - W_T$. The Value-at-Risk $\text{VaR}_{\alpha,T}$ of this portfolio is defined as the $(1 - \alpha)$-quantile of the loss distribution depending on a time interval with length $T$ and is also called the VaR at an $\alpha$ per cent level. A formal definition for the $\text{VaR}_{\alpha,T}$ is

$$\Pr[L \geq \text{VaR}_{\alpha,T}] = \alpha.$$  \hspace{1cm} (1)

In other words $\text{VaR}_{\alpha,T}$ is the loss of the worst case scenario on the investment at a $(1 - \alpha)$ confidence level at time $T$. It is also possible to define the $\text{VaR}_{\alpha,T}$ in a more general way

$$\text{VaR}_{\alpha,T}(L) = \inf \{ \ell \in \mathbb{R} \mid \Pr(L > \ell) \leq \alpha \}.$$  \hspace{1cm} (2)

Although frequently used, VaR has attracted some criticisms. First of all, a drawback of the traditional Value-at-Risk measure is that it does not care about the tail behaviour of the losses. In other words, by focusing on the VaR at, let’s say a 5% level, we ignore the potential severity of the losses below that 5% threshold. This means that we have
no information on how bad things can become in a real stress situation. Therefore, the
important question of ‘how bad is bad’ is left unanswered. Secondly, it is not a coherent
risk measure, as suggested by Artzner et al. (1999). More specifically, it fails to fulfil
the subadditivity requirement which states that a risk measure should always reflect the
advantages of diversifying, that is, a portfolio will risk an amount no more than, and
in some cases less than, the sum of the risks of the constituent positions. It is possible to
provide examples that show that VaR is sometimes in contradiction with this subadditivity
requirement.

Artzner et al. (1999) suggested the use of CVaR (Conditional Value-at-Risk) as risk mea-
sure, which they describe as a coherent risk measure. CVaR is also known as TVaR, or
Tail Value-at-Risk and is defined as follows:

\[ \text{TVaR}_{\alpha,T}(L) = \frac{1}{\alpha} \int_{1-\alpha}^{1} \text{VaR}_{1-\beta,T}(L) d\beta. \]  

This formula boils down to taking the arithmetic average of the quantiles of our loss,
from \( 1 - \alpha \) to 1 on, where we recall that \( \text{VaR}_{1-\beta,T}(L) \) stands for the \( \beta \)-quantile of the loss
distribution, see (1).

A closely related risk measure concerns Expected Shortfall (ESF). It is defined as:

\[ \text{ESF}_{\alpha,T}(L) = E[(L - \text{VaR}_{\alpha,T}(L))_+]. \]  

In order to determine TVaR\(_{\alpha,T}(L)\), we can also make use of the following equality:

\[ \text{TVaR}_{\alpha,T}(L) = \text{VaR}_{\alpha,T}(L) + \frac{1}{\alpha} \text{ESF}_{\alpha,T}(L) \]

\[ = \text{VaR}_{\alpha,T}(L) + \frac{1}{\alpha} E[(L - \text{VaR}_{\alpha,T}(L))_+]. \]

This formula already makes clear that TVaR\(_{\alpha,T}(L)\) will always be larger than \( \text{VaR}_{\alpha,T}(L) \).
If moreover the cumulative distribution function of the loss is continuous, TVaR is also
equal to the Conditional Tail Expectation (CTE) which for the loss \( L \) is calculated as:

\[ \text{CTE}_{\alpha,T}(L) = E[L \mid L > \text{VaR}_{\alpha,T}(L)]. \]

This is for example the case in the bond hedging problem that we consider in the subse-
quent sections, when bond prices are lognormally distributed.

3 The bond hedging problem

Analogously to Ahn et al. (1999), we assume that we have, at time zero, one zero-coupon
bond with maturity \( S \) and we will sell this bond at time \( T \), which is prior to \( S \). In case of an increase in
interest rates, not hedging can lead to severe losses. Therefore, the company decides to spend an amount $C$ on hedging. This amount will be used to buy one or part of a bond put option with the bond as underlying, so that, in case of a substantial decrease in the bond price, the put option can be exercised in order to prevent large losses. The remaining question now is how to choose the strike price. We will find the optimal strike prices which minimize VaR and TVaR respectively for a given hedging cost. An alternative interpretation of our setup is that it can be used to calculate the minimal hedging budget the firm has to spend in order to achieve a specified VaR or TVaR level, a setup which was followed in the paper by Miyazaki (2001) in another setting.

### 3.1 Zero-coupon bond

Let us assume that the institution has at date zero an exposure to a bond, $P(0, S)$, with principal $N = 1$, which matures at time $S$, and that the company has decided to hedge the bond value by using a percentage $h$ ($0 < h < 1$) of one put option $ZBP(0, T, S, X)$ with strike price $X$ and exercise date $T$ (with $T \leq S$).

Further, we assume a short rate model for $r(T)$ with an affine term structure such that the zero-coupon bond price $P(T, S)$ can be written in the form

$$P(T, S) = A(T, S)e^{-B(T, S)r(T)},$$

with parameters $A(T, S) (> 0)$ and $B(T, S) (> 0)$ independent of $r(T)$.

This assumption covers a range of commonly used interest rate models such as Vasicek, one-factor Hull-White and one-factor Heath-Jarrow-Morton with deterministic volatility, see e.g. Brigo and Mercurio (2001).

In Heyman et al. (2006) we treat this problem theoretically in a more general framework. We make no assumption on $r(T)$, we only assume that the cumulative distribution function of $P(T, S)$ is known.

Analogously as in the paper of Ahn et al. (1999), we can look at the future value of the hedged portfolio that is composed of the bond $P$ and the put option $ZBP(0, T, S, X)$ at time $T$ as a function of the form

$$H_T = \max(hX + (1 - h)P(T, S), P(T, S)).$$

In a worst case scenario — a case which is of interest to us — the put option finishes in-the-money. Then the future value of the portfolio equals

$$H_T = (1 - h)P(T, S) + hX.$$

Taking into account the cost of setting up our hedged portfolio, which is given by the sum of the bond price $P(0, S)$ and the cost $C$ of the position in the put option, we get for the
value of the loss:

\[ L = P(0, S) + C - ((1 - h)P(T, S) + hX), \]

and this under the assumption that the put option finishes in-the-money.

In view of the assumption on the form of \( P(T, S) \), this loss of the portfolio equals a strictly increasing and continuous function \( f \) of the random variable \( r(T) \):

\[
f(r(T)) := L = P(0, S) + C - ((1 - h)A(T, S)e^{-B(T,S)r(T)} + hX).
\]

(8)

**VaR minimization**

We first look at the case of determining the optimal strike \( X \) when minimizing the VaR under a constraint on the hedging cost.

**Lemma 1** Under the assumption of an affine term structure such that the zero-coupon bond price \( P(T, S) \) is given by (7), the Value-at-Risk at an \( \alpha \) percent level of a position \( H = \{P, h, ZBP\} \) consisting of the bond \( P(T, S) \) and \( h \) put options ZBP on this zero-coupon bond (which are assumed to be in-the-money at expiration) with a strike price \( X \) and an expiry date \( T \) is equal to

\[
\text{VaR}_{\alpha,T}(L) = P(0, S) + C - ((1 - h)A(T, S)e^{-B(T,S)F_{r(T)}^{-1}(1-\alpha)} + hX),
\]

where \( F_{r(T)} \) denotes the cumulative distribution function (cdf) of \( r(T) \) and \( F_{r(T)}^{-1} \) stands for the inverse of this cdf and is defined in the usual way:

\[
F_{r(T)}^{-1}(p) = \inf \left\{ x \in \mathbb{R} \mid F_{r(T)}(x) \geq p \right\}, \quad p \in [0,1].
\]

(10)

**PROOF.** We start from the general definition (2) of VaR, use definition (8) of the function \( f \), the fact that \( f \) is strictly increasing and the definition (10) of the inverse cdf to obtain consecutively:

\[
\begin{align*}
\text{VaR}_{\alpha,T}(L) &= \inf \{ \ell \in \mathbb{R} \mid \Pr(L > \ell) \leq \alpha \} \\
&= \inf \{ \ell \in \mathbb{R} \mid \Pr(f(r(T)) > \ell) \leq \alpha \} \\
&= \inf \{ \ell \in \mathbb{R} \mid \Pr(r(T) > f^{-1}(\ell)) \leq \alpha \} \\
&= \inf \{ \ell \in \mathbb{R} \mid \Pr(r(T) \leq f^{-1}(\ell)) \geq 1 - \alpha \} \\
&= \inf \{ \ell \in \mathbb{R} \mid F_{r(T)}(f^{-1}(\ell)) \geq 1 - \alpha \} \\
&= f(F_{r(T)}^{-1}(1-\alpha)).
\end{align*}
\]

Finally, invoking again definition (8) of the function \( f \) we arrive at (9). \( \square \)

\(^1\) In case of an unhedged portfolio, take \( C = h = 0 \) in (8) and in (9) to obtain the loss function \( L \) with corresponding \( \text{VaR}_{\alpha,T}(L) \).
Similar to the Ahn et al. problem, we would like to minimize the risk of the future value of the hedged bond $H_T$, given a maximum hedging expenditure $C$. More precisely, we consider the minimization problem

$$\min_{X, h} P(0, S) + C - ((1 - h)A(T, S)e^{-B(T, S)F^{-1}_{T}(1-\alpha)} + hX)$$

subject to the restrictions $C = hZBP(0, T, S, X)$ and $h \in (0, 1)$.

This is a constrained optimization problem with Lagrange function

$$\mathcal{L}(X, h, \lambda) = \text{VaR}_{\alpha,T}(L) - \lambda(C - hZBP(0, T, S, X)),$$

containing one multiplicator $\lambda$. Note that the multiplicators to include the inequalities $0 < h$ and $h < 1$ are zero since these constraints are not binding. Taking into account that the optimal strike $X^*$ will differ from zero, we find from the Kuhn-Tucker conditions

$$\begin{cases}
\frac{\partial \mathcal{L}}{\partial X} = -h + h\lambda \frac{\partial ZBP}{\partial X} (0, T, S, X) = 0 \\
\frac{\partial \mathcal{L}}{\partial h} = -(X - A(T, S)e^{-B(T, S)F^{-1}_{T}(1-\alpha)}) + \lambda ZBP(0, T, S, X) = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} = C - hZBP(0, T, S, X) = 0 \\
0 < h < 1 \quad \text{and} \quad \lambda > 0
\end{cases}$$

that this optimal strike $X^*$ should satisfy the following equation

$$ZBP(0, T, S, X) - (X - A(T, S)e^{-B(T, S)F^{-1}_{T}(1-\alpha)}) \frac{\partial ZBP}{\partial X} (0, T, S, X) = 0. \quad (11)$$

It is well known that, by a change of numeraire, the put option price equals the discounted expectation under the $T$-forward measure of the payoff:

$$ZBP(0, T, S, X) = P(0, T)E^T[(X - P(T, S))_+] \quad (12)$$

When the cumulative distribution function $F^T_{P(T, S)}$ of $P(T, S)$ under this $T$-forward measure has bounded variation and the expectation $E^T[P(T, S)]$ is finite, then by partial integration we find

$$ZBP(0, T, S, X) = P(0, T) \int_{-\infty}^{X} (F^T_{P(T, S)}(p) - 1) dp.$$

Its first order derivative with respect to the strike $X$ leads immediately to

$$\frac{\partial ZBP}{\partial X} (0, T, S, X) = P(0, T)F^T_{P(T, S)}(X). \quad (12)$$

This relation between the cdf and the price of the put option is analogous to a result derived in a Black&Scholes framework in Breeden and Litzenberger (1978). Since the
randomness of $P(T, S)$ is completely due to the randomness of $r(T)$, relation (7) implies the following connection between their cdfs under the $T$-forward measure (indicated by the subscript $T$):

$$
F^T_{P(T, S)}(X) = 1 - F^T_{r(T)} \left( \frac{\ln A(T, S) - \ln X}{B(T, S)} \right).
$$

(13)

Hence, (11) is equivalent to

$$
ZBP(0, T, S, X) = (X - A(T, S)e^{-B(T, S)F_r^{-1}(1-\alpha)})P(0, T) \left[ 1 - F^T_{r(T)} \left( \frac{\ln A(T, S) - \ln X}{B(T, S)} \right) \right].
$$

Important remarks

1. We note that the optimal strike price is independent of the hedging cost $C$. This independence implies that for the optimal strike $X^*$, VaR in (9) is a linear function of $h$ (or $C$):

$$
\text{VaR}_{\alpha, T}(L) = P(0, S) - A(T, S)e^{-B(T, S)F_r^{-1}(1-\alpha)} + h(ZBP(0, T, S, X^*) + A(T, S)e^{-B(T, S)F_r^{-1}(1-\alpha)} - X^*).
$$

So, there is a linear trade-off between the hedging expenditure and the VaR level, see Figure 1 in the application of Section 4. It is a decreasing function since in view of (12) $\frac{\partial ZBP}{\partial X}(0, T, S, X^*) < 1$ and thus according to (11)

$$
X^* - A(T, S)e^{-B(T, S)F_r^{-1}(1-\alpha)} > ZBP(0, T, S, X^*).
$$

(14)

Although the setup of the paper is determining the strike price which minimizes a certain risk criterion, given a predetermined hedging budget, this trade-off shows that the analysis and the resulting optimal strike price can evidently also be used in the case where a firm is fixing a nominal value for the risk criterion and seeks the minimal hedging expenditure needed to achieve this risk level. It is clear that, once the optimal strike price is known, we can determine, in both approaches, the remaining unknown variable (either VaR, either $C$).

2. We also note that the optimal strike price $X^*$ is higher than the bond VaR level

$$
A(T, S)e^{-B(T, S)F_r^{-1}(1-\alpha)}.
$$

This has to be the case since inequality (14) holds with $ZBP(0, T, S, X)$ being positive. This result is also quite intuitive since there is no point in taking a strike price which is situated below the bond price you expect in a worst case scenario.

When moreover the optimal strike is smaller than the forward price of the bond, i.e.

$$
X^* < \frac{P(0, S)}{P(0, T)}.
$$
then the time zero price of the put option to buy will be small.

**TVaR minimization**

In this section, we demonstrate the ease of extending our analysis to the alternative risk measure TVaR (3) by integrating $\text{VaR}_{1-\beta,T}(L)$, given by (9) with $\alpha = 1 - \beta$, with respect to $\beta$:

$$\text{TVaR}_{\alpha,T}(L) = P(0, S) + C - hX - \frac{1}{\alpha}(1 - h) A(T, S) \int_{1-\alpha}^{1} e^{-B(T,S)F^{-1}_{r(T)}(\beta)} d\beta. \quad (15)$$

We again seek to minimize this risk measure, in order to minimize potential losses. The procedure for minimizing this TVaR is analogous to the VaR minimization procedure. The resulting optimal strike price $X^*$ can thus be determined from the implicit equation below:

$$\frac{\partial ZBP}{\partial X}(0, T, S, X) = 0 \quad (16)$$

which is in view of (12)-(13) equivalent to

$$ZBP(0, T, S, X) = P(0, T)[X - \frac{A(T, S)}{\alpha} \int_{1-\alpha}^{1} e^{-B(T,S)F^{-1}_{r(T)}(\beta)} d\beta] \times$$

$$\times [1 - F^{T}_{r(T)} \left( \frac{\ln A(T, S) - \ln X}{B(T, S)} \right)] \quad (17)$$

As for the VaR-case the optimal strike $X^*$ is independent of the hedging cost $C$ and TVaR can be plotted as a linear function of $C$ (or $h$) representing a trade-off between the cost and the level of protection.

For the same reason as in the VaR-case, the optimal strike $X^*$ has to be higher than the bond TVaR level $\frac{1}{\alpha} A(T, S) \int_{1-\alpha}^{1} e^{-B(T,S)F^{-1}_{r(T)}(\beta)} d\beta$.

**Expected shortfall**

Substitution of the expressions (9) and (15) for the VaR and the TVaR in (5) or (6) provides immediately the value of the expected shortfall of the loss $L$:

$$E_{\alpha,T}(L) = \alpha[\text{TVaR}_{\alpha,T}(L) - \text{VaR}_{\alpha,T}(L)]$$

$$= (1 - h) A(T, S)[ae^{-B(T,S)F^{-1}_{r(T)}(1-\alpha)} - \int_{1-\alpha}^{1} e^{-B(T,S)F^{-1}_{r(T)}(\beta)} d\beta]. \quad (18)$$
Summary

The implicit equations (11) and (16) to solve for the optimal strike price $X^*$ in the VaR-case respectively the TVaR-case, have the same structure and only differ by the risk measure level. Hence, we can treat these as one problem when we introduce the notation $RM$ for the risk measures VaR and TVaR. Further we put for the bond risk measure level:

$$RM_{level} = \begin{cases} 
A(T, S)e^{-B(T, S)F_{r(T)}^{-1}(1-\alpha)} & \text{if VaR} \\
\frac{1}{\alpha}A(T, S)\int_{1-\alpha}^{1} e^{-B(T, S)F_{r(T)}^{-1}(\beta)}d\beta & \text{if TVaR.}
\end{cases} \tag{19}$$

Hence, the results that we derived above can be summarized as follows:

**Theorem 2** Under the assumption of an affine term structure such that the zero-coupon bond price $P(T, S)$ is given by (7), the constrained minimization problem:

$$\min_{X, h} RM_{\alpha,T}(L) \tag{20}$$

s.t. $C = hZBP(0, T, S, X)$ and $h \in (0, 1) \tag{21}$

with $RM_{\alpha,T}(L)$ given by (9) or (15), has an optimal solution $X^*$ implicitly given by

$$ZBP(0, T, S, X) = (X - RM_{level})\frac{\partial ZBP}{\partial X}(0, T, S, X). \tag{22}$$

When moreover the cdf of $P(T, S)$ under the $T$-forward measure has bounded variation and $E_T[P(T, S)]$ is finite, the optimal strike $X^*$ solves:

$$ZBP(0, T, S, X) = (X - RM_{level})P(0, T)[1 - F_{r(T)}^{T} \left( \frac{\ln A(T, S) - \ln X}{B(T, S)} \right)]. \tag{23}$$

The corresponding expected shortfall of the loss is given by

$$ESF_{\alpha,T}(L) = (1 - h)\alpha(VaR_{level} - TVaR_{level}).$$

$RM_{level}$, $VaR_{level}$ and $TVaR_{level}$ are defined by respectively (19), (19a) and (19b).

**VaR and TVaR minimization and ESF: lognormal case**

When the short rate $r(T)$ is a normal random variable, then $P(T, S)$ is lognormally distributed and we can further elaborate the relations of Theorem 2 noting that the assumptions are satisfied.

**Theorem 3** Assume that under the risk neutral measure — in which we also express our risk measures — the short rate $r(T)$ is normally distributed with mean $m$ and variance $s^2$. Then $P(T, S)$ in (7) is lognormally distributed with parameters $\Pi(T, S)$ and $\Sigma(T, S)^2$.
given by
\[
\Pi(T, S) = \ln A(T, S) - B(T, S)m, \quad \Sigma(T, S)^2 = B(T, S)^2 s^2,
\] (24)
and the optimal solution \(X^*\) to the constrained minimization problem (20)-(21) satisfies
\[
G(\Phi^{-1}(\alpha)) = \frac{P(0, S)\Phi(-d_1(X))}{P(0, T)\Phi(-d_2(X))},
\] (25)
with
\[
G(\Phi^{-1}(\alpha)) = \begin{cases} 
\epsilon^{T(S) + \Sigma(T, S)\Phi^{-1}(\alpha)} & \text{if VaR} \\
\epsilon^{T(S) + \frac{1}{2}\Sigma(T, S)^2}\Phi(\Phi^{-1}(\alpha) - \Sigma(T, S)) & \text{if TVaR.}
\end{cases}
\] (26a, 26b)
where \(\Phi(\cdot)\) stands for the cumulative standard normal distribution, and with
\[
d_1(X) = \frac{1}{\Sigma(T, S)} \log \left( \frac{P(0, S)}{XP(0, T)} \right) + \frac{\Sigma(T, S)}{2}, \quad d_2(X) = d_1(X) - \Sigma(T, S). \] (27)
The corresponding shortfall of the loss equals:
\[
\text{ESF}_{a,T}(L) = (1 - h)\epsilon^{T(S)}(\alpha e^{\Sigma(T, S)\Phi^{-1}(\alpha)} - \epsilon^{\frac{1}{2}\Sigma(T, S)^2}\Phi(\Phi^{-1}(\alpha) - \Sigma(T, S))].
\] (28)

**PROOF.** When the short rate \(r(T)\) is normally distributed with mean \(m\) and variance \(s^2\) then the parameters \(\Pi\) and \(\Sigma\) of the lognormally distributed \(P(T, S)\) follow immediately from (7) while for the inverse cdf of \(r(T)\) we find
\[
F_{r(T)}^{-1}(p) = m + s\Phi^{-1}(p), \quad p \in [0, 1].
\] (29)
Since \(P(T, S)\) is lognormally distributed, the price at date zero of a European put option with the zero-coupon bond as the underlying security and with strike price \(X\) and exercise date \(T\) \((T \leq S)\), see for example Brigo and Mercurio (2001), is explicitly known:
\[
\text{ZBP}(0, T, S, X) = -P(0, S)\Phi(-d_1(X)) + XP(0, T)\Phi(-d_2(X)),
\] (30)
where \(d_1(X)\) and \(d_2(X)\) are defined in (27).
Its first order derivative with respect to \(X\) is:
\[
\frac{\partial \text{ZBP}}{\partial X}(0, T, S, X) = P(0, T)\Phi(-d_2(X)).
\] (31)
Combining (30) and (31) in (22) will provide the required result (25)-(26) when we have an expression for the RMlevel which is in this lognormal case denoted by \(G(\Phi^{-1}(\alpha))\) to express the dependence on \(\Phi^{-1}(\alpha)\). For the VaR case we substitute (29) in (19a) and use the property \(\Phi^{-1}(1 - \alpha) = -\Phi^{-1}(\alpha)\) to end up with (26a). For the TVaR-expression of \(G(\Phi^{-1}(\alpha))\) we start from the integral in (15) combined with
(29), apply a change of variable, use the properties of the cdf $\Phi(\cdot)$ and invoke the well-known result

$$
\int_{-\infty}^{b} e^{\lambda z} \varphi(z) dz = e^{\frac{1}{2} \lambda^2} \Phi(b - \lambda),
$$

with $\varphi(\cdot)$ the probability density function of a standard normal random variable:

$$
G_{TVaR}(\Phi^{-1}(\alpha)) = A(T, S)e^{-B(T, S)m} \int_{1-\alpha}^{\Phi^{-1}(\alpha)} e^{B(T, S)sz} \varphi(z) dz
$$

Finally we express the right hand-side in terms of the parameters (24) of $P(T, S)$, leading to (26b). The expected shortfall of the loss equals in this lognormal case:

$$
ESF_{\alpha,T}(L) = (1 - h)A(T, S)e^{-B(T, S)m} \times \\
\times [\alpha e^{B(T, S)s\Phi^{-1}(\alpha)} - e^{\frac{1}{2} B(T, S)^2 s^2} \Phi(\Phi^{-1}(\alpha) - B(T, S)s)]
$$

The expected shortfall $ESF_{\alpha,T}(L)$ could also be computed directly by combining (8) and (9) in (4). Hereto, we recall (29) and note that

$$
r(T) \overset{d}{=} m + sZ, \quad Z \sim N(0, 1),
$$

such that we may write

$$
ESF_{\alpha,T}(L) = (1 - h)A(T, S)e^{-B(T, S)m} E[(e^{B(T, S)s\Phi^{-1}(\alpha)} - e^{-B(T, S)sZ})_+].
$$

Then, apply a Black&Scholes like formula and (24) to arrive at (28). □

**Remark 1**  It is easily noticed that the case considered in Ahn et al. (1999) is of the same form as formula (25) when using a Brownian motion process.

**Remark 2** Two factor models like two-factor additive Gaussian model G2++, two-factor Hull-White, two-factor Heath-Jarrow-Morton with deterministic volatilities which result in lognormally distributed bond prices (see Brigo and Mercurio (2001)), are also applicable in our framework.

### 3.2 Coupon-bearing bond

We consider now the case of a coupon-bearing bond paying deterministic cash flows $C = [c_1, \ldots, c_n]$ at maturities $S = [S_1, \ldots, S_n]$. Let $T \leq S_1$. The price of this coupon-bearing
bond in $T$ is expressed as a linear combination (or a portfolio) of zero-coupon bonds:

$$\text{CB}(T, S, C) = \sum_{i=1}^{n} c_i P(T, S_i). \tag{33}$$

As in the previous section, the company wants to hedge its position in this bond by buying a percentage of a put option on this bond with strike $X$ and maturity $T$. In order to determine the strike $X$, the VaR or the TVaR of the hedged portfolio at time $T$ is minimized under a budget constraint. As in the previous section we will be able to treat the VaR-case and the TVaR-case together.

We first have a look at the value of a put option on a coupon-bearing bond as well as at the structure of the loss function.

The prices of the zero-coupon bonds $P(T, S_i)$, given by (7), all depend on the same short rate $r(T)$. Each $P(T, S_i)$ equals a strictly decreasing and continuous function of one and the same random variable $r(T)$, i.e. for all $i$

$$P(T, S_i) = A(T, S_i) e^{-B(T, S_i) r(T)} := g_i(r(T)). \tag{34}$$

Hence the vector $(P(T, S_1), \ldots, P(T, S_n))$ is comonotonic, see Kaas et al. (2000), and a European option on a coupon-bearing bond can be explicitly priced by means of Jamshidian’s decomposition, which was originally derived in Jamshidian (1989) in case of a Vasicek interest rate model. In fact a European option on a coupon-bearing bond decomposes into a portfolio of options on the individual zero-coupon bonds in the portfolio, which gives in case of a put with maturity $T$ and strike $X$:

$$\text{CBP}(0, T, S, C, X) = \sum_{i=1}^{n} c_i \text{ZBP}(0, T, S_i, X_i), \tag{35}$$

with $X_i = g_i(r_X)$ satisfying $\sum_{i=1}^{n} c_i X_i = X. \tag{36}$

Thus $r_X$ is the value of the short rate at time $T$ for which the coupon-bearing bond price equals the strike.

Repeating the reasoning of Section 3.1 we may conclude that in a worst case scenario the loss of the hedged portfolio at time $T$ composed of the coupon-bearing bond (33) and the put option (35) equals a strictly increasing function $f$ of the random variable $r(T)$:

$$L = \text{CB}(0, S, C) + C - ((1 - h) \sum_{i=1}^{n} c_i g_i(r(T)) + hX) := f(r(T)), \tag{37}$$

with $g_i(r(T))$ defined in (34).
VaR and TVaR minimization

The VaR of this loss that we want to minimize under the constraints \(0 < h < 1\) and \(C = h \text{CBP}(0, T, S, C, X)\), is analogously to (9) given by

\[
\text{VaR}_{\alpha,T}(L) = \text{CB}(0, S, C) + C - hX - (1 - h) \sum_{i=1}^{n} c_i g_i(F_{r(T)}^{-1}(1 - \alpha)).
\] (38)

By integrating this relation (38), after replacing \(\alpha\) by \(1 - \beta\), with respect to \(\beta\) between the integration bounds \(1 - \alpha\) and 1, we find for the TVaR of the loss:

\[
\text{TVaR}_{\alpha,T}(L) = \text{CB}(0, S, C) + C - hX - \frac{1}{\alpha}((1 - h) \sum_{i=1}^{n} c_i \int_{1-\alpha}^{1} g_i(F_{r(T)}^{-1}(\beta)) d\beta).
\] (39)

Also here we note the similarity in the expressions for the risk measures (RM) VaR and TVaR which could be collected in one expression:

\[
\text{RM}_{\alpha,T}(L) = \text{CB}(0, S, C) + C - hX - (1 - h) \sum_{i=1}^{n} c_i G_i(F_{r(T)}^{-1}(1 - \alpha))
\] (40)

with

\[
G_i(F_{r(T)}^{-1}(1 - \alpha)) = \begin{cases} 
  g_i(F_{r(T)}^{-1}(1 - \alpha)) = A(T, S_i) e^{-B(T,S_i)F_{r(T)}^{-1}(1-\alpha)} & \text{if VaR} \\
  \frac{1}{\alpha} \int_{1-\alpha}^{1} g_i(F_{r(T)}^{-1}(\beta)) d\beta = \frac{A(T, S_i)}{\alpha} \int_{1-\alpha}^{1} e^{-B(T,S_i)F_{r(T)}^{-1}(\beta)} d\beta & \text{if TVaR}.
\end{cases}
\] (41)

We now want to solve the constrained optimization problem

\[
\min_{X, h} \text{RM}_{\alpha,T}(L) \quad \text{subjected to} \quad C = h \text{CBP}(0, T, S, C, X), \quad 0 < h < 1.
\]

From the Kuhn-Tucker conditions we find that the optimal strike price \(X^*\) satisfies the following equation:

\[
\text{CBP}(0, T, S, C, X) - [X - \sum_{i=1}^{n} c_i G_i(F_{r(T)}^{-1}(1 - \alpha))] \frac{\partial \text{CBP}}{\partial X}(0, T, S, C, X) = 0.
\] (42)

Rewriting this equation in terms of the put options on the individual zero-coupon bonds
cfr. (35) leads to the following equivalent set of equations:

\[
\sum_{i=1}^{n} c_i ZBP(0, T, S_i, X_i) = [X - \sum_{i=1}^{n} c_i G_i(F^{-1}_{r(T)}(1 - \alpha))] \sum_{i=1}^{n} c_i \frac{\partial ZBP}{\partial X_i}(0, T, S_i, X_i) \frac{\partial X_i}{\partial X} \tag{43}
\]
\[
\sum_{i=1}^{n} c_i X_i = X \tag{44}
\]
\[
\sum_{i=1}^{n} c_i \frac{\partial X_i}{\partial X} = 1. \tag{45}
\]

The first equation simplifies by noting that \(\frac{\partial ZBP}{\partial X_i}(0, T, S_i, X_i)\) is independent of \(i\) and by using (45). Indeed, plug (36) in (12)-(13) while recalling (34):

\[
\frac{\partial ZBP}{\partial X_i}(0, T, S_i, X_i) = P(0, T)[1 - F^{T}_{r(T)}(r_{X})] \tag{46}
\]

\[
\Rightarrow \sum_{i=1}^{n} c_i \frac{\partial ZBP}{\partial X_i}(0, T, S_i, X_i) \frac{\partial X_i}{\partial X} = P(0, T)[1 - F^{T}_{r(T)}(r_{X})] \sum_{i=1}^{n} c_i \frac{\partial X_i}{\partial X} \overset{\text{(45)}}{=} P(0, T)[1 - F^{T}_{r(T)}(r_{X})].
\]

Thus in order to find the optimal strike \(X^*\) we proceed as follows:

**Step 1** Solve the following equation, which is equivalent to (43), for \(r_{X}\):

\[
\sum_{i=1}^{n} c_i ZBP(0, T, S_i, g_i(r_{X})) = P(0, T)[1 - F^{T}_{r(T)}(r_{X})] \sum_{i=1}^{n} c_i [g_i(r_{X}) - G_i(F^{-1}_{r(T)}(1 - \alpha))]. \tag{47}
\]

**Step 2** Substitute the solution \(r_{X}^*\) in (44):

\[
X^* = \sum_{i=1}^{n} c_i g_i(r_{X}^*) = \sum_{i=1}^{n} c_i A(T, S_i) e^{-B(T,S_i) r_{X}^*}. \tag{48}
\]

**Remark** In all cases, the optimal strike price is independent of the hedging cost and one can look at the trade-off between the hedging expenditure and the RM level, cfr. Section 3.1.

We summarize these results in the following theorem.

**Theorem 4** Under the assumption of an affine term structure model so that for all \(i\) the zero-coupon bond price \(P(T, S_i)\) is given by (34) and assuming for all \(i\) that the cdf of \(P(T, S_i)\) under the \(T\)-forward measure has bounded variation and that \(E^{T}[P(T, S_i)]\) is...
finite, the hedging problem for a coupon bond (33):

\[
\min_{X, h} \text{RM}_{\alpha, T}(L)
\]

\[
\text{s.t. } C = h \text{CBP}(0, T, S, X) \text{ and } h \in (0, 1)
\]

with \( \text{RM}_{\alpha, T}(L) \) defined by (40)-(41), has an optimal solution \( X^* \) given by (47)-(48).

**VaR and TVaR minimization and ESF: lognormal case**

We consider the special case that \( r(T) \) is a normal random variable cfr. (29) such that the zero-coupon bond prices \( P(T, S_t) \) are lognormally distributed with parameters \( \Pi(T, S_t) \) and \( \Sigma(T, S_t)^2 \) given by (24) for \( S = S_t \). Then the put option prices \( Z_{BP}(0, T, S_t, G_i(r_X)) \) in relation (47) are given by (30) and (27), while \( G_i(F_{r(T)}^{-1}(1 - \alpha)) \) is defined by (26) for \( S = S_t \) and will be denoted \( G_i(\Phi^{-1}(\alpha)) \). The factor \( P(0, T)[1 - F_r(T)(r_X)] \) in (47) equals according to (46) the first order derivative of the put option prices, which is in the lognormal case given by (31):

\[
P(0, T)[1 - F_r(T)(r_X)] = P(0, T)\Phi(-d_2(g_i(r_X))), \quad \text{for any } i.
\]

This implies that \( d_2(g_i(r_X)) \) is independent of \( i \).

Thus the optimal strike \( X^* \) can be found as follows:

**Step 1** Solve the following equation for \( r_X \):

\[
\sum_{i=1}^{n} c_i[-P(0, S_t)\Phi(-d_1(g_i(r_X))) + P(0, T)g_i(r_X)\Phi(-d_2(g_i(r_X)))]
\]

\[
- P(0, T)\Phi(-d_2(g_1(r_X))) \sum_{i=1}^{n} c_i[g_i(r_X) - G_i(\Phi^{-1}(\alpha))] = 0.
\]

**Step 2** Substitute the solution \( r_X^* \) in (48).

The expected shortfall in case of a coupon bearing bond is derived in a similar way as for the zero-coupon bond:

\[
\text{ESF}_{\alpha, T}(L) = \alpha[T\text{VaR}_{\alpha, T} - \text{VaR}_{\alpha, T}]
\]

\[
= \sum_{i=1}^{n} c_iA(T, S_t)[\alpha e^{-B(T, S_i)}F_r^{-1}(1 - \alpha) - \int_{1-\alpha}^{1} e^{-B(T, S_i)}F_r^{-1}(\beta) \, d\beta]
\]

\[
= (1 - h) \sum_{i=1}^{n} c_iA(T, S_t) e^{-B(T, S_i)\alpha} \times
\]

\[
\times [\alpha e^{B(T, S_i)\Phi^{-1}(\alpha)} - e^{\frac{1}{2}B(T, S_i)^2}\Phi(\Phi^{-1}(\alpha) - B(T, S_i)\delta)]
\]

\[
= (1 - h) \sum_{i=1}^{n} c_i e^{\Pi(T, S_t)} [\alpha e^{\Sigma(T, S_i)\Phi^{-1}(\alpha)} - e^{\frac{1}{2}\Sigma(T, S_i)^2}\Phi(\Phi^{-1}(\alpha) - \Sigma(T, S_i))].
\]
The expected shortfall ESF\textsubscript{\alpha,T}(L) can also be computed directly by combining (37) and (38) in relation (4) and by invoking comonotonicity properties (see Kaas et al. (2000)) for calculating a stop-loss premium of a comonotonic sum. This implies that ESF\textsubscript{\alpha,T}(L) is in fact a linear combination of the expressions in the right hand side of (18) with \( S \) replaced by \( S_i \) for \( i = 1, \ldots, n \).

We derived formula (11), (16) and formula (47) combined with (48) to calculate the optimal strike price for the hedging problems under consideration. In all cases, the specification of an interest rate model is necessary. Until now, the optimization has been achieved with the most important modelling assumption that the bond price \( P(T, S) \) has the form (7) such that the term structure is affine. We also looked at a special case that the bond price \( P(T, S) \) is lognormally distributed. We did not yet form concrete beliefs on how the (instantaneous) interest rate will move. By forming these beliefs, or in other words, by specifying a model for the evolution of the interest rate, we also get explicit expressions for the bond and bond option prices, which then enables us to determine the (theoretically) optimal strike price.

In the next section, we will define and explain the specification of the model for the evolution of the instantaneous interest rate.

4 Application

4.1 The Hull-White model

There exists a whole literature concerning interest rate models. For a comprehensive overview we refer for example to Brigo and Mercurio (2001). For our analysis, we focus on the Hull-White one-factor model, first discussed in Hull and White (1990). We choose this model because it is still an often used model in financial institutions for risk management purposes, (see Brigo and Mercurio (2001)). Two main reasons explain this popularity. First of all, it is a model that allows closed form solutions for bond and plain vanilla European option pricing. So, since there are exact pricing formulas, there is no need to run time consuming simulations. But of course, if the model lacks credibility, fast but wrong price computations do not offer any benefit. But that is where the second big advantage of the Hull-White model comes from since it succeeds in fitting a given term structure by having (at least) one time-dependent parameter. Therefore, today’s bond prices can be perfectly matched. It belongs to the class of so called no-arbitrage interest rate models. This means that, in contrast to equilibrium models (such as Vasicek, Cox-Ingersoll-Ross), no-arbitrage models succeed in fitting a given term structure, and thus can match today’s bond prices perfectly.

An often cited critique is that applying the model sometimes results in a negative interest rate, but with up-to-date calibrated parameters which are used for a rather short period, it
can be proved that the probability of obtaining negative interest rates is very small.

Hull and White (1990) assume that the instantaneous interest rate follows a mean reverting process also known as an Ornstein-Uhlenbeck process:

\[
dr(t) = (\theta(t) - \gamma(t)r(t))dt + \sigma(t)dZ(t)
\]  

(52)

for a standard Brownian motion \(Z(t)\) under the risk-neutral measure \(Q\), and with time dependent parameters \(\theta(t), \gamma(t)\) and \(\sigma(t)\). The parameter \(\theta(t)/\gamma(t)\) is the time dependent long-term average level of the spot interest rate around which \(r(t)\) moves, \(\gamma(t)\) controls the mean-reversion speed and \(\sigma(t)\) is the volatility function. By making the mean reversion level \(\theta\) time dependent, a perfect fit with a given term structure can be achieved, and in this way arbitrage can be avoided. In our analysis, we will keep \(\gamma\) and \(\sigma\) constant, and thus time-independent. According to Brigo and Mercurio (2001), this is desirable when an exact calibration to an initial term structure is wanted. This perfect fit then occurs when \(\theta(t)\) satisfies the following condition:

\[
\theta(t) = F^M_t(0, t) + \gamma F^M(0, t) + \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma t}),
\]

where, \(F^M(0, t)\) denotes the instantaneous forward rate observed in the market on time zero with maturity \(t\).

It can be shown (see Hull and White (1990)) that the expectation and variance of the stochastic variable \(r(t)\) are:

\[
E[r(t)] = m(t) = r(0)e^{-\gamma t} + a(t) - a(0)e^{-\gamma t}
\]

(53)

\[
\text{Var}[r(t)] = s^2(t) = \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma t}),
\]

(54)

with the expression \(a(t)\) calculated as follows:

\[
a(t) = F^M(0, t) + \frac{\sigma^2}{2} \left( \frac{1 - e^{-\gamma t}}{\gamma} \right)^2.
\]

Based on these results, Hull and White developed an analytical expression for the price of a zero-coupon bond with maturity date \(S\)

\[
P(t, S) = A(t, S)e^{-B(t, S)r(t)},
\]

(55)

where

\[
B(t, S) = \frac{1 - e^{-\gamma(S-t)}}{\gamma},
\]

(56)

\[
A(t, S) = \frac{P^M(0, S)}{P^M(0, t)}e^{B(t, S)F^M(0, t) - \frac{\sigma^2}{4\gamma}(1 - e^{-2\gamma t})B^2(t, S)},
\]

(57)
with $P^M$ the bond price observed in the market. Since $A(t, S)$ and $B(t, S)$ are independent of $r(t)$, the distribution of a bond price at any given time must be lognormal with parameters $\Pi$ and $\Sigma^2$:

$$\Pi(t, S) = \ln A(t, S) - B(t, S)m(t), \quad \Sigma(t, S)^2 = B(t, S)^2 s^2(t),$$

(58)

with $m(t)$ and $s^2(t)$ given by (53) and (54).

### 4.2 Calibration of the Hull-White model

Until now, we theoretically discussed the issue of minimizing the VaR and TVaR of our investment. If the firm wants to pursue this minimization into practice, it needs credible parameters for the interest rate model it uses. Focusing in particular on the Hull-White model that we discussed above, we need to have parameter values for $\gamma$ and $\sigma$. The process to obtain these parameters is calibration. The most common way to calibrate the Hull-White model is by using interest rate options, such as swaptions or caps. The goal of the calibration is to find the model parameters that minimize the relative difference between the market prices of these interest rate options and the prices obtained by applying our model.

Suppose we have $M$ market prices of swaptions or caps, then we search the $\gamma$ and $\sigma$ such that the sum of squared errors between the market and model prices are minimized. Formally,

$$\min_{\gamma, \sigma} \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left( \frac{\text{model}_i - \text{market}_i}{\text{market}_i} \right)^2}.$$

Interest rate caps are instruments that provide the holder protection against a specified interest rate (e.g. the three month EURIBOR, $R_L$) rising above a specified level (the cap rate, $R_C$). Suppose a company issued a floating rate note with as reference rate the three month EURIBOR. When EURIBOR rises above the cap rate, a payoff is generated such that the net payment of the holder only equals the cap rate. One cap consists of a series of caplets. These caplets can be seen as call options on the reference rate. The maturity of the underlying floating interest rate of these call options equals the tenor, which is the time period between two resets of the reference rate. In our case, this is three months, or 0.25 year.

If in our case, at time $t_k$, the three month EURIBOR rises above the cap rate, the call will be exercised, which leads to a payoff at time $t_{k+1}$ (0.25 year later) that can be used to compensate the increased interest payment on the floating rate note. Formally, the payoff at time $t_{k+1}$ equals (see Hull (2003)):

$$\max(0.25(R_L - R_C), 0).$$
This is equivalent to a payoff at time $t_k$ of

$$
\text{max}(0.25(R_L - R_C), 0) \over 1 + 0.25R_L.
$$

This can be restated as:

$$
\text{max} \left( 1 - \frac{1 + 0.25R_C}{1 + 0.25R_L}, 0 \right).
$$

This is the payoff of a put option with strike 1, expiring at $t_k$, on a zero-coupon bond with principal $1 + 0.25R_C$, maturing at $t_{k+1}$. This means that each individual caplet corresponds to a put option on a zero-coupon bond. Thus, a cap can be valued as a sum of zero-coupon bond put options. Since these put options can be valued using the Hull-White model, this offers us a way to fit our model to the market data. The market data we have used are to be found in Table 1 where cap maturities are listed, along with the volatility quotes of these caps and the cap rate. The data are obtained on 11 April 2005 and have as reference rate EURIBOR. Note that the volatility quotes have the traditional humped relation with respect to the maturity of the cap: the volatility reaches its peak at the 2 year cap and then decreases steadily as the maturity increases. Although the cap rate can be freely determined, it is most common to put it equal to the swap rate for a swap having the same payment dates as the cap. The volatility quotes that are provided are based on Black’s model. This means that we first have to use Black’s formula for valuing bond options in order to arrive at the prices of the caps. These prices are shown in the fourth column. Now we still have to calculate the model prices. Therefore, we use, for each caplet, the following formula:

$$
\text{ZBP}(0, T, S, X, N) = -NP(0, S)\Phi(-d_1(X)) + XP(0, T)\Phi(-d_2(X)), \quad (59)
$$

As strike price $X$ we take 1, and as principal $N$ we take $1 + 0.25R_C$. $P(0, T)$ and $P(0, S)$ can be read from the term structure.

Taking the sum of all the caplets in a given cap, we get an expression for which we need to seek the parameters that, globally, make the best fit. The calibration procedure results in the following parameter values:

$$
\gamma = 0.31621 \quad \sigma = 0.011631.
$$
<table>
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<th>Cap maturity</th>
<th>Volatility (in %)</th>
<th>Cap rate (in %)</th>
<th>Cap price</th>
</tr>
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<td>2.200</td>
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<tr>
<td>3Y</td>
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<td>2.741</td>
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<td>2.911</td>
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<tr>
<td>20Y</td>
<td>15.465745</td>
<td>4.093</td>
<td>0.11080</td>
</tr>
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Table 1: Overview cap data

4.3 VaR and TVaR minimization

Supposing we have gone through this calibration procedure, the next step in our hedging programme would then be to provide this protection to our portfolio. This can basically be achieved in two ways: first of all, by buying a put option, or secondly, by replicating this option. In the first approach, we are also facing two possibilities: either we buy the put option at a regulated exchange market, either we buy it over the counter (OTC). If options are bought as protection against interest rate risk, it is most common to buy them OTC. Genuine bond options are only available at a restricted number of exchanges. Furthermore, at these exchanges, trading in bond options is usually very thin. The second approach, replicating the option synthetically, involves quite some follow up and adjustment in positions, and can entail a considerable amount of transaction costs. Therefore, it is not unreasonable to consider the OTC market as the only viable possibility for a firm to buy protection. A major advantage of buying over the counter is that we can completely tailor the option to our needs. What is of utter importance to the firm is that the option can be bought at any desired strike. This opposes to buying options on an exchange market, where options can only be bought at predetermined strike prices. A source of uncertainty is the discrepancy between the theoretical option prices that were calculated and the option price that has to be paid over the counter. Therefore, the firm could perform the
optimization procedure using the prices of the financial institution. However, this restricts
the possible calculation methods to using formula (42). The combination of (51) and (48)
cannot be used since this requires the knowledge of \(d_1\) and \(d_2\), which we clearly not have,
since we only have the price of the option. So, it is necessary to use formula (42). The
difference in optimal strike price in both approaches is an empirical question and will be
dealt with in this part.

For our numerical illustration, we suppose the firm has an OLO 35. OLO (which stands
for Obligation Lineair/Lineaire Obligatie) are debt instruments issued by the Belgian gov-
ernment, and as such, believed to be risk-free. OLOs have a fixed coupon. The OLO we
consider was issued on 28 Sept 2000 and will mature on 28 Sept 2010, so the maturity
is 10 years, i.e. \(S = 10\). It pays a yearly coupon of 5.75 %, on 28 Sept of each year, i.e.
\(c_i = 0.0575\) for all \(i\). As there are no traded options for this kind of bond, we have to pro-
tect by buying OTC options. Therefore, we got OTC prices from a financial institution.
The date on which these data were delivered, is 30 Sept 2005. This means that the bond
then has a remaining maturity of 4.99 years, and coupons will be paid out at \(S_1 = 0.99\),
\(S_2 = 1.99\), \(S_3 = 2.99\), \(S_4 = 3.99\) and \(S_5 = 4.99\). At that particular date, 30 Sept 2005,
the bond had a market price of 1.1393. We received the option prices for a wide range
of strikes: going from a strike price of 1.05 to a strike of 1.199, with steps of 0.001. The
option maturity is exactly one year, i.e. \(T = 1\).

This means that the maturity of the option lies between the first and second coupon pay-
ment, whereas when deriving optimal strike price, we supposed that the option matured
before the first coupon payment. This problem can easily be solved by reducing our
coupon payment vector to the last four observations.

We now have **three** methods of computing the optimal strike price.

1. The first method is solving equation (51) and substituting in (48).
2. The second method still uses the theoretical option prices, but solves equation (42)
   and approximates the first derivative of the option price with respect to the strike
   price by the difference quotient of the changes in the option prices to the changes in
   the strike price.
3. The third is equivalent to the second approach, but uses the option prices received
   from the financial institution.

Using a 5% level, the bond VaR level for a holding period of one year (in other words, a
worst case expectation of the evolution of the bond price) is 1.0716. Using this number,
we can calculate the optimal strike price in the **three** different methods. Note that VaR
has to be calculated under the true probability measure. Since we have calibrated our
interest rate model using option prices, the parameters we obtained are under the risk-
neutral measure. So, in order to know the parameters under the true probability measure,
we would need to estimate the market price of risk. However, as quite often done (see
Stanton (1997)), we assumed the market price of risk to be zero.

1. The first method results in an optimal strike price of 1.0833.
2. The second method yields an optimum which is very close to this: 1.084.
(3) The last method finds as an optimum a slightly higher strike price: 1.087.

In all three cases, the optimum is situated above the VaR level of the bond as predicted by the theory. The close correspondence between the first two methods is evident, since the difference can only be attributed to approximation errors in the second method. Although not dramatic, we observe a difference between the first two and the last method. The third method is resulting in an option that is a bit more in the money.

Using a 1% level, a comparable picture emerges. The bond VaR level of course is lower this time, namely 1.0561. This results in lower optimal strike prices: in the first method, we obtain an optimal strike price of 1.0649. The second method shows an optimum of 1.065. The third method again shows a higher optimum, this time the strike price amounts to 1.068.

For both levels the results are summarized in Table 2:

<table>
<thead>
<tr>
<th></th>
<th>equation (52) &amp; (49)</th>
<th>equation (43) theoretical prices</th>
<th>equation (43) empirical prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR</td>
<td>TVaR</td>
<td>VaR</td>
<td>TVaR</td>
</tr>
<tr>
<td>1%</td>
<td>1.0649</td>
<td>1.0537</td>
<td>1.0650</td>
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<tr>
<td>5%</td>
<td>1.0833</td>
<td>1.0717</td>
<td>1.0840</td>
</tr>
</tbody>
</table>

Table 2: Optimal strike prices for one and five percent levels, for different calculation methods.

As stated earlier, the firm that wishes to hedge its exposure is now facing a linear trade-off between VaR and hedging expenditure. This is illustrated in Figure 1. On this graph, the firm can clearly see the consequence of choosing a particular hedging cost. Alternatively, it can read the hedging cost required to obtain a certain protection, expressed in VaR terms. Note that the hedging cost is restricted to the range $[0, 0.003171]$, with the left hand side of the range corresponding to no hedging, and the right hand side corresponding to buying an entire put option (at the OTC price) at the optimal strike price (so, $h = 1$). No hedging leads to a VaR of 0.0677. Buying an entire option at the optimal strike price reduces the VaR to 0.0557. It is clear that the exact position a firm takes, is determined by both the budget and the risk aversion or appetite of the firm, which we cannot judge. Furthermore, it makes economic sense to execute the hedge since we observe that the hedging cost is smaller than the reduction in VaR you get by hedging.

Conclusions are comparable when performing a TVaR minimization. Of course, the bond TVaR level lies below the VaR level. For the 5% level, it is situated at 1.0621. The first method results in an optimum of 1.0717. The second method finds 1.072 as optimal strike price, and the third method (taking into account the OTC prices) produces an optimum of 1.075. Again we observe the difference between the first two methods and the third
method.

For the 1% level, the bond TVaR is 1.0485. The optimum in the first method is now at 1.0537. The second method results in an optimum of 1.056. Using the OTC prices, an optimal strike price is reached at a level of 1.059.

We can thus conclude that, based on OTC prices, the optimum is situated slightly higher than the optimum reached under theoretical prices. This conclusion is robust for different risk criteria and different levels.

5 Conclusions

We provided a method for minimizing the risk of a position in a bond (zero-coupon or coupon-bearing) by buying (a percentage of) a bond put option. Taking into account a budget constraint, we determine the optimal strike price, which minimizes a Value-at-Risk or Tail-Value-at-Risk criterion. Alternatively, our approach can be used when a nominal risk level is fixed, and the minimal hedging budget to fulfil this criterion is desired. From the class of short rate models which result in lognormally distributed future bond prices, we have selected the Hull-White one-factor model for an illustration of our optimization. This Hull-White model is calibrated to a set of cap prices, in order to obtain credible parameters for the process. We illustrated our strategy using as investment asset a Belgian government bond, on which we want to buy protection. We calculated the optimal strike price of the bond option that we use, both with theoretical Hull-White prices, and with real market prices. The results are comforting in the sense that the optimal strike prices in both approaches show a close correspondence. The strike price based on real prices is
only slightly higher than the one based on theoretical prices. Further research possibilities are mainly situated in two directions. First of all, we can consider other instruments to hedge our investment. The use of a swaption to hedge a swap is very widespread in the financial industry. It should be possible to determine the optimal swap rate to hedge the swap. The second direction concerns the interest rate models that can be used in our analysis. It is often stated that two-factor models are better suited to capture interest rate behaviour. Such a model cannot be used here to hedge an investment in a coupon-bearing bond. The reason is that the Jamshidian decomposition cannot be applied. An alternative could be the comonotonicity approach of Dhaene et al. (2002a) and Dhaene et al. (2002b), which results in a lower and upper bound for the bond put option. As an alternative for a two-factor model, a model with a jump component can be considered. Johannes (2004) finds evidence for the importance of adding a jump term to interest rate models. The use of jump models, however, raises new pricing and hedging issues.

Acknowledgements

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Chapter 5:

The use of options by individual investors
The use of options by individual investors

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Abstract

This paper discusses the use of options by individual investors. We report descriptive statistics concerning the main characteristics of options, namely whether call or put options are traded, whether the positions initiated concern a long or short position, and what the moneyness and time to maturity are. We document the performance investors achieve on the transactions they execute. We show that a difference exists between long and short positions. On average, short positions lose money, whereas the average return for long options is positive. This is explained by the difference in attainable returns between short and long positions. We further show that as the duration of the trade increases, the performance decreases. This study provides novel insights in the dynamics of option trading behaviour by individual investors.

1. Introduction

The pathbreaking article of Black \& Scholes (1973) on the pricing of options initiated a stream of interesting research on the topic of options. A substantial amount of these papers focuses on pricing and hedging issues. Researchers departed from the assumptions of the Black \& Scholes model in order to achieve a greater fit between theoretical and observed prices of options (see for example Cox \& Rubinstein (1986), Heston (1993), Kou (2002) and Carr \& Wu (2004)). Furthermore, the pricing of non plain vanilla options was first tackled by amongst others Margrabe (1978) and Johnson (1987). Later, all sorts of exotic options have been discussed and priced (see e.g. Kyprianou et al. (2005)). Another strand of the literature discusses option strategies and documents the profitability that implementing these strategies would have yielded. Important contributions in this area are made by Chance...
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(1998), Coval & Shumway (2001), Santa Clara & Saretto (2005). However, what the real performance attached to option trading for individual investors is remains largely unknown. Research on this topic is important and interesting in several ways.

First of all, options can provide protection to a portfolio in order to prevent the portfolio value from falling below a certain threshold. This aspect of the use of options refers to the hedging characteristic of options. A textbook example of such a protection is a protective put. In this strategy, an investment in a particular stock is protected by buying a put option on that same stock. When the stock price falls below the strike price of the option, the option will become profitable and will compensate for the loss made on the stock investment. Chapters two to four discuss this hedging motive (for a bond) in more detail.

Alternatively, options can be used as a speculative tool. Indeed, a small change in the value of the underlying often results in a change of the option value of much higher magnitude. Therefore, an investor who is convinced of knowing the direction that a particular stock will move into in the near future, could invest in an option on that stock, instead of directly investing in the stock itself. If the stock price indeed evolves in the expected way, the gain will be substantial and will exceed the gain of the underlying stock itself. In this sense, the investor engages in directional trading, because he expects a particular price movement. Another rationale for trading is for volatility reasons. In this strategy, the investor is confident that the volatility of the underlying stock will change and tries to anticipate on that volatility change. Such volatility trading can be implemented by for example straddles and strangles. However, Lakonishok et al. (2007) examine trades on the Chicago Board Options Exchange (CBOE) and conclude that only a very limited amount of the trades can be assumed to be volatility trading. They therefore state that speculating on and hedging the direction of underlying stock price movements are the main drivers of option market activity.

The complexity of options, both in terms of understanding the drivers of option prices, as in terms of possible payoffs, makes clear that the investor should have a minimum level of option knowledge when trading in options. Especially shorting options exposes the investor to a significant amount of risk, due to the (theoretically) unlimited loss potential.

Furthermore, options provide an additional way of taking advantage of a declining stock market. Similar to shorting stocks, buying put options also results in a profit when
prices are decreasing. Since shorting stocks is argued to be complex and expensive (see Jones & Lamont (2002) and Lakonishok et al. (2007)), options are perhaps the most effective way of benefiting from stock price drops.

This paper provides evidence on the use of options by individual investors, and the (transactional) return performance of these investors. We retrieve data from a sample of 2,254 online clients engaging in option trades. Data is provided by a major Dutch bank and covers two years (2006-2007).

The rest of the paper is structured as follows. First, we describe the data set that we use. Afterwards we show descriptive statistics concerning several variables of interest. In the next section, we explain the calculation of returns on the option transactions, report the return results and interpret them. We end with conclusions.

2. Data

The data set is provided by a major Dutch bank and contains monthly investment holdings as well as individual transactions of 5,465 online clients. These holding and transaction files contain different asset classes like common stock and mutual funds, but also investments in derivative products like options and turbos for the period January 2006-December 2007. Out of the 5,465 online investors, 2,254 investors (41%) engage in at least one option position during the observation period. This percentage is substantial and suggests that Dutch online investors are well experienced in option/derivative trading.1

In total, over the two-year period 2006-2007, we find that these 2,254 investors trade 18,254 different types of option contracts. Option contracts differ by the following four characteristics: call/put, underlying stock, expiration date and strike price. Option contracts might differ by only one or more of these characteristics. For instance, a call option on ING with maturity date August 2007 and strike price 25 EUR is a different contract than a call

1 This evidence of high importance of derivative trading activity is consistent with findings from Bauer et al. (2007). These authors show that option trades account for about 49% of all trades in the period 2000-2005.
option on ING with maturity date August 2007 and strike price 24 EUR. Of these 18,254 contracts, 17,387 contracts relate to a domestic (Dutch) underlying stock or index, which suggests a strong home bias in option trading activity. The remaining contracts were predominantly written on an American underlying.

For further calculations further in the paper, we use time-series data on the different option contracts that are traded. We therefore make use of the option module of the Datastream database. We were able to retrieve information on 13,383 (77%) option contracts in Datastream. The reason for this imperfect match is twofold. First of all, Datastream does not contain data on the American contracts. Secondly, and more importantly with respect to our research, is that Datastream does not keep all matured (termed ‘dead’ in Datastream) contracts in their database. As a consequence, option contracts which matured before May 2006 could not be retrieved, resulting in a loss of about 23% of our observations.

Further, we also retrieve data on the assets underlying the option contracts. In total, we identify 53 different underlyings, of which 52 relate to common stocks and the remainder is the AEX, i.e., the Dutch market index. As stated earlier, our period of observation comprises two calendar years. Although this is a rather limited period and does not correspond to a full business cycle, we believe the remaining sample is large and representative enough to provide an answer to our research questions. We briefly comment on the evolution of the stocks and the index that underlie the option contracts. Figure 1 plots the evolution of the AEX index.

Over the two year period, the index realized a gain of 18.08%. However, we clearly notice some periods of declining prices. For the stocks underlying the option contracts, the average return over the period was 22.33%, with a standard deviation of 40.81%. Moreover, 14 (26%) of the 53 underlyings had a net negative performance over the observed time period.
3. Descriptive Statistics

Column (1) of Table 1 lists for the different percentiles the number of trades made. Several investors are executing a rather limited number of trades over our observation period. The median number of trades amounts to 19. We observe a large jump in the number of trades made between the 90th percentile and the maximum value. Whereas the 90th percentile only amounts to 208, the maximum number of trades jumps to 8,115 trades. The expectation that our sample contains some very frequent traders, is confirmed in the third column of Table 1, which gives the cumulative percentage of trades made up to a particular percentile. As is often perceived, the 20/80 rule also holds in our sample, since the top 20% highest traders account for 81.73% of the trades.

*** Insert Table 1 about here***

Table 2 reports descriptive statistics of the trading behaviour of the option investors (N=2,254), where a distinction is made between long and short positions and put and call positions.

*** Insert Table 2 about here***

The average number of trades is high (93.54) and diverges substantially from the median (19). Since only a specific subset of investors trades very actively in options (Max.=8,110) we focus on median values for descriptive purposes. Most option trading takes place on the call side. The median investor buys nine and sells two calls over the observation period. This frequency of call trading is higher than the activity level on the put side, where the median investor buys three and sells five puts over the sample period. The popularity of calls is also confirmed when we look at the proportion of investors that engage at least once in a specific derivative product. We show that more than 86 percent of the option investors trades at least once in calls, both on the buy and sell side. The frequency for puts, however, is substantially lower and approximates 62%.
Table 3 shows a detailed split up of the transactions along two axes: firstly, whether the transaction involves a call or a put, and secondly, whether it relates to a buy or a sell. Further, this table contains four panels, in which we report the proportion of (i) call versus put contracts and (ii) buy versus sell contracts, leading to four quadrants. We report absolute occurrences of each combination in panel A; relative occurrences to the grand total in panel B; relative as a proportion of call, resp. put contracts in panel C; and relative as a proportion of buy, resp. sell contracts in panel D. Further, each panel contains results for seven different samples. The first sample is the total sample; the second contains contracts on domestic (Dutch) options; the third sample is for the foreign contracts only. Sample 4 to 6 are samples that challenge the robustness of our results. Further, Sample 4 contains the trades belonging to the group of the 20% investors that trade the most. Sample 5 contains the trades belonging to the group of the 80% investors that trade the least. Sample 6 contains those transactions for which the option contract data could be retrieved from Datastream. The final sample (7) is created by selecting only the first transaction of a particular investor in a particular option contract. We call it the starting transaction. This can shed light on the expectations of the investor. Suppose an investor performs two transactions in a particular put option contract. In the first transaction the put option contract is shorted. After some period, this shorted put option contract is bought back. If we assume that the investor has traded in the option contract because of expectations of a directional move of the underlying, one can deduce that the investor is expecting a price increase, and simply wants to cash in the option premium.2

*** Insert Table 3 about here***

Panel A then simply shows occurrences. First of all, we clearly see an overwhelming majority of domestic options. We also notice that the Datastream limitations reduce our initial sample of 210,385 transactions to 168,414 transactions. Panel B is obtained by each time dividing the occurrences of a particular option combination by the relevant sample size. For each subsample, it holds true that the activity on the call side dominates the activity on the put side. The largest difference is observed for the sample of infrequent traders, where the proportion of call activity is twice the proportion of the put activity. The difference between

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2 We are aware that option trading can also happen because of volatility expectations, however, as stated in the introduction, Lakonishok et al. (2007) find a very limited role for volatility trading.
both proportions is smallest for the frequent traders (55.81% versus 44.19%). A possible explanation could be that infrequent traders choose for the option type of which the payoff corresponds in directional sense to the payoff of buying a stock. Indeed, call option payoffs are the most easy to understand.

We note no major differences between the numbers in the total sample and the sample based on the Datastream contracts, which is reassuring. We also observe that the buying activity and selling activity are almost equal for four out of the seven samples. Only for the foreign options, buying activity dominates the selling activity. For the sample of infrequent investors, selling is slightly higher than buying. The difference however is to be neglected. This cannot be said for the sample of starting transactions. There we notice that the majority of positions are initiated by a sell transaction. Especially the selling of put options seems to happen frequently. Indeed, when we look back at panel A, we observe that the occurrences of short puts as a starting transaction is close to the occurrences of long calls (20,582 versus 21,163). Third in terms of popularity is the short call. The least common starting transaction is by far the purchase of a put option.

The first row of panel C is calculated by dividing the first row from panel A by the total number of calls of the relevant sample. The second row is obtained by dividing the second row of panel A by the total number of puts. For each sample, we notice that for calls, the buying activity dominates the selling activity (values ranging from 50.70% for the sample of infrequent traders to 59.69% for the starting transactions). Except for the foreign contracts, a reverse picture emerges for the put side. In that segment, selling put options is for six of the samples the most popular activity. This panel could be an indication of the optimism of the investors regarding the future price evolution of the stock market. Long call options are yielding high returns in booming markets, whereas selling (i.e., writing) put options give an option premium with no future obligations if prices exceed the target (strike) price.

The first column of panel D is calculated by dividing the first column of panel A by the total number of buys (calls and puts aggregated) of the relevant sample. The second column is obtained by dividing the second column of panel A by the total number of sells. For each sample, we find more concentrated buying behaviour than selling behaviour. Buying occurs predominantly in call contracts (more than 58%), whereas the selling is more evenly spread across calls and puts.
Since we observe no explicit differences between the total sample and the sample based on the contracts that could be retrieved in Datastream, we feel quite confident that the limitations in our data collection will not fundamentally change our conclusions.

A next variable of interest concerns moneyness of options at trading dates. We identify the stock price of the underlying share or index at the date of trade, and then link this to the strike price. We define moneyness as the ratio between the strike price and the underlying stock price. In line with Lakonishok et al. (2007), we define a moneyness smaller than 0.9 as being in the money for calls and out of the money for puts, and a moneyness above 1.1 as being out of the money for calls, and in the money for puts. A moneyness ratio that lies between 0.9 and 1.1 results in an option that is at the money. Due to data unavailability, we restrict the analysis to the sample of the contracts available in Datastream. We then again split this sample in the frequent and less frequent traders and also consider the starting transactions. The result of this analysis is reported in Table 4.

*** Insert Table 4 about here***

It does not come as a surprise that for each of the four possible combinations of short/long and call/put, around three quarters of the trades occur in the at the money band.

For long calls, we then notice that the out of the money options is second most popular. The difference with the in the money calls is quite substantial for all samples. For short calls, and for three out of the four samples, the proportion of in the money options dominates the proportion of the out of the money options. The last sample, composed of the starting transactions, shows a dramatically different pattern. There, virtually no options are written in the money.

When examining long put options, in the money option trades occur more frequently than out of the money trades. The difference is highest for the sample of starting transactions. For short put options, the out of the money frequency is higher than the in the money frequency. This also holds for the sample of starting transactions. The obtained result for the short calls does not longer hold true here.
Table 5 describes the distribution of the trades with respect to a third feature of an option contract, namely the expiration date. We calculate the remaining maturity of the traded option by taking the difference (in calendar days) between the expiration date and the trade date, and then divide this number by seven. This results in the number of weeks until expiration. Again in accordance with Lakonishok et al. (2007), we define short term as less than six weeks, medium term as between six and eighteen weeks, and long term as more than eighteen weeks.

The entire sample and the sample of frequent traders show, qualitatively, the same tendency. Short term trading happens most often, followed by long term trading. This is consistent for each type, except for the long puts in the whole sample. There, medium term exceeds long term. In a later phase, we will show that indeed a substantial amount of transactions are initiated and closed within a limited period of time.

A clearly different situation is observed in the sample of infrequent traders. There, except for short calls, the long term transactions dominate the short term transactions. Infrequent traders perhaps opt more for a buy and hold type strategy, in which they buy a long term option, which they hold until (or sell just before) the expiration date. As witnessed in the sample of starting transactions, the majority of transactions start when the option is still relatively far away from expiration. Only long puts are at odds with this observation. In that case, more than 60% of the trades start less than six weeks away from maturity. Only 15.19% starts with a remaining maturity of 18 weeks. This can be seen as evidence that, most of the time, investors were quite optimistic concerning the performance of the underlying.

In Table 6, we establish the link between the remaining maturity and the moneyness of the trade. We only report the results for the DS sample, but results are similar for the other samples. A consistent pattern across the subgroups formed by call/put and short/long is that the further away from maturity, the more important the away from the money categories become. This of course is at the cost of the at the money category.
4. Return calculation

We have chosen to calculate returns on a transactional base, which allows us to precisely calculate the gain/loss incurred by an investor on a particular transaction. For each investor, we select all executed trades. Then we identify the different contracts that are traded by each investor. For each unique contract traded, we select the list of transactions executed by the investor in that particular contract. We compute at each transaction date the cumulative number of units in that contract and calculate a return each time the absolute number of units outstanding decreases. Note that this in fact means that a return is calculated, a) when a sell transaction occurs at a moment that the investor is long in the contract, or b) when a buy transaction occurs at a moment that the investor is short in a contract. If at some point, a particular transaction results in a switch of sign of the cumulative number of units, this means that the investor has switched from a long position in the option to a short position, or vice versa. Then, in order to still be able to calculate the return, we split up this particular transaction in two parts. The first part closes the existing position in the option contract, and has as number of units the number of contracts that was still open. The second transaction opens the new position in the particular contract. The settlement prices for these two transactions are calculated proportionally to their weight in the initial transaction.

If at the end of the transaction list, the cumulative number of units is not equal to zero, then either the contract is still alive at the end of the sample, either it has expired. In each case, we add a closing transaction. If the contract has expired, we take as settlement price the number of open units times the price of the option at expiration. If the contract has not matured at the end date of our sample (31/12/2007), we take as settlement price the number of units times the price of the option at 31/12/2007.4

3 To recap on what we said earlier, a call option on ING with maturity date August 2007 and strike price 25 EUR is a different contract than a call option on ING with maturity date August 2007 and strike price 24 EUR.
4 Our findings are robust to excluding the transactions that are still alive at the end of our sample.
When multiple purchases of an option occur, the basis for computing the gain or loss associated with the sale is computed using the FIFO (first-in first-out) accounting method. Consider an investor who purchased 10 option contracts of ING for 60 EUR on January 6, 2007 and then 20 option contracts of ING at 90 EUR on February 10, 2007. A sale of 15 option contracts of ING options on December 21, 2007 by this same investor is assumed to consist of 10 option contracts purchased on January 6 and 5 shares purchased on February 10. This sale then entails two return calculations.

Since the manipulation of the transactions and the return calculations are perhaps not that easy to understand, we provide an example which captures all the possibilities, and should help to clearly understand how we proceeded.

*** Insert Table 7 about here ***

Panel A of Table 7 shows the list of transactions a particular investor has made in a particular contract. This investor initially shorts 10 units in transaction 1. In transaction 2 these 10 units are bought back, which brings the cumulative number back to zero. In transaction 3, a long position of 10 units is built up. This position is again closed in transaction 4. The holdings again equal zero. In transaction 5, the investor shorts 12 units again. In transaction 6, 20 units are bought, which means that the investor switches from a short position to a long position. In transaction 7 and 8, the long position is each time increased by 5 units. In transaction 9 and 10 the long position is each time decreased with 5 units.

In panel B of Table 7, we show the transactions that we have added for the sake of our return calculation. We now see that transaction 6 is split up in two transactions. The first consists of a buy of 12 units, which closes the short position. The second consists of a buy of 8 units, which initiates the long position. We also add a closing transaction at the end, because we notice that the cumulative holdings were not zero. Therefore, we sell the remaining units at
the price of that particular contract on the expiration date. Then, after having transformed
the transactions, we can calculate returns.

The return \( RET_i \) of transaction \( i \) over the transaction period going from \( t_1 \) until \( t_2 \) is now
calculated as:

\[
RET_i = \frac{P_{t_2,i} - P_{t_1,i}}{P_{t_1,i}} \times \text{sign}(\text{HOLD}_{t_1,i}),
\]

with \( P \) the price of the option contract and \( \text{sign}(\text{HOLD}_{t_1,i}) \) the sign of the number of
holdings at time \( t_1 \), corresponding to transaction \( i \). In order to correctly account for short
positions, we multiply with the sign of the number of holdings at time period \( 1 \). This means
that we multiply with minus one in case of a short position. We here briefly touch upon the
divergence in attainable returns between short and long positions. Indeed, the maximum
loss on a long position is 100\%, and the potential gain is theoretically unlimited. The reverse
holds true for short positions: no more than 100\% can be gained, whereas the possible loss is
not bounded.

This first return measure is straightforward to understand, namely the return achieved over
the time period of the transaction. However, the average return is an average across all
transactions, regardless of their duration. This means that it is possible that the average
return on all transactions is positive, simply because some very long dated transactions have
a positive return, even though the majority of trades (with short maturity) is negative. An
example should clarify this. Suppose we record five transactions in total. Four of them are on
two days and make losses of 1\%, 2\%, 3\% and 4\%. The last trades 10 days and gains 20\%. The
average return is positive and amounts to 2\%. However, most trades ended badly. To tackle
this problem, we make the returns over the different holding periods comparable by
calculating a second return measure, which is obtained by dividing each \( RET_i \) by the number
of calendar days that the investment is in place. If the transaction is initiated and closed
within the same day, we of course divide by one.
Mathematically,

\[ RETD_i = \frac{RET_i}{N}, \]

with \( N \) the number of calendar days that the investment is in place.

In our example, daily returns are then -0.5\%, -1\%, -1.5\%, -2\%, and 2\%. The average daily return now amounts to -1\%.

We also check the significance of the obtained results. This is done by one or two sample T tests, depending on what we want to test. Although, as we discuss later, the distribution of the returns cannot be assumed to be normal, we still perform statistical inference of the returns by way of a classical T test. When the number of observations is large, which is the case in all of our samples and subsamples, the central limit theorem should apply and we can still use the T test.

5. Results

We perform the calculation of returns for different samples, in order to ascertain the robustness of our results and in order to document return differences and origins.

*** Insert Table 8 about here***

As a first sample we take all the transactions for which a return can be calculated. This sample totals 107,314 observations, of which 53,587 are short transactions and 53,727 are long transactions. In other words, there is equilibrium in the amounts of short and long transactions.

We notice, in the first column of panel A of Table 8 that the average return is severely negative and amounts to -10.55\%, and is significantly different from zero. The median transaction results in a gain of 7.35\%, but due to severe downside risk, the average becomes negative. We also notice that the returns are negatively skewed and that the return
distribution has fat tails, as witnessed by the high kurtosis. In short, the return distribution is far from normal.

In columns 2 and 3 we split our sample into short and long positions. We see a clearly different picture between these two columns. Shorting on average leads to a loss of nearly 42%, whereas the average long transaction leads to a gain of nearly 21%. The median values show a reverse picture: the median return on short transactions is above 16%, whereas the median return on long transaction is less than 1%. However, this median ignores the fact that on the short side, a high percentage of trades results in a loss of 100%, whereas the maximum gain is bounded to one. The maximum loss for a short position is -20,769.3% and the maximum gain is 1. For the long positions, the downside risk is much smaller and the upward potential is unlimited. The maximum loss for a long position is 99.82%, and the maximum gain is 13,868.2%. Since the return distribution on short options is bounded to the right, it is evident that the distribution is left skewed. The left bound on the return distribution of long position induces a right skew.

Panel B shows the results for our second return measure. As stated earlier, we calculate the daily return of each transaction. The results reveal a somewhat different picture. Now we see that on average, the daily return is positive and amounts to 0.66%. The median is also positive and equals 0.26%. The left skew and excess kurtosis found in the first return measure are again present. When splitting the sample into long and short positions, we again notice the poor average performance of short positions. On average, a daily loss of 4.26% is incurred. The average return on a long position is positive: it amounts to 5.6%. Again, we observe some very high extreme returns.

It is hard to ascertain whether the extreme returns were indeed actually achieved, or are due to some data error. In order to be sure that the results are not driven by these outliers, we apply an outlier filtering. The top and bottom 0.5% of the return observations are discarded. Note that this type of outlier filtering should positively influence the return on short positions and negatively affect the long positions. The reason is that the outlier filtering is executed on the whole sample. The lowest half percentile of the return distribution is entirely populated by short options, since nearly ten percent of the whole sample loses 100% of the
investment, which can only occur in a short position. The filtering on the upper side of the distribution will only remove long position, since only long positions can gain more than 100%. The total effect of the outlier filtering on the whole return sample is unknown a priori and is an empirical matter.

*** Insert Table 9 about here***

In a qualitative sense, the results remain the same. We refer to panel A of Table 9. We observe that the mean return increases to -6.93%. The median return is of course unaffected. Average returns for short positions increase to -22.24%. The median slightly increases to 17.46%. For the long options, the average return declines to 8.35%. The median decreases to 0.16%. We again observe the left skew for returns on short options and the right skew for returns on long options. The entire distribution is still left skewed. The extremes for this return measure now amount to -1,014.3% for the minimum and 627.07% for the maximum. These figures seem very reasonable in an option environment, and strengthen the impression that outliers were not driving the initial results. A comparable picture then is observed for the daily return measure. This is illustrated in panel B of Table 9. The mean is slightly below the mean of the complete sample (0.63% in this sample versus 0.66% in the original sample). The mean of the short and long positions have evolved in the expected directions and are now at a level of respectively -2.59% and 3.83%.

In a next robustness check, we only allow those transactions which are actively closed by the investor and thus for which we do not had to add a closing transaction (see description of procedure above). The results on the transaction period return measure are found in panel A of Table 10.

*** Insert Table 10 about here***

Note that this reduces our sample size by more than 30,000 transactions. We now observe a difference in number of transactions between the short and long side. Long transactions are
more often closed actively. The average return is again negative, but only by 1.54%, and not
significantly different from zero. The median increased to 11.56%. When looking at the
subsamples formed by the short position and long positions, we notice that the returns
became much more extreme. For long positions, the average return increases to 49.23%, and
for the short positions, the average return decreases to -55.99%. One noteworthy difference
with the prior sample is that now the median return for the long transactions exceeds the
median return for the short transactions (13.34% versus 9.38).

The same picture is shown in panel B of Table 10, which presents results for the daily
return measure. As compared to the original sample, returns on the short side are worse, and
those on the long side are better.

*** Insert Table 11 about here***

Before trying to explain these results, we first focus on another subsample, which is made up
by the expired transactions. This sample can be viewed as the complement of the sample
formed by the closed transactions. Results are found in Table 11. Note that we have 30,732
transactions which were not actively closed and thus expired. Here we report more short
than long expired transactions. Also note that the sum of the sample sizes of the closed and
expired transactions does not add up to the number of observations of the entire sample. The
reason is that we left out those transactions for which the holdings are not zero at the end of
our sample period, and for which the option contract still exists.

The first panel of Table 11 shows that the performance is very bad: the mean falls
back to -32.70%. The median is also very poor, and very close to the average value: it
amounts to -32.43%. When examining columns 2 and 3, we see a reverse picture with what
we previously documented. Now the performance on the short transactions is better than the
performance on the long transactions. The mean for the short transactions is of -11.47%,
whereas the mean for the long transactions dwindles to -57.59%. We find a very high and
positive median for short transactions, namely 66.04%. The median for the long transactions
is -90%, which is very bad. In fact, for the short side, more than 60% of the transactions end
in a gain, whereas for long transactions, this is the case for less than 10%. When considering
the daily return measure, we now observe that the daily average return for all the expired transactions is negative (-2.43%). This is mainly driven by the bad performance of the long positions (-4.72%). The average return on the short position is also negative (-0.47%), but is much smaller in absolute value than the return on the long positions. In order to be sure that these results are not caused by the outliers in the sample, Table 12 and Table 13 show that the same conclusions hold when we restrict our analysis to the filtered sample. Table 12 reports results for the actively closed transactions. Table 13 shows results for the expired transactions.

*** Insert Table 12 & 13 about here***

In a sense, these results are logical. When focusing on the results for the expired transactions, we assume that people that have initiated a short position and then observe a positive evolution of the sold option contract, in the sense that it becomes (more) out of the money, just let the contract expire. On the other hand, investors that have bought an option, and then observe an unfavourable evolution of the contract, will not sell the contract and also simply let it expire.

We further document return differences by examining the return profitability over trades with varying durations. Figure 2 shows a histogram of trade durations.

*** Insert Figure 2 about here***

The bulk of the trades happens on less than 50 days. In fact, there is quite some intraday trading. About 9,210 trades are initiated and closed within the same day, which is quite substantial. The average duration of a trade is 57.25 days, whereas the median duration amounts to 26. There seems to be a difference in trading days between long and short positions. The mean (median) duration of a long position equals 45.59 days (17 days). The duration for short positions is substantially longer: the mean amounts to 69.03 days, and the median is 37 days.
Chapter 5

We form three subsamples, based on the duration of the trades: the first sample consists of trades closed within the week (so, the trade duration should not exceed 7 days). The next sample consists of trades with duration longer than one week, but shorter than one month (so, the trade duration should be higher than 7 days but below 31 days). The last sample consists of trades with duration longer than one month (longer than 30 days). Table 14 shows, for the different samples, the return characteristics. The short term trades indeed are the most profitable. The longer the duration, the worse the performance. Differences are statistically significant. We ascertain whether the poor performance of the long term trades is not driven by the poor results of the expired transactions. Indeed, the expired transactions are predominantly situated in the sample of longer term trades. We exclude expired contracts and only focus on transactions actively closed by investors. The results however, remain the same in qualitative sense. We also calculate the correlation between the return measure and the number of days that a trade is in place. For both return measures, the Pearson correlation coefficient is negative and significant. The origin of these return differences is not totally clear. One possibility is that a disposition effect shows up (see Shefrin and Statman (1985), Odean (1999)). Individual investors have the tendency to hang on to losers too long and sell winners too early. This behaviour is consistent with our finding that the longer trades are held, the worse the performance becomes. A major difference between options and stocks is the limited life time of an option. Whereas stocks can be kept in the portfolio of an investor as long as the company that issued the stock is listed, options automatically cease to exist after the expiration date. This could have implications for the way losses in the portfolio are treated.

Finally, we examine the influence of the number of trades executed by an investor, and the profitability achieved by this investor. More specifically, we consider the nominal value an investor made or lost due to his trading activities over the sample period. For each trade a particular investor made, the gain or loss (in EUR) is calculated. Then, we sum these amounts over the different trades. This indicates whether the trades of the investor were profitable or detrimental to his wealth. We form deciles based on the number of trades per investor and calculate per decile the percentage of investors for which trading has resulted in a gain. Results are reported in Table 15. For the lower three deciles, the percentage of
investors for which option trading was profitable, remains limited and does not exceed the 40%. As we move on, we observe an increasing trend. For the highest decile, the percentage climbs to 73%. These results make sense we believe. The lowest decile contains investors who execute only one trade. Once burned, twice shy probably holds for these investors. An unfavourable first option transaction could render the investor disillusioned and make him shy away from making further option investments. Results for the higher deciles imply that probably there are learning effects in option trading. Indeed, since options are described as complex products, it is perhaps the type of investment that requires some form of experience.

6. Risk

In this part, we consider the risk attached to the option trades. It is indeed evident that option prices and returns are much more volatile than stock prices. This is already obvious from the standard deviations reported in the return tables. We provide more proof on the riskiness of option trading by calculating option betas at the start of the transactions. The option beta is obtained by multiplying the omega of the option with the beta of the underlying stock. The option omega is defined as the elasticity of the option price with respect to the underlying stock price and can be calculated by dividing the delta of the option by the ratio of the price of the option contract and the price of the underlying stock. (see Cox & Rubinstein (1985) and Ni (2007)). Mathematically,

$$\beta_o = \Omega_o \beta_s$$

$$\text{and } \Omega_o = \frac{\delta_o}{P_o / P_s},$$

with \(\beta_o, \beta_s\) the option beta, respectively the stock beta, 
\(\Omega_o\) the option omega, 
\(\delta_o\) the option delta 
\(P_o, P_s\) the option price, respectively the stock price.

We calculate the monthly stock beta as the slope coefficient from a CAPM regression of the previous 60 monthly stock returns on the monthly excess market return. The monthly excess
market return is calculated as the difference between the monthly return on the AEX index and the EURIBOR one month interest rate. Daily option deltas, option prices and stock prices are obtained from Datastream. For each day and option contract, we can thus calculate an option beta. Since all the calculated stock betas are positive, it follows that call options will have positive betas, and put options negative betas. This is due to the positive deltas for call options, and negative deltas for put options.

We consider the riskiness of the option transactions at the moment of initiation. Therefore, we retrieve for each transaction, of which we have calculated the return, the starting date, and calculate the corresponding option beta. Table 16 shows descriptive statistics for the various samples. Since we want to prevent option betas from call options and put options to cancel out, we report mean and median values based on absolute values. The mean of 23.22 is high. Indeed, a 1% change of the AEX index will lead to an average change of the option price of 23.22%. The median is considerably lower and amounts to 13.01. The minimum is 1,239.28. The maximum amounts to 500,005. In the next two columns, we show descriptive statistics for the short positions (column three) and the long positions (column four). The riskiness of short positions seems to be lower than the riskiness of long positions. Both mean and median point in this direction.

We also perform the analysis on the subsamples formed by the duration split-up. The statistics are shown in the last three columns of table 16. We observe that the riskiness is highest for the short term trades, and then decreases for the transactions with longer durations. The differences are quite substantial. So, in line with the return calculation, we demonstrate that also the risk profile is quite different for the different subsamples.6

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5 Removing the 0.5% highest and lowest values results in a mean of 21.24, and a median of 12.81.
6 We are cautious on drawing risk-return conclusions, since for that goal, each transaction should be risk-adjusted in some way.
7. Concluding remarks and further research

This paper discusses the use of options by individual investors. We analyze a data set containing options trades of online investors at a large Dutch bank, over the period 2006-2007. We report descriptive statistics concerning the main characteristics of options, namely whether call or put options are traded, whether the positions initiated concern a long or short position, and what the moneyness and time to maturity are. Call trading seems to be most popular, especially for the infrequent (inexperienced) traders. We document the performance investors achieve on the transactions they execute. We show that a difference exists between long and short positions. On average, short positions lose money, whereas the average for long options is positive. This is explained by the difference in attainable returns between short and long position. A substantial amount of short positions result in a loss of (much) more than 100%. This is not possible for long options, for which the maximum loss is 100%.

We further show that as the duration of the trade increases, the performance decreases. We also demonstrate that experience, as measured by the number of trades, seems to matter. The more trades executed by an investor, the higher the odds that option trading is resulting in a profit. We also calculate the risk (proxied by the option beta) of the transaction at initiation, and also report differences between various subsamples.

Future research should go in several directions. Complementing the transaction level analysis, a portfolio level analysis could shed more light on the profitability of option trading. We also need to further document the risk-return trade-off, and perhaps focus on other non-CAPM measures. Indeed Leland (1999) points at the malfunctioning of CAPM as risk measure in the presence of skewness, which clearly is the case here.

Having ignored the stock transactions of investors, we assume all trading is speculative. However, hedging motives can also be present. Therefore, the link between the stock and option portfolio of an investor should be examined.
Chapter 5

Acknowledgements

We thank the data provider. We also thank Jan Annaert, Ann De Schepper, Frank Dejonghe, Michael Frömmel and Michèle Vanmaele for helpful comments. Dries Heyman acknowledges financial support from BOF project 011/155/04 of Ghent University.

References

- Heston, S., 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, Review of Financial Studies 6, 327-343


Chapter 5

Tables

Table 1: Trade dispersion
Column (1) shows for different percentiles the number of trades executed. Column (2) shows the cumulative percentage of trades executed up to a particular percentile.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Number of trades</th>
<th>Cumulative % of trades</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>1</td>
<td>0.00%</td>
</tr>
<tr>
<td>10%</td>
<td>2</td>
<td>0.13%</td>
</tr>
<tr>
<td>20%</td>
<td>4</td>
<td>0.42%</td>
</tr>
<tr>
<td>30%</td>
<td>7</td>
<td>0.96%</td>
</tr>
<tr>
<td>40%</td>
<td>12</td>
<td>1.92%</td>
</tr>
<tr>
<td>median</td>
<td>19</td>
<td>3.52%</td>
</tr>
<tr>
<td>60%</td>
<td>31</td>
<td>6.10%</td>
</tr>
<tr>
<td>70%</td>
<td>55</td>
<td>10.47%</td>
</tr>
<tr>
<td>80%</td>
<td>100</td>
<td>18.27%</td>
</tr>
<tr>
<td>90%</td>
<td>208</td>
<td>33.46%</td>
</tr>
<tr>
<td>max</td>
<td>8,110</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Table 2: Trades per investor
Table 2 shows descriptive statistics (mean, median, min, max, and percentage of occurrences) of the number of trades per investors. Descriptive statistics are shown for the total sample of trades, and for subsamples consisting of long calls, short calls, long puts and short puts.

<table>
<thead>
<tr>
<th></th>
<th>All contracts</th>
<th>Call, long</th>
<th>Call, short</th>
<th>Put, long</th>
<th>Put, short</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>93.54</td>
<td>28.38</td>
<td>25.74</td>
<td>18.75</td>
<td>20.67</td>
</tr>
<tr>
<td>Median</td>
<td>19</td>
<td>9</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Min</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Max</td>
<td>8,110</td>
<td>1,553</td>
<td>2,030</td>
<td>2,270</td>
<td>2,257</td>
</tr>
<tr>
<td>% Occurrences</td>
<td>100.00%</td>
<td>86.96%</td>
<td>89.80%</td>
<td>61.89%</td>
<td>61.45%</td>
</tr>
</tbody>
</table>
Table 3: Sample split up

Table 3 contains 4 panels, in which we report the proportion of (i) call versus put contracts and (ii) buy versus sell contracts, leading to 4 quadrants. We report absolute occurrences of each combination in Panel A; relative occurrences to the grand total in panel B; relative as a proportion of call. resp. put contracts in panel C; and relative as a proportion of buy. resp. sell contracts in panel D. Further, each panel contains results for seven different samples. The first sample is the total sample; the second contains contracts on domestic (Dutch) options; the third sample is for the foreign contracts only. Further, sample 4 contains the trades belonging to the group of the 20% investors that trade the most. Sample 5 contains the trades belonging to the group of the 80% investors that trade the least; Sample 6 contains those transactions for which the option contract data could be retrieved from Datastream. The final sample (7) is created by selecting only the first transaction of a particular investor in a particular option contract.

<table>
<thead>
<tr>
<th>A. Total sample</th>
<th>B. Total sample</th>
<th>C. Total sample</th>
<th>D. Total sample</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
</tr>
<tr>
<td>Buy 63,961</td>
<td>Sell 58,007</td>
<td>Buy 30.34%</td>
<td>Sell 27.51%</td>
</tr>
<tr>
<td>Sum 121,968</td>
<td>Sum 88,867</td>
<td>Sum 57.85%</td>
<td>Sum 42.15%</td>
</tr>
<tr>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
</tr>
<tr>
<td>Buy 52.44%</td>
<td>Sell 47.56%</td>
<td>Buy 100.00%</td>
<td>Sell 100.00%</td>
</tr>
<tr>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<tr>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
</tr>
<tr>
<td>Buy 62,380</td>
<td>Sell 56,824</td>
<td>Buy 30.34%</td>
<td>Sell 27.51%</td>
</tr>
<tr>
<td>Sum 119,204</td>
<td>Sum 87,329</td>
<td>Sum 57.85%</td>
<td>Sum 42.15%</td>
</tr>
<tr>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
</tr>
<tr>
<td>Buy 52.33%</td>
<td>Sell 47.67%</td>
<td>Buy 100.00%</td>
<td>Sell 100.00%</td>
</tr>
<tr>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
</tr>
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</table>

<table>
<thead>
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<th>Foreign</th>
<th>Foreign</th>
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<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
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<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
</tr>
<tr>
<td>Buy 1,380</td>
<td>Sell 1,056</td>
<td>Buy 32.08%</td>
<td>Sell 24.55%</td>
</tr>
<tr>
<td>Sum 2,436</td>
<td>Sum 3,102</td>
<td>Sum 56.62%</td>
<td>Sum 43.38%</td>
</tr>
<tr>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
</tr>
<tr>
<td>Buy 56.65%</td>
<td>Sell 43.35%</td>
<td>Buy 100.00%</td>
<td>Sell 100.00%</td>
</tr>
<tr>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
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</table>

<table>
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<tr>
<th>Frequent traders</th>
<th>Frequent traders</th>
<th>Frequent traders</th>
<th>Frequent traders</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
</tr>
<tr>
<td>Buy 50,769</td>
<td>Sell 45,179</td>
<td>Buy 29.53%</td>
<td>Sell 26.28%</td>
</tr>
<tr>
<td>Sum 95,948</td>
<td>Sum 75,976</td>
<td>Sum 55.81%</td>
<td>Sum 44.19%</td>
</tr>
<tr>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
</tr>
<tr>
<td>Buy 52.91%</td>
<td>Sell 47.09%</td>
<td>Buy 100.00%</td>
<td>Sell 100.00%</td>
</tr>
<tr>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total sample</th>
<th>Buy 63,961</th>
<th>Sell 58,007</th>
<th>Buy 60.21%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
</tr>
<tr>
<td>Buy 42,270</td>
<td>Sell 46,597</td>
<td>Buy 20.05%</td>
<td>Sell 22.10%</td>
</tr>
<tr>
<td>Sum 88,867</td>
<td>Sum 88,867</td>
<td>Sum 42.15%</td>
<td>Sum 47.57%</td>
</tr>
<tr>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
</tr>
<tr>
<td>Buy 47.57%</td>
<td>Sell 52.43%</td>
<td>Buy 100.00%</td>
<td>Sell 100.00%</td>
</tr>
<tr>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total sample</th>
<th>Buy 106,231</th>
<th>Sell 104,604</th>
<th>Buy 100.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
<td><strong>Put</strong></td>
<td><strong>Call</strong></td>
</tr>
<tr>
<td>Buy 103,848</td>
<td>Sell 102,685</td>
<td>Buy 100.00%</td>
<td>Sell 100.00%</td>
</tr>
<tr>
<td>Sum 206,533</td>
<td>Sum 206,533</td>
<td>Sum 100.00%</td>
<td>Sum 100.00%</td>
</tr>
</tbody>
</table>

Chapter 5
<table>
<thead>
<tr>
<th>Infrequent traders</th>
<th>Infrequent traders</th>
<th>Infrequent traders</th>
<th>Infrequent traders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call 13192 12828 26020</td>
<td>Call 33.90% 32.97% 66.87%</td>
<td>Call 50.70% 49.30% 100.00%</td>
<td>Call 69.06% 64.76%</td>
</tr>
<tr>
<td>Put 5909 6982 12891</td>
<td>Put 15.19% 17.94% 33.13%</td>
<td>Put 45.84% 54.16% 100.00%</td>
<td>Put 30.94% 35.24%</td>
</tr>
<tr>
<td>19101 19810 38911</td>
<td>49.09% 50.91% 100.00%</td>
<td>100.00% 100.00%</td>
<td>100.00% 100.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DS sample</th>
<th>DS sample</th>
<th>DS sample</th>
<th>DS sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call 51401 47366 98767</td>
<td>Call 30.52% 28.12% 56.65%</td>
<td>Call 52.04% 47.96% 100.00%</td>
<td>Call 60.91% 56.37%</td>
</tr>
<tr>
<td>Put 32986 36661 69647</td>
<td>Put 19.99% 21.77% 41.35%</td>
<td>Put 47.36% 52.64% 100.00%</td>
<td>Put 39.09% 43.63%</td>
</tr>
<tr>
<td>84387 84027 168414</td>
<td>50.11% 49.89% 100.00%</td>
<td>100.00% 100.00%</td>
<td>100.00% 100.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample of starting transactions</th>
<th>Sample of starting transactions</th>
<th>Sample of starting transactions</th>
<th>Sample of starting transactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call 21582 14573 36155</td>
<td>Call 33.96% 22.93% 56.89%</td>
<td>Call 59.69% 40.31% 100.00%</td>
<td>Call 74.89% 41.95%</td>
</tr>
<tr>
<td>Put 7237 20163 27400</td>
<td>Put 11.39% 31.73% 43.11%</td>
<td>Put 26.41% 73.59% 100.00%</td>
<td>Put 25.11% 58.05%</td>
</tr>
<tr>
<td>28819 34736 63555</td>
<td>45.34% 54.66% 100.00%</td>
<td>100.00% 100.00%</td>
<td>100.00% 100.00%</td>
</tr>
</tbody>
</table>
Table 4: Relative importance of levels of moneyness

This table gives for each category the percentage of trades that occur in, at, or out of the money. Moneyness is defined by dividing the strike price by the stock price of the underlying at the trade date. K/S < 0.9 means in the money for calls and out of the money for puts. 0.9 < K/S < 1.1 means at the money. K/S > 1.1 means out of the money for calls and in the money for puts. Results are reported for various samples: the sample of the contracts available in Datastream, the sample of frequent traders, the sample of infrequent traders, and the sample of starting transactions.

<table>
<thead>
<tr>
<th></th>
<th>Call, long</th>
<th>Call, short</th>
<th>Put, long</th>
<th>Put, short</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DS sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>8.80%</td>
<td>18.01%</td>
<td>11.88%</td>
<td>8.87%</td>
</tr>
<tr>
<td>AT</td>
<td>76.48%</td>
<td>72.04%</td>
<td>76.60%</td>
<td>75.15%</td>
</tr>
<tr>
<td>OUT</td>
<td>14.72%</td>
<td>9.95%</td>
<td>11.52%</td>
<td>15.98%</td>
</tr>
<tr>
<td><strong>Frequent traders</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>8.39%</td>
<td>17.85%</td>
<td>11.73%</td>
<td>8.46%</td>
</tr>
<tr>
<td>AT</td>
<td>77.79%</td>
<td>72.41%</td>
<td>78.29%</td>
<td>74.91%</td>
</tr>
<tr>
<td>OUT</td>
<td>13.83%</td>
<td>9.74%</td>
<td>9.97%</td>
<td>16.62%</td>
</tr>
<tr>
<td><strong>Infrequent traders</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>10.39%</td>
<td>18.99%</td>
<td>16.73%</td>
<td>11.25%</td>
</tr>
<tr>
<td>AT</td>
<td>71.47%</td>
<td>69.77%</td>
<td>70.89%</td>
<td>76.51%</td>
</tr>
<tr>
<td>OUT</td>
<td>18.14%</td>
<td>11.24%</td>
<td>12.38%</td>
<td>12.24%</td>
</tr>
<tr>
<td><strong>Starting transactions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>8.20%</td>
<td>1.51%</td>
<td>12.07%</td>
<td>13.41%</td>
</tr>
<tr>
<td>AT</td>
<td>75.51%</td>
<td>77.47%</td>
<td>82.09%</td>
<td>71.67%</td>
</tr>
<tr>
<td>OUT</td>
<td>16.29%</td>
<td>21.02%</td>
<td>5.84%</td>
<td>14.92%</td>
</tr>
</tbody>
</table>
Table 5: Relative importance of levels of remaining maturity

This table gives for each category the percentage of trades that expire at short term, medium term or long term. Maturity is calculated by dividing the difference (in calendar days) between expiration date and trade date by seven. Maturity less than 6 weeks is short term, between 6 weeks and 18 weeks is medium term, above 18 weeks is long term. Results are reported for various samples: the sample of the contracts available in Datastream, the sample of frequent traders, the sample of infrequent traders and the sample of starting transactions.

<table>
<thead>
<tr>
<th></th>
<th>Call, long</th>
<th>Call, short</th>
<th>Put, long</th>
<th>Put, short</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DS sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MT&lt; 6 weeks</td>
<td>42.78%</td>
<td>55.42%</td>
<td>42.89%</td>
<td>39.21%</td>
</tr>
<tr>
<td>6 weeks &lt; MT &lt; 18 weeks</td>
<td>24.17%</td>
<td>23.09%</td>
<td>25.11%</td>
<td>23.20%</td>
</tr>
<tr>
<td>MT &gt; 18 weeks</td>
<td>33.05%</td>
<td>21.49%</td>
<td>32.00%</td>
<td>37.59%</td>
</tr>
<tr>
<td><strong>Frequent traders</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MT&lt; 6 weeks</td>
<td>45.98%</td>
<td>56.65%</td>
<td>46.58%</td>
<td>40.03%</td>
</tr>
<tr>
<td>6 weeks &lt; MT &lt; 18 weeks</td>
<td>24.40%</td>
<td>22.88%</td>
<td>24.51%</td>
<td>23.14%</td>
</tr>
<tr>
<td>MT &gt; 18 weeks</td>
<td>29.62%</td>
<td>20.47%</td>
<td>28.91%</td>
<td>36.83%</td>
</tr>
<tr>
<td><strong>Infrequent traders</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MT&lt; 6 weeks</td>
<td>30.25%</td>
<td>47.99%</td>
<td>29.99%</td>
<td>34.36%</td>
</tr>
<tr>
<td>6 weeks &lt; MT &lt; 18 weeks</td>
<td>23.30%</td>
<td>24.34%</td>
<td>27.22%</td>
<td>23.56%</td>
</tr>
<tr>
<td>MT &gt; 18 weeks</td>
<td>46.44%</td>
<td>27.67%</td>
<td>42.79%</td>
<td>42.08%</td>
</tr>
<tr>
<td><strong>Starting transactions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MT&lt; 6 weeks</td>
<td>29.91%</td>
<td>28.74%</td>
<td>60.28%</td>
<td>22.96%</td>
</tr>
<tr>
<td>6 weeks &lt; MT &lt; 18 weeks</td>
<td>27.19%</td>
<td>29.87%</td>
<td>24.53%</td>
<td>27.78%</td>
</tr>
<tr>
<td>MT &gt; 18 weeks</td>
<td>42.91%</td>
<td>41.39%</td>
<td>15.19%</td>
<td>49.26%</td>
</tr>
</tbody>
</table>
Table 6: Relative importance of maturity-moneyness combinations

Table 6 reports for each category of remaining maturity (as defined in Table 5), the percentage of trades that occurs at, in, and out of the money (as defined in Table 4). Results are only reported for the sample of the contracts available in Datastream.

<table>
<thead>
<tr>
<th></th>
<th>Call, Long</th>
<th></th>
<th></th>
<th>Call, short</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ST</td>
<td>MT</td>
<td>LT</td>
<td>IN</td>
<td>2.81%</td>
<td>6.53%</td>
<td>14.50%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>AT</td>
<td>91.90%</td>
<td>79.74%</td>
<td>63.72%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>OUT</td>
<td>5.29%</td>
<td>13.73%</td>
<td>21.78%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IN</td>
<td>9.36%</td>
<td>15.59%</td>
<td>19.13%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>AT</td>
<td>75.34%</td>
<td>64.28%</td>
<td>60.34%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>OUT</td>
<td>15.30%</td>
<td>20.13%</td>
<td>20.54%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IN</td>
<td>7.63%</td>
<td>9.42%</td>
<td>11.31%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>AT</td>
<td>81.32%</td>
<td>76.25%</td>
<td>71.11%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>OUT</td>
<td>11.05%</td>
<td>14.33%</td>
<td>17.58%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IN</td>
<td>9.91%</td>
<td>11.31%</td>
<td>17.48%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>AT</td>
<td>74.59%</td>
<td>71.11%</td>
<td>65.84%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>OUT</td>
<td>15.50%</td>
<td>17.58%</td>
<td>16.68%</td>
</tr>
</tbody>
</table>
Table 7: Transaction example

Panel A of Table 7 reports a transaction list for a particular investor in a particular contract, as retrieved from the dataset. Panel B reports the transaction list after having transformed the list for return calculation purposes.

### A. Transactions from the transaction file

<table>
<thead>
<tr>
<th># Units</th>
<th>Settlement price</th>
<th>Buy(1) or sell(-1)</th>
<th>Option ID</th>
<th>Transaction date</th>
<th>Price per unit</th>
<th>Cumulative holdings</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>16,161</td>
<td>-1</td>
<td>6178</td>
<td>732736</td>
<td>1616.10</td>
<td>-10</td>
</tr>
<tr>
<td>10</td>
<td>19,280</td>
<td>1</td>
<td>6178</td>
<td>732795</td>
<td>1928.00</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>6,880</td>
<td>1</td>
<td>6178</td>
<td>732816</td>
<td>688.00</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>8,170</td>
<td>-1</td>
<td>6178</td>
<td>732827</td>
<td>817.00</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>9,080</td>
<td>-1</td>
<td>6178</td>
<td>732827</td>
<td>756.67</td>
<td>-12</td>
</tr>
<tr>
<td>20</td>
<td>5,255</td>
<td>1</td>
<td>6178</td>
<td>732843</td>
<td>262.75</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>4,336</td>
<td>1</td>
<td>6178</td>
<td>732904</td>
<td>867.20</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>5,899</td>
<td>1</td>
<td>6178</td>
<td>732918</td>
<td>1179.80</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>5,901</td>
<td>-1</td>
<td>6178</td>
<td>732920</td>
<td>1180.20</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>6,040</td>
<td>-1</td>
<td>6178</td>
<td>732925</td>
<td>1208.00</td>
<td>8</td>
</tr>
</tbody>
</table>

### B. Transformed transactions

<table>
<thead>
<tr>
<th># Units</th>
<th>Settlement price</th>
<th>Buy(1) or sell(-1)</th>
<th>Option ID</th>
<th>Transaction date</th>
<th>Price per unit</th>
<th>Cumulative holdings</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>16,161</td>
<td>-1</td>
<td>6178</td>
<td>732736</td>
<td>1616.10</td>
<td>-10</td>
</tr>
<tr>
<td>10</td>
<td>19,280</td>
<td>1</td>
<td>6178</td>
<td>732795</td>
<td>1928.00</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>6,880</td>
<td>1</td>
<td>6178</td>
<td>732816</td>
<td>688.00</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>8,170</td>
<td>-1</td>
<td>6178</td>
<td>732827</td>
<td>817.00</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>9,080</td>
<td>-1</td>
<td>6178</td>
<td>732827</td>
<td>756.67</td>
<td>-12</td>
</tr>
<tr>
<td>12</td>
<td>3,153</td>
<td>1</td>
<td>6178</td>
<td>732843</td>
<td>262.75</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>2,102</td>
<td>1</td>
<td>6178</td>
<td>732843</td>
<td>262.75</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>4,336</td>
<td>1</td>
<td>6178</td>
<td>732904</td>
<td>867.20</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>5,899</td>
<td>1</td>
<td>6178</td>
<td>732918</td>
<td>1179.80</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>5,901</td>
<td>-1</td>
<td>6178</td>
<td>732920</td>
<td>1180.20</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>6,040</td>
<td>-1</td>
<td>6178</td>
<td>732925</td>
<td>1208.00</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>12,440</td>
<td>-1</td>
<td>6178</td>
<td>732935</td>
<td>1555.00</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 8: Return characteristics (sample = all transactions).

Panel A of Table 8 reports percentiles and the first four moments of the return distribution, when the return is calculated as the transaction return. The sample for the first column consists of all transactions, whereas the samples for the second and third column consist of respectively the short and long transactions. T statistics for the mean are reported in parentheses below the mean. The last line reports the mean difference between short and long transactions, and the corresponding T statistic. Panel B replicates panel A for the daily return measure. The input sample for Table 8 is the entire sample.

### Complete sample

<table>
<thead>
<tr>
<th></th>
<th>A. TRANSACTION BASED RETURN MEASURE</th>
<th>B. DAILY RETURN MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>min</td>
<td>-20,769.25%</td>
<td>-20,769.25%</td>
</tr>
<tr>
<td>10%</td>
<td>-98.49%</td>
<td>-200.95%</td>
</tr>
<tr>
<td>20%</td>
<td>-80.06%</td>
<td>-79.96%</td>
</tr>
<tr>
<td>30%</td>
<td>-38.13%</td>
<td>-25.57%</td>
</tr>
<tr>
<td>40%</td>
<td>-8.45%</td>
<td>2.16%</td>
</tr>
<tr>
<td>median</td>
<td>7.35%</td>
<td>16.59%</td>
</tr>
<tr>
<td>60%</td>
<td>21.43%</td>
<td>35.59%</td>
</tr>
<tr>
<td>70%</td>
<td>42.59%</td>
<td>58.16%</td>
</tr>
<tr>
<td>80%</td>
<td>72.55%</td>
<td>81.48%</td>
</tr>
<tr>
<td>90%</td>
<td>94.68%</td>
<td>92.88%</td>
</tr>
<tr>
<td>max</td>
<td>13,868.16%</td>
<td>100.00%</td>
</tr>
<tr>
<td>mean</td>
<td>-10.55%</td>
<td>-42.00%</td>
</tr>
<tr>
<td>(std)</td>
<td>(-13.34)</td>
<td>(-31.33)</td>
</tr>
<tr>
<td>skewness</td>
<td>-11.92</td>
<td>-20.74</td>
</tr>
<tr>
<td>kurtosis</td>
<td>1,080.31</td>
<td>933.64</td>
</tr>
</tbody>
</table>

Difference between mean return of long and short positions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.81%</td>
<td>(21.5)</td>
<td>9.84%</td>
</tr>
</tbody>
</table>
Table 9: Return characteristics (sample = outlier filtered sample).

Panel A of Table 9 reports percentiles and the first four moments of the return distribution, when the return is calculated as the transaction return. The sample for the first column consists of all transactions, whereas the samples for the second and third column consist of respectively the short and long transactions. T statistics for the mean are reported in parentheses below the mean. The last line reports the mean difference between short and long transactions, and the corresponding T statistic. Panel B replicates panel A for the daily return measure. The input sample for Table 9 is constructed by discarding the 1% extreme observations from the entire sample.

Filtered sample (1% outliers)

<table>
<thead>
<tr>
<th></th>
<th>A. TRANSACTION BASED RETURN MEASURE</th>
<th>B. DAILY RETURN MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>min</td>
<td>-1,014.29%</td>
<td>-1,014.29%</td>
</tr>
<tr>
<td>10%</td>
<td>-98.14%</td>
<td>-183.93%</td>
</tr>
<tr>
<td>20%</td>
<td>-78.88%</td>
<td>-73.69%</td>
</tr>
<tr>
<td>30%</td>
<td>-37.44%</td>
<td>-22.77%</td>
</tr>
<tr>
<td>40%</td>
<td>-8.22%</td>
<td>2.99%</td>
</tr>
<tr>
<td>median</td>
<td>7.35%</td>
<td>17.46%</td>
</tr>
<tr>
<td>60%</td>
<td>21.28%</td>
<td>36.45%</td>
</tr>
<tr>
<td>70%</td>
<td>42.11%</td>
<td>58.74%</td>
</tr>
<tr>
<td>80%</td>
<td>71.52%</td>
<td>81.68%</td>
</tr>
<tr>
<td>90%</td>
<td>94.22%</td>
<td>92.96%</td>
</tr>
<tr>
<td>max</td>
<td>627.07%</td>
<td>100.00%</td>
</tr>
<tr>
<td>mean</td>
<td>-6.93%</td>
<td>-22.25%</td>
</tr>
<tr>
<td></td>
<td>(-17.41)</td>
<td>(-34.06)</td>
</tr>
<tr>
<td>std</td>
<td>1.30</td>
<td>1.50</td>
</tr>
<tr>
<td>skewness</td>
<td>-1.79</td>
<td>-2.86</td>
</tr>
<tr>
<td>kurtosis</td>
<td>15.64</td>
<td>13.44</td>
</tr>
<tr>
<td>N</td>
<td>106,240</td>
<td>53,050</td>
</tr>
</tbody>
</table>

Difference between mean return of long and short positions

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30.60%</td>
<td>(35.80)</td>
</tr>
<tr>
<td></td>
<td>6.42%</td>
<td>(38.71)</td>
</tr>
</tbody>
</table>
Table 10: Return characteristics (sample = sample of closed transactions).

Panel A of Table 10 reports percentiles and the first four moments of the return distribution, when the return is calculated as the transaction return. The sample for the first column consists of all transactions, whereas the samples for the second and third column consist of respectively the short and long transactions. T statistics for the mean are reported in parentheses below the mean. The last line reports the mean difference between short and long transactions, and the corresponding T statistic. Panel B replicates panel A for the daily return measure. The input sample for Table 10 is constructed by selecting, from the entire sample, the transactions that were closed by the investor.

### Closed transactions

#### A. TRANSACTION BASED RETURN MEASURE

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>-20,769.25%</td>
<td>-20,769.25%</td>
<td>-99.56%</td>
</tr>
<tr>
<td>10%</td>
<td>-94.71%</td>
<td>-212.43%</td>
<td>-58.78%</td>
</tr>
<tr>
<td>20%</td>
<td>-47.50%</td>
<td>-92.32%</td>
<td>-27.00%</td>
</tr>
<tr>
<td>30%</td>
<td>-15.52%</td>
<td>-35.75%</td>
<td>-6.20%</td>
</tr>
<tr>
<td>40%</td>
<td>2.39%</td>
<td>-4.04%</td>
<td>5.13%</td>
</tr>
<tr>
<td>median</td>
<td>11.58%</td>
<td>9.38%</td>
<td>13.34%</td>
</tr>
<tr>
<td>60%</td>
<td>23.75%</td>
<td>22.22%</td>
<td>25.00%</td>
</tr>
<tr>
<td>70%</td>
<td>40.71%</td>
<td>39.02%</td>
<td>42.75%</td>
</tr>
<tr>
<td>80%</td>
<td>63.71%</td>
<td>57.66%</td>
<td>75.00%</td>
</tr>
<tr>
<td>90%</td>
<td>93.14%</td>
<td>79.81%</td>
<td>154.16%</td>
</tr>
<tr>
<td>max</td>
<td>13,868.16%</td>
<td>99.79%</td>
<td>13,868.16%</td>
</tr>
<tr>
<td>mean</td>
<td>-1.54%</td>
<td>-55.99%</td>
<td>49.23%</td>
</tr>
<tr>
<td>std</td>
<td>2.79</td>
<td>3.32</td>
<td>2.06</td>
</tr>
<tr>
<td>skewness</td>
<td>-13.38</td>
<td>-22.08</td>
<td>22.88</td>
</tr>
<tr>
<td>kurtosis</td>
<td>1,103.28</td>
<td>941.93</td>
<td>1,150.12</td>
</tr>
<tr>
<td>N</td>
<td>76,227</td>
<td>36,779</td>
<td>39,448</td>
</tr>
</tbody>
</table>

#### B. DAILY RETURN MEASURE

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>-17,135.88%</td>
<td>-17,135.88%</td>
<td>-97.99%</td>
</tr>
<tr>
<td>10%</td>
<td>-8.64%</td>
<td>-12.14%</td>
<td>-6.32%</td>
</tr>
<tr>
<td>20%</td>
<td>-2.61%</td>
<td>-3.81%</td>
<td>-1.70%</td>
</tr>
<tr>
<td>30%</td>
<td>-0.62%</td>
<td>-1.19%</td>
<td>-0.28%</td>
</tr>
<tr>
<td>40%</td>
<td>0.09%</td>
<td>-0.12%</td>
<td>0.27%</td>
</tr>
<tr>
<td>median</td>
<td>0.53%</td>
<td>0.32%</td>
<td>0.99%</td>
</tr>
<tr>
<td>60%</td>
<td>1.21%</td>
<td>0.70%</td>
<td>2.24%</td>
</tr>
<tr>
<td>70%</td>
<td>2.49%</td>
<td>1.35%</td>
<td>4.48%</td>
</tr>
<tr>
<td>80%</td>
<td>5.26%</td>
<td>2.69%</td>
<td>8.97%</td>
</tr>
<tr>
<td>90%</td>
<td>13.25%</td>
<td>6.50%</td>
<td>21.58%</td>
</tr>
<tr>
<td>max</td>
<td>5,433.16%</td>
<td>90.11%</td>
<td>5,433.16%</td>
</tr>
<tr>
<td>mean</td>
<td>1.93%</td>
<td>-5.98%</td>
<td>9.31%</td>
</tr>
<tr>
<td>std</td>
<td>0.88</td>
<td>1.08</td>
<td>0.63</td>
</tr>
<tr>
<td>skewness</td>
<td>-97.72</td>
<td>-117.17</td>
<td>37.62</td>
</tr>
<tr>
<td>kurtosis</td>
<td>19,786.64</td>
<td>17,683.02</td>
<td>2,498.78</td>
</tr>
<tr>
<td>N</td>
<td>76,227</td>
<td>36,779</td>
<td>39,448</td>
</tr>
</tbody>
</table>

Difference between mean return of long and short positions

|       | 105.22% | (23.71) | 15.29% | (52.07) |
Table 11: Return characteristics (sample = sample of expired transactions).

Panel A of Table 11 reports percentiles and the first four moments of the return distribution, when the return is calculated as the transaction return. The sample for the first column consists of all transactions, whereas the samples for the second and third column consist of respectively the short and long transactions. T statistics for the mean are reported in parentheses below the mean. The last line reports the mean difference between short and long transactions, and the corresponding T statistic. Panel B replicates panel A for the daily return measure. The input sample for Table 11 is constructed by selecting, from the entire sample, the transactions that were not closed by the investor.

**Expired transactions**

<table>
<thead>
<tr>
<th></th>
<th>A. TRANSACTION BASED RETURN MEASURE</th>
<th></th>
<th>B. DAILY RETURN MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>min</td>
<td>-5,837.50%</td>
<td>-5,837.50%</td>
<td>-99.82%</td>
</tr>
<tr>
<td>10%</td>
<td>-98.92%</td>
<td>-174.50%</td>
<td>-98.34%</td>
</tr>
<tr>
<td>20%</td>
<td>-96.15%</td>
<td>-49.43%</td>
<td>-97.05%</td>
</tr>
<tr>
<td>30%</td>
<td>-90.88%</td>
<td>-5.38%</td>
<td>-95.61%</td>
</tr>
<tr>
<td>40%</td>
<td>-74.14%</td>
<td>27.29%</td>
<td>-93.52%</td>
</tr>
<tr>
<td>median</td>
<td>-32.43%</td>
<td>66.04%</td>
<td>-90.00%</td>
</tr>
<tr>
<td>60%</td>
<td>3.33%</td>
<td>85.71%</td>
<td>-83.66%</td>
</tr>
<tr>
<td>70%</td>
<td>58.33%</td>
<td>91.12%</td>
<td>-68.85%</td>
</tr>
<tr>
<td>80%</td>
<td>89.21%</td>
<td>94.28%</td>
<td>-35.98%</td>
</tr>
<tr>
<td>90%</td>
<td>95.19%</td>
<td>96.56%</td>
<td>-4.92%</td>
</tr>
<tr>
<td>max</td>
<td>7,223.94%</td>
<td>100.00%</td>
<td>7,223.94%</td>
</tr>
<tr>
<td>mean</td>
<td>-32.70%</td>
<td>-11.47%</td>
<td>-57.59%</td>
</tr>
<tr>
<td>std</td>
<td>2.00</td>
<td>2.28</td>
<td>1.56</td>
</tr>
<tr>
<td>skewness</td>
<td>-0.29</td>
<td>-6.94</td>
<td>23.76</td>
</tr>
<tr>
<td>kurtosis</td>
<td>219.69</td>
<td>91.24</td>
<td>821.46</td>
</tr>
<tr>
<td>N</td>
<td>30,732</td>
<td>16,584</td>
<td>14,148</td>
</tr>
</tbody>
</table>

Difference between mean return of long and short positions

|                  | -46.13% | (-16.28) | -4.26% | (-20.91) |
Table 12: Return characteristics (sample = outlier filtered sample of closed transactions).

Panel A of Table 12 reports percentiles and the first four moments of the return distribution, when the return is calculated as the transaction return. The sample for the first column consists of all transactions, whereas the samples for the second and third column consist of respectively the short and long transactions. T statistics for the mean are reported in parentheses below the mean. The last line reports the mean difference between short and long transactions, and the corresponding T statistic. Panel B replicates panel A for the daily return measure. The input sample for Table 12 is constructed by selecting, from the outlier filtered sample, the transactions that were closed by the investor.

Closed transactions (filtered)

<table>
<thead>
<tr>
<th></th>
<th>A. TRANSACTION BASED</th>
<th>B. DAILY RETURN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RETURN MEASURE</td>
<td>MEASURE</td>
</tr>
<tr>
<td></td>
<td>(1) (2) (3)</td>
<td>(1) (2) (3)</td>
</tr>
<tr>
<td>min</td>
<td>-1,014.29% -1,014.29%</td>
<td>min -998.41%</td>
</tr>
<tr>
<td></td>
<td>-99.56%</td>
<td>-998.41%</td>
</tr>
<tr>
<td>10%</td>
<td>-92.75% -195.97% -59.39%</td>
<td>-8.31% -11.06%</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>-6.40%</td>
</tr>
<tr>
<td>20%</td>
<td>-46.46% -86.34% -27.65%</td>
<td>-2.52% -3.50%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>-1.76%</td>
</tr>
<tr>
<td>30%</td>
<td>-15.17% -32.90% -6.82%</td>
<td>-0.60% -1.08%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>-0.30%</td>
</tr>
<tr>
<td>40%</td>
<td>2.43% -2.65% 4.74% 40%</td>
<td>0.10% -0.08% 0.25%</td>
</tr>
<tr>
<td>median</td>
<td>11.51% 9.97% 12.71%</td>
<td>median 0.53%</td>
</tr>
<tr>
<td></td>
<td>60%</td>
<td>0.34% 0.93%</td>
</tr>
<tr>
<td></td>
<td>62.67% 58.18% 70.78%</td>
<td>80% 5.13% 8.46%</td>
</tr>
<tr>
<td></td>
<td>80%</td>
<td>2.73% 8.46%</td>
</tr>
<tr>
<td></td>
<td>90%</td>
<td>12.73% 6.59% 19.94%</td>
</tr>
<tr>
<td>max</td>
<td>627.07% 99.79% 627.07%</td>
<td>max 622.90% 90.11% 622.90%</td>
</tr>
<tr>
<td>mean</td>
<td>1.56% -34.09% 34.87%</td>
<td>1.81% -3.80% 7.04%</td>
</tr>
<tr>
<td></td>
<td>(3.28) (-43.98) (68.29)</td>
<td>15.3 -22.73 43.47</td>
</tr>
<tr>
<td>std</td>
<td>1.30 1.48 1.01 std</td>
<td>0.32 0.32 0.32</td>
</tr>
<tr>
<td>skewness</td>
<td>-1.69 -2.83 2.43 skewness</td>
<td>-1.87 -12.30 7.63</td>
</tr>
<tr>
<td>kurtosis</td>
<td>15.75 13.17 11.09 kurtosis</td>
<td>157.48 234.39 90.73</td>
</tr>
<tr>
<td>N</td>
<td>75,366 36,402 38,964 N</td>
<td>75,366 36,402 38,964</td>
</tr>
</tbody>
</table>

Difference between mean return of long and short positions

|                | 68.96% (74.35) 10.85% (46.54) |

Chapter 5
Table 13: Return characteristics (sample = outlier filtered sample of expired transactions).

Panel A of Table 13 reports percentiles and the first four moments of the return distribution, when the return is calculated as the transaction return. The sample for the first column consists of all transactions, whereas the samples for the second and third column consist of respectively the short and long transactions. T statistics for the mean are reported in parentheses below the mean. The last line reports the mean difference between short and long transactions, and the corresponding T statistic. Panel B replicates panel A for the daily return measure. The input sample for Table 12 is constructed by selecting, from the outlier filtered sample, the transactions that were closed by the investor.

Expired transactions (filtered)

<table>
<thead>
<tr>
<th></th>
<th>A. TRANSACTION BASED RETURN MEASURE</th>
<th>B. DAILY RETURN MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>min</td>
<td>-1,013.45%</td>
<td>-1,013.45%</td>
</tr>
<tr>
<td>10%</td>
<td>-98.81%</td>
<td>-155.68%</td>
</tr>
<tr>
<td>20%</td>
<td>-96.05%</td>
<td>-45.50%</td>
</tr>
<tr>
<td>30%</td>
<td>-90.74%</td>
<td>-3.34%</td>
</tr>
<tr>
<td>40%</td>
<td>-73.08%</td>
<td>29.37%</td>
</tr>
<tr>
<td>median</td>
<td>-32.18%</td>
<td>68.22%</td>
</tr>
<tr>
<td>60%</td>
<td>3.65%</td>
<td>86.11%</td>
</tr>
<tr>
<td>70%</td>
<td>58.33%</td>
<td>91.23%</td>
</tr>
<tr>
<td>80%</td>
<td>89.17%</td>
<td>94.31%</td>
</tr>
<tr>
<td>90%</td>
<td>95.12%</td>
<td>96.58%</td>
</tr>
<tr>
<td>max</td>
<td>624.42%</td>
<td>100.00%</td>
</tr>
<tr>
<td>mean</td>
<td>-27.68%</td>
<td>3.70%</td>
</tr>
<tr>
<td></td>
<td>(-38.55)</td>
<td>(3.10)</td>
</tr>
<tr>
<td>std</td>
<td>1.25</td>
<td>1.53</td>
</tr>
<tr>
<td>skewness</td>
<td>-2.20</td>
<td>-3.10</td>
</tr>
<tr>
<td>kurtosis</td>
<td>16.02</td>
<td>15.07</td>
</tr>
<tr>
<td>N</td>
<td>30,520</td>
<td>16,425</td>
</tr>
</tbody>
</table>

Difference between mean return of long and short positions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>67.94%</td>
<td>(51.62)</td>
<td>5.07%</td>
</tr>
</tbody>
</table>

Chapter 5
Table 14: Return characteristics for trades with varying duration (full sample)

Table 14 reports percentiles and the first four moments of the return distribution, when the return is calculated as the transaction return. Column one consists of the trades for which the duration is not longer than 7 days. Column two consists of the trades for which the duration is between 7 and 31 days. Column three consists of trades with a duration exceeding 30 days. The input sample for Table 14 is the input sample of Table 8.

<table>
<thead>
<tr>
<th></th>
<th>Less than 8 days</th>
<th>between 8 and 30 days</th>
<th>Longer than 30 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>-1003.26%</td>
<td>-1014.29%</td>
<td>-1013.87%</td>
</tr>
<tr>
<td>10%</td>
<td>-75.61%</td>
<td>-98.53%</td>
<td>-115.79%</td>
</tr>
<tr>
<td>20%</td>
<td>-30.38%</td>
<td>-78.26%</td>
<td>-92.28%</td>
</tr>
<tr>
<td>30%</td>
<td>-9.89%</td>
<td>-40.28%</td>
<td>-63.61%</td>
</tr>
<tr>
<td>40%</td>
<td>1.57%</td>
<td>-8.82%</td>
<td>-26.38%</td>
</tr>
<tr>
<td>median</td>
<td>6.77%</td>
<td>9.13%</td>
<td>7.01%</td>
</tr>
<tr>
<td>60%</td>
<td>13.21%</td>
<td>23.86%</td>
<td>31.23%</td>
</tr>
<tr>
<td>70%</td>
<td>22.99%</td>
<td>41.49%</td>
<td>58.76%</td>
</tr>
<tr>
<td>80%</td>
<td>41.01%</td>
<td>66.32%</td>
<td>85.29%</td>
</tr>
<tr>
<td>90%</td>
<td>82.40%</td>
<td>93.29%</td>
<td>95.19%</td>
</tr>
<tr>
<td>max</td>
<td>625.86%</td>
<td>627.07%</td>
<td>623.68%</td>
</tr>
<tr>
<td>mean</td>
<td>7.32%</td>
<td>-6.20%</td>
<td>-15.24%</td>
</tr>
<tr>
<td></td>
<td>(12.55)</td>
<td>(-8.97)</td>
<td>(-23.15)</td>
</tr>
<tr>
<td>std</td>
<td>0.99</td>
<td>1.31</td>
<td>1.43</td>
</tr>
<tr>
<td>skewness</td>
<td>-0.78</td>
<td>-1.65</td>
<td>-1.90</td>
</tr>
<tr>
<td>kurtosis</td>
<td>22.60</td>
<td>15.45</td>
<td>13.09</td>
</tr>
<tr>
<td>N</td>
<td>28936</td>
<td>28110</td>
<td>49194</td>
</tr>
</tbody>
</table>

Difference of means

<table>
<thead>
<tr>
<th></th>
<th>(1)-(2)</th>
<th>(1)-(3)</th>
<th>(2)-(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)-(2)</td>
<td>13.52%</td>
<td>(18.63)</td>
<td></td>
</tr>
<tr>
<td>(1)-(3)</td>
<td>22.56%</td>
<td>(30.87)</td>
<td></td>
</tr>
<tr>
<td>(2)-(3)</td>
<td>9.04%</td>
<td>(9.16)</td>
<td></td>
</tr>
</tbody>
</table>
### Table 15: Wealth creation by option trading

Table 15 reports for deciles based on the number of trades made by an investor, the percentage of each decile for which option trading resulted in a creation of wealth, defined as having an aggregate gain on the trades.

<table>
<thead>
<tr>
<th>Deciles</th>
<th>Successful traders</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>31.90%</td>
</tr>
<tr>
<td>20%</td>
<td>37.14%</td>
</tr>
<tr>
<td>30%</td>
<td>40.48%</td>
</tr>
<tr>
<td>40%</td>
<td>39.52%</td>
</tr>
<tr>
<td>50%</td>
<td>49.05%</td>
</tr>
<tr>
<td>60%</td>
<td>50.00%</td>
</tr>
<tr>
<td>70%</td>
<td>49.05%</td>
</tr>
<tr>
<td>80%</td>
<td>51.90%</td>
</tr>
<tr>
<td>90%</td>
<td>54.29%</td>
</tr>
<tr>
<td>100%</td>
<td>73.81%</td>
</tr>
</tbody>
</table>
Table 16: Option betas

Table 16 reports descriptive statistics of the option betas for various samples. Sample (1) is the total sample of transactions, sample (2) is the sample of short transactions, sample (3) the sample of long transactions. Sample (4) contains the trades with a duration below 8 days, sample (5) contains the trades with a duration between 8 and 30 days. Sample (6) contains trades with a duration of more than 30 days.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (absolute value based)</td>
<td>23.22</td>
<td>18.16</td>
<td>27.63</td>
<td>43.71</td>
<td>24.12</td>
<td>11.02</td>
</tr>
<tr>
<td>median (absolute value based)</td>
<td>13.02</td>
<td>11.06</td>
<td>17.07</td>
<td>33.58</td>
<td>16.39</td>
<td>8.31</td>
</tr>
<tr>
<td>min</td>
<td>-1,239.26</td>
<td>-794.80</td>
<td>-1,239.26</td>
<td>-794.80</td>
<td>-703.03</td>
<td>-1,239.26</td>
</tr>
<tr>
<td>max</td>
<td>500.00</td>
<td>500.00</td>
<td>500.00</td>
<td>500.00</td>
<td>500.00</td>
<td>500.00</td>
</tr>
</tbody>
</table>
Chapter 5

Figures

Figure 1: Evolution of AEX index between 01/01/2006 and 31/12/2007
Chapter 5

Figure 2: Histogram of trade duration
Chapter 6:

Overconfidence and disposition effect in institutional trades
Disposition bias and overconfidence in institutional trades

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Abstract

Using a unique data set of mutual fund transactions, this paper examines two widely acknowledged behavioural biases: overconfidence in trading and disposition behaviour. We test for the first bias by comparing the ex post profitability of the purchased and sold securities by mutual funds. Our empirical results show that the returns on the purchased securities are not worse than the returns on the sold securities, implying that the trades of mutual fund managers do not erode performance. The disposition bias, i.e. the reluctance of investors to sell losing stocks, is tested by the widely accepted methodology of Odean (1998). In contrast to Odean’s findings for individual investors, using the average purchase price to assess gains and losses, we reject the disposition hypothesis for our entire mutual fund sample and instead document a propensity of mutual fund managers to cut losses early. Nonetheless, some regional differences are observed. After splitting our sample into geographical subsamples, we observe disposition behaviour for the sample of UK (oriented) funds. However, the impact of this disposition behaviour seems to be limited, since we also find that the purchases by UK funds perform significantly better than the sold securities in the post-trade period. The results from the disposition analysis are sensitive to the choice of the reference point. While no disposition effect is observed when average, first, last, or highest purchase prices serve as reference point, we document disposition behaviour for our entire sample when historical peak prices are used as reference point. This underlines the importance of the reference point in tests of disposition behaviour.
1. Introduction

Traditional finance theory assumes that markets are efficient and investors have rational expectations and take decisions that maximize their expected utility. Nevertheless, several trading patterns have been observed that do not concur with this rationality assumption and which have been recognized as behavioural biases. For instance, individual investors seem to trade more than can be rationally justified (see e.g. Barber and Odean (2000); Odean (1999)). Several explanations have been advanced to explain the excessive trading volume observed in financial markets. As with other patterns of investor behaviour, it is difficult to interpret this excessive trading volume from a traditional perspective of rational investor behaviour. From a behavioural point of view, ‘overconfidence’ has been proposed as the main reason for this trading activity (see e.g. Statman, Thorley and Vorkink (2006)). Overconfidence is modelled amongst others by the behavioural model of Daniel, Hirshleifer and Subrahmanyam (1998) in which investors overreact to private information, while underreacting to public information. Moreover, this overconfidence may be enforced through ‘biased self-attribution’, i.e. investors attribute successful investment performance to their own skills, which further strengthens their overconfidence (see also Gervais and Odean (2001)).

Apart from this irrational trading behaviour, the ‘disposition bias’ predicts that investors sell winners too early and ride losers too long (see Shefrin and Statman (1985)). Such behaviour complies with Kahneman and Tversky’s (1979) prospect theory suggesting that investors are averse to realize their losses. More specifically, under prospect theory, investors assess potential losses and gains using an S-shaped value function quantifying gains and losses rather than levels of wealth as in standard expected utility theory. In other words, this theory models the responsiveness to changes in wealth rather than to absolute levels. Potential losses and gains are defined according to a reference point. The value function displays concavity in the domain of gains and convexity in the domain of losses and is steeper for losses than for gains (i.e. loss aversion). Note that the disposition bias reflects an investor’s viewpoint on the individual stocks in his portfolio and their realized performance, whereas the overconfidence bias reveals investors’ beliefs about the future performance of the stocks under consideration. Moreover, disposition behaviour will only affect the decision to sell a security, whereas overconfidence will have an impact on both the buying and selling behaviour.

The present paper tests for the presence of overconfidence and disposition behaviour in institutional trades. In theory, institutional investors should be less receptive to behavioural biases than individual traders, although their trades may be motivated by more
agency-related issues or incentives.\footnote{1}{For example, institutional traders may engage in noise trading to address the moral hazard problem where the principal is unable to monitor the effort level of the agent (see e.g. Dow and Gorton (1997)). Moreover, fund managers tend to alter the composition of their portfolios around disclosure dates (i.e. engage in window dressing) in order to receive a positive evaluation from investors (see e.g. Lakonishok, Shleifer, Thaler and Vishny (1991)). Furthermore, compensation in the mutual fund industry is typically based on relative rankings, which triggers low-ranked funds to alter their portfolios in response to their mid-year position in the ranking (see e.g. Sirri and Tufano (1998), Brown, Harlow and Starks (1996); Chevalier and Ellison (1997)). Likewise, relative performance structures may induce fund managers to base their asset allocation decisions on the trades of other managers (i.e. herding).}

We examine the disposition bias and test the overconfidence hypothesis in an institutional trading context using a data set of mutual fund transactions, which is unique in the sense that the data set comprises daily transactions over the period August 2002 to April 2007. In our setting, transactional data have a clear advantage over holding data. First of all, we do not need to infer the institutional trades from changes in quarterly holdings. Instead, we observe flows directly, i.e. we know the trading volume and exact transaction date corresponding to each fund trade, which gives insight into the dispersion of the fund trades. Secondly, we do not need to make assumptions on the direction of the trades, since the transaction type (e.g. a purchase or sale) is identified. Thirdly, we know the exact price the fund paid or received for a particular trade. Moreover, rather than focusing on a single market, our data set covers an international spread. This permits us to test whether the behavioural bias is a global effect or a region-specific trend. In addition, our dataset allows extending the extant literature on the disposition effect, which predominantly concentrates on US investors.\footnote{2}{From an international perspective, we refer to Grinblatt and Keloharju (2001) for an analysis of the disposition effect among Finnish investors, and Shapira and Venezia (2001) for an analysis among Israeli investors. Chen, Kim, Nofsinger and Rui (2004), and Feng and Seasholes (2005) find supporting evidence for the disposition effect in Chinese stock markets. Barber, Lee, Liu and Odean (2007) report that the individual investors trading on the Taiwan Stock Exchange exhibit the disposition bias, while foreign investors and mutual funds do not.} 

Our results show that the fund managers in our sample do not exhibit overconfidence in their trading behaviour. Generally, the trades they execute are not detrimental in the sense that the purchased securities do not underperform the sold securities over a short-term horizon of 21, 42, 63, or 84 days after the trade. Therefore, we dismiss the overconfidence hypothesis. When using as reference point for assessing gains and losses the average
purchase price, we also find no evidence of disposition behaviour in mutual fund trades, albeit that disposition behaviour is observed in the subsample of UK fund managers. The results suggest that, rather than holding on to losing stocks, institutionals seem to cut losses early. However, we show that results are sensitive to the reference point. When average, first, last, or highest purchase prices are implemented as reference point, we generally reject the disposition hypothesis. Conversely, when considering maximum (historical peak) prices (over various time horizons), we confirm the disposition hypothesis. In short, the results are beneficial for institutional investors. They seem less prone to behavioural biases often documented for individual investors.

This paper is insightful for several reasons. First of all, it adds to the literature concerning the added value of mutual funds. Indeed, since Jensen (1968) pointed out that actively managed mutual funds do not outperform passive benchmarks, there is considerable debate on the usefulness of mutual funds. Next, by showing the sensitivity of results on disposition behaviour to the reference point used, we underline the need for further research in the domain of loss aversion in general and benchmarking in particular. In our opinion, it is worthwhile to examine whether reference points differ for different types of investors (private investors versus professional investors).

The remainder of the paper is organized as follows. Section 2 provides an overview of the existing literature. In section 3 the mutual fund data set is described. Section 4 explains the methodology used to test both behavioural biases. In section 5 the results are discussed and concluding remarks are given in section 6.

2. Review of the existing literature

2.1. Overconfidence in institutional trades

If institutional investors possess managerial skills, the trades they execute should add value to the fund portfolio. More specifically, they should be able to correctly assess the future return on the securities they scrutinize. Ideally, the future return on the securities they buy will exceed the future return on the securities they sell. For trades to be profitable, the difference in return between buys and sells should at least exceed the associated trading costs. If this is not the case, the trade is detrimental to the fund’s performance and does not add any value to the portfolio. Odean (1999) shows that for a sample of individual investors, the profitability of the purchases does not exceed the profitability of the sales when trading costs are ignored. Apparently, these investors ‘overestimate the precision of their information’. In addition to this, the author finds that investors exhibit ‘overconfidence with respect to their ability to interpret information’. Due to this overconfidence, individuals
execute trades for which the difference in returns between the bought and sold securities cannot even cover the associated trading costs.

2.2. Disposition behaviour in institutional trades

Intuitively, institutional investors (unlike individual investors) are not expected to be vulnerable to behavioural biases such as the disposition effect. First of all, they are expected to act more rationally, due to better education and training. Dhar and Zhu (2006), for example, relate the disposition bias to investor characteristics and find that the propensity to sell winners and reluctance to sell losers is significantly smaller for individuals who are wealthier and work in professional occupations. Since institutional investors trade on behalf of their clients and have more trading experience and training, it is possible that the trading behaviour of these investors diverges from that of individual investors. Secondly, an increasing amount of mutual funds is managed in an environment where trades are dictated by a computerized search for anomalies and arbitrage opportunities. For these funds, human intervention is very limited. And even if it happens, the mutual fund manager probably has to fundamentally account for it. Thirdly, even if decisions are not based on automated trading rules, protective mechanisms such as stop loss can force the mutual fund manager to liquidate the incurred losses.

On the empirical side, the majority of the literature tests the disposition behaviour of individual investors (see amongst others Odean (1998)). Evidence on this topic in an institutional trading context remains limited (see Locke and Mann (2005); Grinblatt and Keloharju (2001)). Moreover, the scarce empirical evidence on this bias in an institutional context provides mixed results. Using a unique data set on the Finnish stock market that covers a variety of investor types, Grinblatt and Keloharju (2001) do not only find evidence supporting the disposition effect for individual but also for institutional investors. Shapira and Venezia (2001) examine the behaviour of Israeli investors and conclude that the disposition effect is present both at the individual and institutional level in Israel, although it is weaker for professional than for individual investors. Garvey and Murphy (2004) analyse the trades of US proprietary stock traders and find confirming evidence for the disposition effect. Likewise, Jin and Scherbina (2006) show that US mutual fund managers are susceptible to the disposition bias and illustrate that new fund managers are less reluctant to sell the losers from the inherited portfolio than continuing managers. Examining high-frequency transactions data, Locke and Mann (2005) report a reluctance to sell losers among futures traders on the Chicago Mercantile Exchange. Frazzini (2006) observes a disposition bias among US mutual funds and links this bias to post-announcement price drifts. More specifically, the author argues that upward price trends will trigger disposition investors to
realize the gain, thereby suppressing the stock price temporarily to move to the news-updated price level. Analogously, negative news prevents disposition investors with a capital loss to realize their losses, thereby impeding the price to fully adjust to the lower price level.

In contrast to the above-mentioned supporting evidence of the disposition bias, various papers have pointed out that institutional investors are less prone to the disposition effect. For instance, O’Connell and Teo (2004) examine the currency trading decisions of institutional investors, but find no verification of disposition effects. Instead, the authors find that institutional investors cut losses while riding gains. According to Feng and Seasholes (2005), sophistication and trading experience eliminate the reluctance to realize losses, but only partly remove the propensity to realize gains. Using quarterly portfolio holdings data of US equity mutual funds, Cici (2005) observes a ‘reverse’ disposition effect, i.e. unlike retail investors, mutual fund managers realize losses more eagerly than gains. Motivated by the contrasting evidence on the disposition bias in an institutional context, this paper attempts to shed light on this matter using a unique data set of mutual fund transactions.

3. The Data
The data set in this study was provided by a major global custodian and contains mutual fund transactions on a daily basis. It covers all daily transactions from mutual funds that have assigned the custodian to manage their transactions. The mutual funds have an international spread and trade securities from various markets. Each transaction is characterized by a mutual fund code, a trade date, an ISIN code, the price of the transaction, the number of securities traded, the transaction type, the country in which the trade was executed and the currency in which the trade was settled. The mutual fund code allows us to identify which fund is trading. Note that the mutual funds in our dataset are completely anonymous. The ISIN code allows us to infer which security is traded. By dividing the broker amount of a transaction by the number of securities traded, we can calculate the unit price of the security for a particular trade. Transaction type is mostly purchase or sale, i.e. ownership of the securities is exchanged for cash. However, in some cases, a free delivery or receipt of securities is recorded. An example of a free delivery (where a mutual fund hands back securities to the broker without receiving money) is a merger. Free delivery implies here that the mutual fund returns the ‘old’ shares. Afterwards, shares of the newly created company are received by means of a free receipt. Another typical example of a free receipt or free delivery is a stock split and a reverse stock split. Apart from the transaction types mentioned above, our data set comprises lending and borrowing transactions, which are
used when mutual funds short securities. However, the number of instances of these types of transactions is restricted.

To set minds, a typical transaction reads as: On the 15th of October 2006, mutual fund X buys, on the Finnish market, 10,000 shares of Nokia and pays 600,000 EUR to the broker. We then calculate the unit price of a Nokia stock as 60 EUR. We double-check the correctness of the calculated prices in our data set, by comparing the price of each trade to the Datastream unadjusted low and high price of the traded security on that trading day. Theoretically, the calculated price should fall in-between. However, when performing this check, we observe that 7.53% of the observations are not situated within these bounds. Allowing a deviation of 1%, the number of outliers falls back to 1.74%. In order to prevent these outliers from disturbing our analysis, censoring is applied. More specifically, if the share price in our data set is below the intraday low, we set it equal to the intraday low. Likewise, if it is above the intraday high, we set it equal to the intraday high.

Spanning the 2002-2007 period, the data base comprises 1,666,449 transactions executed by 1,741 different mutual funds. The dominant asset class is stocks, with 10,031 different stocks traded. The second most traded assets are bonds, with 7,976 bonds traded. Other, far less important (in terms of occurrences), asset classes include asset backed securities, units trusts, and closed end funds. The analysis in this paper is restricted to equity trades executed by equity funds. The data set comprises 1,064,440 equity transactions, which are executed by 1,041 mutual funds. Out of these funds, we identify 571 funds as equity funds. Since funds are anonymous, we have to define a criterion for funds to be classified as equity funds. We label a fund as ‘equity fund’ if more than 70% of its trades involves an equity transaction.

Some funds do not execute sale transactions. In our opinion, such funds do not represent typical equity funds, so we choose to remove these funds from our sample. This further reduces our sample to 519 funds. Our analysis in the next sections is centred on these 519 equity funds. In addition, we eliminate the lending and borrowing transactions from our sample and focus only on buy transactions, sell transactions, and receipt and delivery free transactions for the construction of the holdings and computation of purchase prices. Table 1 reports descriptive statistics for the 519 equity funds in our sample. Table 1 reveals that the majority of transactions are buy transactions. On average, a fund performs 785.94 buy transactions over the 2002-2007 period. We find 1214 unique trading days, suggesting that the average fund performs less than one buy transaction per day. Moreover, the average fund has 685.90 sell transactions over the sample period. Looking at the number of trades per day executed by our total sample of mutual funds, we find that on average 336 purchases occur per day, whereas 293.23 sell transactions take place.
We split the full sample into geographical subsamples, based on the different countries in which trades were executed: the Euro countries (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain), the Pacific region, rest of Europe, emerging markets, the UK, and the US. Table 2 displays summarizing trade statistics for each of these markets. We observe that most of the trading in our sample (41.89%) takes place on the UK market. The US market contains 23.48% of all trades, followed by the Euro countries and the pacific region. Note that relatively few trades are settled on the BRIC and other emerging markets (only 2.38%). For some analyses in this paper, we will consider only subsamples of funds that focus on respectively the Euro market, the UK market, and the US market. A fund is allocated to a particular subsample if more than 2/3 of its equity trades are performed on the respective geographical markets. Examining the currencies of the trades in our data set, we observe that 36.44% of the trades is in GBP, while 21.92% of the trades is in USD and 12.79% is in EUR. These figures broadly, but not exactly, correspond to the percentages from the country analysis. This implies that some trades that happen in a particular country are not settled in the currency of that country. Throughout the paper, local currencies are converted into euro.

4. Methodology

4.1. Overconfidence

Under rational expectations, institutional investors should purchase securities for which the (risk-adjusted) returns equal or exceed the (risk-adjusted) returns on the sold securities. In line with Odean’s (1999) analysis for individual investors, we calculate the average return on a buy (sell) portfolio, by examining the returns over a particular holding period following the purchase (sale) of a security to assess the future profitability of the fund trades. Unlike the author, we choose to consider shorter term holding periods than a quarter, one year, or two years, since we believe that fund managers are mainly interested in the short-term profitability of their trades. Indeed, since managers are often evaluated based on their recent performance, the profitability of their trades in the 21, 42, 63, or 84 days subsequent to the trade is more relevant. Jin and Kogan (2005) emphasize that since new fund inflows are related to the short run performance of mutual funds, fund managers tend to adopt short run horizons.

More specifically, let $N$ denote the total number of purchases (sales) and $T$ the holding period. We next identify a buy (sell) transaction $i$ in which a security $j_i$ is traded on trading date $t_i$. The holding period return corresponding to this security $j_i$ and transaction date $t_i$ is defined as $R_{j_i,t_i,T}$. This return is calculated by dividing the return index value (in EUR) for the traded security $j_i$ on day $T$ by the return index value on the transaction date
Chapter 6

t_i and subtracting one. Repeating this procedure for each buy (sell) transaction, the average return on the buy (sell) portfolio over a holding period of T trading days following the transaction is calculated as:

\[ R_{p,T} = \frac{\sum_{i=1}^{N} R_{i,t_i,t_i+T}}{N} \] (1)

For example, to compute the average return on the buy portfolio for a holding period of 84 days we adopt the following procedure. For each buy transaction i in our sample, the return index values (in EUR) for the traded security \( j_i \) on day \( t_i \) and 84 days later are retrieved from Datastream. Next, the simple holding period return is calculated by dividing the return index value on day \( t_i + 84 \) by the return index value on the transaction day \( t_i \) and subtracting one. After repeating this procedure for each buy transaction, the average is taken over all transactions to find the net average return on the buy portfolio.

If a particular stock is not traded any longer during the holding period, the return index in Datastream will stay at the same level. Therefore the returns for the days after which the stock stopped trading, are zero. Multiple purchases or sales of the same stock on the same day and by the same fund are only counted as one transaction, i.e. these transactions correspond to only one holding period return.

In order to check whether one portfolio outperforms another, we apply the bootstrapping procedure suggested in Odean (1999). This accounts for both return dependence and risk adjustment. Indeed, the significance test for this analysis should take into account that the returns on the traded stocks are not necessarily independent. Herding behaviour may induce several fund managers to trade the same stocks simultaneously, so the returns on these trades over the subsequent period are not independent. Furthermore, the replacement of one stock by another may be motivated by risk reducing incentives. So, it is possible that the return on the bought securities is lower than the return on the sold securities, but that meanwhile the risk-adjusted return for the bought securities exceeds the risk-adjusted return on the sold securities. We assess risk based on the Fama and French (1993) size and book-to-market correction.

More specifically, by repeatedly drawing replacement securities for the traded securities and computing average returns, an empirical distribution can be constructed of the average return difference between the bought and sold securities. Since our trades have an international spread, we first need to define the set of replacement securities. In order to keep the data manipulations feasible, we focus on the three biggest regions in our sample, being the Euro countries, UK and US. This captures the majority of trades for our sample. We
construct a replacement universe for each geographical market and for each trading day. In particular, for each trading day, we select all shares (both alive and dead) that were available on Datastream for the particular geographical market and download the corresponding return index value, market value and price-to-book ratio (all in EUR) for these stocks. Next, we construct size deciles and price-to-book quintiles for each trading day and each geographical subsample. The bootstrapping procedure then requires drawing a security (with replacement) from the set of replacement securities of the same size decile and same price-to-book quintile as the original security on that trading day. For example, for each stock traded by a European fund, we draw a replacement security from the set of all European stocks that belongs to same size decile and same price-to-book quintile as the original stock on the day that the trade was executed. Next, holding period returns over the 21, 42, 63 and 84 trading days subsequent to the trading date of the original stock are computed and returns are averaged over all purchased (sold) securities. Next, the average return difference between the purchased and sold securities is calculated. Repeating this procedure 1000 times, we can construct an empirical distribution of risk-adjusted return differences between the buy and the sell portfolio. This empirical distribution can then be used to position the actual observed difference and to ascertain whether indeed outperformance of one portfolio vis-à-vis another can be concluded.

A second calculation method for examining the profitability of purchases and sales is provided by the calendar time method, originally proposed by Jaffe (1974) and Mandelker (1974). We construct calendar-time portfolios consisting of all purchase (sale) events during a portfolio formation period of one, two, three, or four months. More specifically, for each purchase (sale) of a security during the formation period, we assign this security to the calendar-time portfolio. If several funds buy (sell) the same security, the security accounts for more than one observation. Next, an equally-weighted portfolio return is calculated for the calendar month subsequent to the formation period. Rolling forward the formation period by one month, a time-series of calendar-time portfolio returns for month \(t+1\) is obtained. According to Lyon et al. (1999), this procedure eliminates the problem of cross-sectional dependence among sample firms, because the returns on sample firms are aggregated into a single portfolio. Since the number of securities in the calendar time portfolio varies from one month to the next, inference is done based on Newey-West \(t\)-statistics, in order to account for heteroscedasticity.

### 4.2. Disposition effect

To facilitate comparison with previous work on the disposition effect and to ensure that potential divergences in results cannot be attributed to model differences, we implement the methodology of Odean (1998). As in Grinblatt and Keloharju (2004) and Odean (1998), we
define capital gains and losses only for those sell transactions for which holds that the number of stocks sold in that transaction does not exceed the number of shares accumulated (by purchase or receipt free) before the sell transaction. In addition, split trades are aggregated, i.e. trades of the same fund in the same stock that are spread over the day, are combined to avoid double-counting of realized gains or losses.

Prices adjusted for stock splits and dividends are obtained from Datastream. On each day and for each mutual fund portfolio, we compute both realized and paper gains and losses with respect to a particular reference point. The former relates to the gain or loss resulting from the sale of stocks, whereas the latter indicates the hypothetical gain or loss that could have been realized if the stock had been sold instead of held. More specifically, on each day that a mutual fund performs a sell transaction, not only the realized gain or loss from this sell transaction is computed, but also the paper gain or loss resulting from the sales of the remaining stocks in the portfolio is computed (see Odean (1998)). Realized gains and losses for sold stocks are computed by comparing the calculated sell price to the average purchase price of the stock. For the remaining stocks in the mutual fund portfolio, paper results are calculated by comparing the Datastream high and low price on that day to the average purchase price. If both prices exceed the average purchase price, the paper result is labelled as a paper gain, whereas if both prices are below the average purchase price, a paper loss is counted. Following among others Odean (1998) and Lim (2006), gains and losses are defined relative to the volume-weighted average purchase price (i.e. the reference point) from the buy transactions preceding the date of the sale transaction. Average purchase prices are computed using adjusted purchase prices to account for corporate actions (e.g. stock splits). Adjusted prices are calculated using the Datastream adjustment factors on the day of the sell transaction and the days of the buy transactions.

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3 Alternatively, paper results can be calculated by comparing the Datastream closing price to the average purchase price (see e.g. Lim (2006)). The results for both procedures are qualitatively the same.

4 Suppose a 1:5 stock split occurred and instead of holding 100 stocks each valued $20, a fund suddenly holds 500 stocks, now worth $4 per share. Using Datastream adjustment factors we correctly revalue the purchase price. After the stock split, a sale of these stocks at $10 per share would result in a realized gain of $6 per share ($10-$4). However, if we did not take the adjustment of the purchase price into account, a realized loss of -$10 ($10-$20) would be incorrectly identified rather than a $6 gain. Mathematically, the purchase price is brought to the same level as the intraday sell price by multiplying this price by the ratio of the adjustment factor on the purchase day to the adjustment factor on the sell day.
As stated earlier, not all of the trades in our data set are pure buy transactions. More specifically, ‘receipt free’ transactions involve no cash exchange. The absence of a purchase price for these stocks implies that we cannot compute a realized gain or loss. We address this issue by considering the Datastream unadjusted closing price of the particular stock prevailing on the day of the free receipt as a proxy for the purchase price.

To test the hypothesis that mutual fund managers are reluctant to realize their losses, we calculate the ratio of realized gains and realized losses (Odean (1998)):

\[
PGR = \frac{n_{\text{realized gains}}}{n_{\text{realized gains}} + n_{\text{paper gains}}}
\]

\[
PLR = \frac{n_{\text{realized losses}}}{n_{\text{realized losses}} + n_{\text{paper losses}}},
\]

where \(PGR\) and \(PLR\) denote the proportion of gains realized and the proportion of losses realized. In this computation, the number of paper gains \(n_{\text{paper gains}}\) or paper losses \(n_{\text{paper losses}}\) is aggregated cross-sectionally and over time. Likewise, \(n_{\text{realized gains}}\) \(n_{\text{realized losses}}\) denotes the total number of gains (losses) realized by all the funds in our sample over the entire sample period. Under the alternative hypothesis, a negative difference is observed between the \(PLR\) and \(PGR\) ratio \((H_1: PLR - PGR < 0)\). A \(t\)-test on difference in proportions is used to evaluate the statistical significance of the results. More specifically, the \(t\)-statistic is calculated as follows:

\[
t - \text{statistic} = \frac{(PLR - PGR) - 0}{\sqrt{\frac{PGR(1 - PGR)}{n_{\text{realized gains}} + n_{\text{paper gains}}} + \frac{PLR(1 - PLR)}{n_{\text{realized losses}} + n_{\text{paper losses}}}}}
\]

We also check for the possibility that investors frequently realize small gains, and occasionally take big losses. In our traditional disposition measure, this would lead to a disposition effect (much more gains than losses are counted), but in terms of realized nominal gain and loss, the numbers could be comparable. We account for this possibility by summing the money amount earned or lost. Therefore, for the realized results, each time a gain or loss is recorded, we multiply the size of the gain per share with the number of shares sold in the particular trade. For the paper results, we multiply with the number of shares that are still in the portfolio of the mutual fund. This procedure is similar to Odean (1998), Cici (2005) and Frazzini (2006).
Implicit in our entire setup (and in the prospect theory proposed by Kahneman and Tversky (1979)) is the assumption that the mutual fund managers assess their gains and losses on a transactional, one by one base. Alternatively, it could be that mutual fund managers rather focus on the return on their entire investment portfolio. In fact, this discussion boils down to the behavioural bias of narrow framing, as discussed by Thaler (1985).

Under the prospect-theoretic explanation of the disposition effect, investors’ tendency to frame decisions at the individual stock level instead of the portfolio level influences the decisions to sell portfolio winners and losers. Investors who are sensitive to the performance of individual stocks instead of the overall portfolio performance would exhibit a greater propensity to sell the winners, relative to the losers in their portfolios. In contrast, investors who adopt a broader decision frame and evaluate the overall performance of their portfolios are less likely to exhibit this asymmetry.

Empirically, Kumar and Lim (2005) proxy the framing mode by the clustering of trades. Investors with highly clustered trades are assumed to be broad framers. They document that individual investors performing more clustered trades, show a lower disposition effect. O’Connel and Teo (2004) examine this issue for a sample of institutional currency traders and reach the conclusion that indeed these traders are perceived as narrow framers.

5. Results

5.1. Do institutional investors perform profitable trades?

5.1.1. Holding period returns

Table 3 reports the returns over various horizons preceding and following institutional transactions. The first row shows the profitability of the buy portfolio, while the second row shows the profitability of the sell portfolio. The third row then shows the difference between both. The fourth row shows the percentage of bootstrapped return differences that are smaller than the actual observed difference. Values below the traditional significance thresholds of 1% or 5% would indicate that, on a risk-adjusted base, purchased securities perform worse than sold securities. Values above 95% or 99% would indicate that, on a risk-adjusted base, purchased securities perform better than sold securities. Values in between would indicate that there is actually no difference between the performance of both portfolios.

Figures 1 to 4 provide a graphical representation of the average holding period returns in the trading days prior and subsequent to a purchase or sale for different geographical subsamples. Results are reported in Table 3, where panels A to D represent the results for the total sample, and the European, UK, and US subsample respectively. For the European subsample (panel B in Table 3), raw return differences are positive for one and two
months before the trades, and negative for three and four months before the trade. No consistent picture emerges from that analysis. However, taking into account the risk adjustment via the bootstrapping procedure, European funds seem to replace stocks with an inferior performance by stocks with a superior performance. For the three closest months, it indeed holds that on a risk-adjusted base, the purchases perform better than the sales (see also Figure 2). Only in the most distant period (84 days), there is no difference between the profitability of purchases and sales, after having corrected for risk. Whether this strategy was very sensible, is questionable. When examining after transaction returns, sales consistently have a higher absolute return than purchases. However, accounting for risk, no portfolio dominates another in terms of profitability.

Results for the UK subsample are shown in panel C. Apparently, UK funds sell stocks that enjoyed a solid performance over the last 42, 63, and 84 days. They are replacing these stocks with stocks for which the performance over the mentioned periods was inferior, as illustrated in Figure 3. This holds for both the raw and risk-adjusted return differences. This strategy is profitable, as demonstrated by the post transaction returns. For each month, the purchases seem to outperform the sales. Moreover, the funds predict quite well at what point to switch the stocks in their portfolio. Indeed, 21 days before the trade, the purchases become more profitable than the sales (see also Figure 3). This pattern holds until (at least) four months after the trade.

US funds seem to pursue the reverse strategy. It is clear from Figure 4 that they are selling the losers in their portfolio, and replace them by winners. This is evidenced by the positive return difference between the purchases and sales before the transaction occurs. Outperformance is quite substantial and amounts to 1.5% and more. This strategy seems to be rewarding only if the purchases are held for a short period of time. The outperformance of purchases versus sales only holds for 21 days after the sale. For the two following months, purchases and sales perform equally well (on a risk-adjusted base). After four months, the purchases underperform with respect to the sales, as illustrated in Figure 4.

Results for the entire sample then of course should be a weighted average of the results for the individual blocks, taking into account both the number of observations and the magnitude of the return differences. For example, focusing on the first column (21 days before the transaction), the return difference between the buy and sell portfolio is positive (0.20%). However, the largest block of observations (UK) had a negative performance (-0.19%). This can be explained by noting that the US has a positive difference with a much higher absolute value (1.55%). This return difference more than compensates the difference in number of observations. This makes deducing a particular strategy for the entire group of funds quite difficult, since it is a mixture of the strategies of the three building blocks. Purchases outperform sales for the two nearest periods before transaction (on a risk-adjusted base). If the holding period is 63 or 84 days, the raw return difference is each time negative,
but fails to hold when risk adjustments are taken into account. In the post-trade period, return differences are always positive, and each time indicate risk-adjusted outperformance of the buy portfolio versus the sell portfolio. Here, results are driven by the good performance of the UK funds. They counterbalance the inferior performance of both European and US funds.

In conclusion, our results reveal that UK funds are making profitable decisions concerning stock trades. This cannot be confirmed for European and US funds. Overall for these funds, we observe no risk-adjusted difference between the buy and the sell portfolio.

Odean (1998) quite heavily focuses on the role of transaction costs. He estimates the size of the transactions costs for a round-trip trade to be 5.9%. However, compared to our research, two important differences should be noted. Firstly (and most importantly), we are dealing with institutional investors. It is common knowledge that the transaction costs they face are much smaller than transaction costs for individual investors. Furthermore, due to technological developments, transaction costs have decreased severely. Therefore, we do not take these transaction costs into account, since their size is negligible.

Another remark is to be made on the magnitude of the returns made on the different portfolios. For both the European and the UK sample, it holds that the returns for the short term are at a level of around 8%. This is substantially lower for US funds. There, we observe returns of around 4%. Also for other holding periods, it shows that the absolute levels of return diverge between on the one hand the European and UK funds and on the other hand, the US funds. This difference can originate from two sources. First of all, the return evolution of the markets (as a whole) can differ, and secondly, exchange rate evolutions can play an important role. Figure 5 further documents these possible causes. We plot for each region, the evolution of the return index of an important indicator of the market evolution, over the time period of our sample. For Europe, we take the MSCI Europe index excluding UK. For the UK, we focus on the FTSE 100 index. For the US, we consider the S&P500 index. Panel A then shows the evolution of the return index (standardized at 100 at the start of our sample) in local currencies. Graphical examination shows that over this period, the S&P500 initially outperformed the other two benchmarks. After some time however, the other two benchmarks catch up and eventually the MSCI Europe excluding UK ends at a higher level than the other benchmarks. Panel B then takes the evolution of the exchange rate into account, since return indices are expressed in EUR in this panel. Now, we observe a clear pattern: the worst performance is for the S&P500. Second comes the FTSE, and again the best performance is achieved by the MSCI Europe excluding UK. These figures provide further insight into the low values for the US buy and sell portfolios.

Table 4 reports the results when the fund sample is divided into subgroups according to their trading frequency. More specifically, group 1 comprises the 30% funds that trade the
least, while group 3 consists of the 30% funds with the highest trading frequency. Group 2 comprises the remaining 40% funds. In this analysis, we restrict the fund sample to funds with a minimum of 60 trades over the sample period (i.e. funds that have one trade per month on average). Intuitively, one would expect frequent traders to be more sophisticated and thus be able to correctly assess the future return on the examined securities. Nevertheless, the literature on ‘churning’ (see e.g. Brown (1996)), casts doubt on the profitability of funds that trade excessively in an effort to chase commissions. The results in Table 4 indicate that funds with a high trading frequency (group 3, panel C in Table 4) purchase securities that outperformed the sold securities in the 21 or 42 days prior to the trade. Moreover, these purchases follow the same positive trend in the months following the trade, in line with the momentum theory of Jegadeesh and Titman (1993). Conversely, funds with a low trading frequency (group 1, panel A in Table 4) act more ‘contrarian’, i.e. they replace the sold stocks by stocks with inferior performance in the period prior to the trade. This strategy seems to be rewarding over a holding period of 42, 63, or 84 days after the trade, since the purchases perform better than the sales (on a risk-adjusted base) in the post-trade period. In sum, we find that both groups execute profitable purchases, even though the purchases of both groups exhibit contrasting return patterns in the period prior to the trade. Furthermore, although we do not find evidence of a particular strategy for the middle group, it seems that they are also executing profitable trades. Panel B indeed reveals that purchases consistently outperformed sales in the post-trade period.

In Table 5 we distinguish trades in December from trades during the rest of the year to account for window dressing practices. Indeed, in December fund managers may be inclined to buy stocks which have recently increased in value to brighten up their portfolios. However, the results in panel A contradict this hypothesis. Focusing on the period prior to the December trade, we find that the securities purchased in December did not outperform the sold securities. In contrast, as panel B reveals, we cannot reject that the securities purchased during the rest of the year have experienced better returns than the sold securities in the 21 or 42 days prior to the trade. Moreover, the results in panel A indicate that the securities purchased in December do not outperform the sold securities in the 42, 63, or 84 days subsequent to the trade. Only over a short horizon of 21 days, the stocks purchased in December outperform the stocks sold during the month. Conversely, for the non-December trades, purchased securities earn better returns than the sold securities in the post-trade period.
5.1.2. Calendar-time portfolios

Table 6 displays the results for the calendar-time portfolios for formation periods of one, two, three and four months. For the subsamples formed by the European, UK, and US funds, the directions of the return differences between the buy and sell portfolios are largely consistent with the results revealed in Table 3. However, the results fail to pass the significance test, as revealed by the t-statistics. For European funds, it holds that the return difference is negative for each formation period. This is consistent with the holding period return analysis. However, also consistent is the fact that we cannot reject that there is no difference between the returns on the two portfolios. In line with Table 3, UK funds show the best performance, at least simply by looking at the sign of the return differences. The performance of the buy portfolio is each time better than the performance of the sell portfolio. Nevertheless, in contrast to the significance results in Table 3, the return differences in panel C are not significant. Likewise, the results in panel D for the US funds corroborate the holding period results in Table 3. When considering a formation period of one month, the buy portfolio performs better than the sell portfolio. This does not hold true for the next formation periods. Each time, a negative difference is recorded. Again, differences are not significant. Considering the entire sample of funds, the results in Table 4 reveal a different picture than the one that emerged from the holding period analysis. The return difference is each time negative, but insignificant, which contrasts with the holding period analysis (where purchased securities significantly outperformed sold securities).

Next, the relative performance of the buy versus sell portfolio is examined using a CAPM regression of the monthly return difference between the buy and sell portfolio on the market risk premia:

$$R_{bt} - R_{st} = \alpha + \beta_{EUR} (R_{mt_{EUR}} - R_{ft_{EUR}}) + \beta_{UK} (R_{mt_{UK}} - R_{ft_{EUR}}) + \beta_{US} (R_{mt_{US}} - R_{ft_{EUR}}) + \varepsilon_{t} \quad (5)$$

The market index for European, UK, and US funds is represented by the MSCI Europe excluding the UK, the FTSE, and S&P500 index respectively, all expressed in EUR. Market risk premia are obtained by subtracting the Euribor 1 month from the market benchmarks. Table 7 shows that Jensen’s alpha is not significantly different from zero for this regression, affirming our calendar-time portfolio results in Table 6 that, for the entire mutual fund sample, the calendar-time buy portfolio is not significantly different from the sell portfolio.
5.2. Are professional traders reluctant to realize their losses?

Table 8 provides an overview of how much gains and losses are realized by the equity funds in our data set. In general, more gains than losses are realized, but the funds also hold relatively more paper gains than paper losses. This should not come as a surprise, since in general the 2002-2007 period was very beneficial for stock investments. The results for the PLR and PGR ratios show that funds sell more losses than gains relative to their opportunities to do so. The PLR statistically significantly exceeds the PGR ratio (revealed by the t-statistic = 30.98). This finding suggests that we cannot reject that the spread between the PLR and PGR ratio is greater than or equal to zero. In other words, the professional investors in our data set do not exhibit the disposition effect, but instead cut losses. Our results corroborate the results of Cici (2005), Ben-David and Doukas (2006), and Xu (2007) for US institutional investors, but diverge from the results of Odean (1998) for retail investors. Barber, Lee, Liu and Odean (2007) examine the trading activity on the Taiwan Stock Exchange and find that individual investors (representing 90% of all trading volume) exhibit a disposition bias, while foreign investors and domestic mutual funds (each representing less than 5% of all trading volume in their data set) do not. Similar to our results, the mutual funds trading on the Taiwan Stock Exchange display a modest tendency to realize losses more eagerly than gains. In their analysis, the disposition spread amounts to 0.26%, which is slightly below the spread of 0.39% that we report. Overall, these findings would suggest that more sophisticated investors are less prone to behavioural biases, perhaps because their trades are more motivated by incentives. In order to account for size differences between the realized gains and losses, we also report the PGR and PLR measure based on money amounts. Both measures are at a slightly lower level, but the PLR is still exceeding the PGR, i.e. the results remain qualitatively the same. This finding is consistent with the results found in Odean (1998), Cici (2005) and Frazzini (2006).

Table 9 reports the average returns corresponding to the paper results and realized results in Table 8. In contrast to Odean (1998), we do not observe that the returns on realized losses are substantially better than those on paper losses. Again, this substantiates the claim that institutional investors are less reluctant to realize their losers than individual investors.

Next, we split our sample into three geographical subsamples (see Table 10): the Euro countries, the UK, and the US. A fund is classified into each of these groups if more than 2/3 of its trades occurs on the specific market. In line with our previous results, we do not observe a disposition bias for European and US mutual funds. However, a low disposition

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5 Results are reported only for the equity funds in our sample. Replicating the analysis for the entire sample of funds yields qualitatively the same results.
bias shows up for the funds with a majority of trades on the UK market. da Silva Rosa, To and Walter (2005) also find evidence of a disposition effect for UK managed funds. The reason for finding a disposition effect for the UK oriented funds is not so clear. Ben David and Doukas (2006) document that disposition tends to be higher when information ambiguity is higher. Information ambiguity is measured by four concepts: idiosyncratic risk, accounting-based risk, analyst dispersion and whether or not a dividend is paid. Even without having checked this overall level of information ambiguity on the different markets, we would be surprised to see that information ambiguity differs substantially between the regions we discuss. After all, all regions have a high level of information disclosure. Therefore, we do not assume this reason to be valid as an explanation for our results.

5.2.1. Choice of the reference point
Although the value function in Kahneman and Tversky’s (1979) prospect theory clearly has a typical S-shape, less clarity exists on the location of the reference point. Indeed, in the identification of the disposition effect, the role of the reference point should not be understated, as noted among others by Heath, Huddart and Lang (1999). While the larger part of the extant literature on the disposition effect typically focuses on the average purchase price as reference point, few papers have tested different locations of the reference point. Odean (1998) still finds supporting evidence for the disposition effect when the reference point in his analysis shifts from the average purchase price to the highest, the first, or the most recent purchase price. However, Köszegi and Rabin (2006) argue that expectations represent a better reference point than historical purchase prices.

Given that the financial press typically reports the maximum price of a stock over the past year, investors may be inclined to use this price as a benchmark to evaluate the profitability of their trades. According to experimental evidence of Gneezy (2005), people use the historical peak as a reference level to evaluate gains and losses. Arkes et al. (2008) explain this observation by proving that reference price adaptation is asymmetric in the sense that investors tend to move reference points upward after gains more than downward after losses. Therefore, after a number of periods, it will eventually approach the past price peak. An additional argument for using the price peaks is that in an institutional setting, average purchase prices are perhaps not that appropriate, by the sheer size of the number of purchases that occurs. Perhaps a fund manager will not start calculating the average prices, but will just focus on one, easy to determine, price. In line with Ben-David and Doukas (2006), we test whether our results are influenced by the choice of the average purchase price as reference point and consider the historical peak price as an alternative. More specifically,
we set the reference point equal to the maximum price\textsuperscript{6} defined over the recent three- or six-month period, the past year or two years. Following Huddart, Lang and Yetman (2006), each evaluation period ends 20 trading days before the evaluation day, to ensure that enough observations can exceed the prior maximum. Table 11 displays the results for this sensitivity analysis. To facilitate comparison, the first column in Table 11 repeats the results with the average purchase price as benchmark. Investors are neither prone to the disposition effect when gains and losses are coded relative to the highest purchase price, the first purchase price, nor the most recent purchase price.

Surprisingly however, disposition behaviour shows up when prior maxima are used as reference point. Regardless of the period over which this prior maximum is computed, a significantly negative difference between the PLR and PGR ratio is observed when prior maxima serve as benchmark. This finding corroborates the results of Ben-David and Doukas (2006) for US investors, who find evidence of a disposition effect once the historical peak price serves as the reference point. Moreover, it underlines the importance of the reference point in coding gains and losses.

Assuming that institutional investors assess gains and losses in a different way than individual investors, we test a few other reference points. For example, since the compensation of professional traders is linked to their past performance and the number of assets under management, we suggest taking the last trading day of December of the year before transaction as reference point, from which they start again with a clean slate. So for example, the sell price of a stock in a particular transaction in July 2006 is compared to the price of that stock at the end of December 2005. The last column in Table 11 points out that institutional investors are not prone to the disposition bias when the last trading day of December is used as benchmark to define gains and losses.

The performance evaluation of a great deal of mutual funds is related to the performance of a benchmark index. For these funds, the performance of the benchmark index can be used as reference point to code gains and losses (see Table 12). After splitting our sample into three geographical subsamples (the Euro countries, the UK, and the US), we consider the S&P500 as a benchmark for US oriented funds, and the FTSE and MSCI Europe

\textsuperscript{6} In this procedure, the maximum is taken over a range of Datastream \textit{adjusted} closing prices (i.e. prices are calibrated to the current stock price level). However, given that the intraday sell price on a particular trading day is \textit{non-adjusted}, we need to bring both prices to the same level by adjusting the maximum closing price. Therefore, we divide this maximum price by the Datastream adjustment factor prevailing on the sell day for the ISIN traded, in order to bring back the maximum price to a historical value, i.e. the price level prevailing on the sell day.
excluding UK as benchmarks for the UK and the Euro countries respectively. Excess returns are calculated for each sell transaction to define whether the transaction resulted in a gain or loss. To this end, we first calculate the return on the realized sell transaction (using the volume-weighted average purchase price as reference point) and next subtract the return on the benchmark from this return. The return on the benchmark index is computed using the index value on the day of the sell transaction and an average purchase price of this index. This index purchase price is determined using the weights used in the calculation of the average purchase price of the sell transaction and combining these weights with the index values prevailing on the day of the buy transactions preceding the date of the sell transaction. In line with the results reported above, we find no evidence of a disposition bias for European and US mutual funds when we use a geographical benchmark as reference point. Again, our results point at a low disposition effect for UK oriented funds.

Of course, because mutual funds are anonymous in our sample, we can only use a general regional benchmark. An alternative would be to focus on the trades (and holdings) of the fund, to infer whether they are trading particular stocks (e.g. stocks belonging to a particular industry, or small stocks). Then we would be able to choose a benchmark that is of more importance, and more appropriate to asses the performance of the fund. However, this is beyond the scope of our paper.

5.2.2. Influence of trading frequency

We also check whether the results depend on the trading frequency of the mutual funds in the sample. To this end, we split the sample into three groups of traders: infrequent traders, moderate traders, and frequent traders. Each group contains approximately 33% of all stock sells. Results are reported for the analysis where the reference price is the average purchase price (see Table 13). The PLR and PGR ratios in Table 15 suggest that none of the three groups exhibits a disposition bias, but instead a small tendency to realize losses rather than gains is observed. The same conclusion holds when the highest purchase price, the first purchase price, or the most recent purchase price serves as reference point. However, we do find evidence of a disposition bias for each group once prior maxima over the past one or two years are used as reference point. Using the historical peak level over the past three or six months shows a disposition bias for the first two groups only. This analysis shows that our findings concerning the disposition bias and more importantly, the importance of the reference point to judge the disposition behaviour, are not dependent on a particular subset of mutual funds.
6. Concluding remarks

In this paper two behavioural biases are examined in an institutional trading context, namely overconfidence and disposition behaviour. First, we test whether mutual fund managers execute trades that are profitable. We check whether the ex-post risk-adjusted profitability of their purchases is different from the risk-adjusted profitability of their sales. Our results reveal that, when the entire mutual fund sample is considered, the fund trades do not erode performance, since the returns on the purchased securities are not worse than the returns on the sold securities. Nevertheless, some regional differences are observed when the fund sample is divided into geographical subsamples. Apparently, UK funds are making profitable decisions concerning stock trades, since the purchased securities perform significantly better than the sold securities in the post-trade period. This cannot be confirmed for European and US funds. For the latter funds, we observe no risk-adjusted difference between the buy and the sell portfolio.

In the second part of this paper, we focus on the selling activity of the mutual funds in our sample. In contrast to earlier findings for retail investors, we document a propensity to cut losses rather than a reluctance to hold on to losing stocks for the fund managers in our sample, when the average purchase price is used as reference point to assess gains and losses. Nevertheless, when the fund sample is split up into geographical subsamples, disposition behaviour is detected for UK oriented funds. Furthermore, we show that the results from the disposition analysis are sensitive to the choice of the reference point. Generally, the disposition hypothesis is rejected when average, first, last, or highest purchase prices serve as reference point. In contrast, disposition behaviour is observed when historical peaks act as reference point.

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References


### Table 1 – Descriptive statistics of the database

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td># equity funds (i.e. &gt;70% trades involve equity transactions)</td>
<td>519</td>
</tr>
<tr>
<td># buy transactions</td>
<td>407,902</td>
</tr>
<tr>
<td># sell transactions</td>
<td>355,981</td>
</tr>
<tr>
<td># receipt free transactions</td>
<td>54,849</td>
</tr>
<tr>
<td># delivery free transactions</td>
<td>51,628</td>
</tr>
<tr>
<td># of unique transaction days</td>
<td>1,214</td>
</tr>
<tr>
<td># of unique equities traded</td>
<td>9,214</td>
</tr>
<tr>
<td>Average # of buys per fund</td>
<td>785.94</td>
</tr>
<tr>
<td>Average # of sells per fund</td>
<td>685.90</td>
</tr>
<tr>
<td>Average # of buys per trading day</td>
<td>336.00</td>
</tr>
<tr>
<td>Average # of sells per trading day</td>
<td>293.23</td>
</tr>
</tbody>
</table>

### Table 2 – International spread

<table>
<thead>
<tr>
<th>Geographical region</th>
<th>Countries included</th>
<th>Number of trades on this market</th>
<th>Percentage of trades on this market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro countries</td>
<td>(Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain)</td>
<td>130,134</td>
<td>14.95%</td>
</tr>
<tr>
<td>UK</td>
<td>UK</td>
<td>364,630</td>
<td>41.89%</td>
</tr>
<tr>
<td>US</td>
<td>US</td>
<td>204,330</td>
<td>23.48%</td>
</tr>
<tr>
<td>Pacific region</td>
<td>(Australia, Canada, Hong Kong, Japan, New Zealand, Singapore)</td>
<td>111,271</td>
<td>12.78%</td>
</tr>
<tr>
<td>Rest of Europe</td>
<td>(Denmark, Euromarket, Norway, Sweden, Switzerland)</td>
<td>39,257</td>
<td>4.51%</td>
</tr>
<tr>
<td>Emerging markets</td>
<td>(Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Israel, Jordan, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Republic of Korea, Russia, South Africa, Taiwan, Thailand, Turkey)</td>
<td>20,738</td>
<td>2.38%</td>
</tr>
</tbody>
</table>
Table 3- Average holding period returns in the trading days prior and subsequent to purchases and sales: short term

Table 3 reports average returns over the 21, 42, 63 and 84 trading days prior and subsequent to a purchase or sale. Returns are obtained from Datastream. Local currencies are converted into euro. P values are reported corresponding to the percentage of bootstrapped return differences that are smaller than the actual observed return difference in the data set.

Panel A: All European, UK and US transactions

<table>
<thead>
<tr>
<th>Holding period</th>
<th>HPR return over t trading days before transaction</th>
<th>HPR return over t trading days after transaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
<td>42</td>
</tr>
<tr>
<td>Purchases</td>
<td>1.92%</td>
<td>3.91%</td>
</tr>
<tr>
<td>Sales</td>
<td>1.72%</td>
<td>3.99%</td>
</tr>
<tr>
<td>Difference</td>
<td>0.20%</td>
<td>-0.08%</td>
</tr>
<tr>
<td>P value</td>
<td>1.000</td>
<td>0.9490</td>
</tr>
</tbody>
</table>

Panel B: European subsample

<table>
<thead>
<tr>
<th>Holding period</th>
<th>HPR return over t trading days before transaction</th>
<th>HPR return over t trading days after transaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
<td>42</td>
</tr>
<tr>
<td>Purchases</td>
<td>1.42%</td>
<td>3.38%</td>
</tr>
<tr>
<td>Sales</td>
<td>1.42%</td>
<td>3.19%</td>
</tr>
<tr>
<td>Difference</td>
<td>0.00%</td>
<td>0.19%</td>
</tr>
<tr>
<td>P value</td>
<td>0.9910</td>
<td>0.9980</td>
</tr>
</tbody>
</table>
### Table 3- Continued

#### Panel C: UK subsample

<table>
<thead>
<tr>
<th>Holding period</th>
<th>HPR return over t trading days before transaction</th>
<th>HPR return over t trading days after transaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
<td>42</td>
</tr>
<tr>
<td>Purchases</td>
<td>2.37%</td>
<td>4.56%</td>
</tr>
<tr>
<td>Sales</td>
<td>2.56%</td>
<td>5.38%</td>
</tr>
<tr>
<td>Difference</td>
<td>-0.19%</td>
<td>-0.82%</td>
</tr>
<tr>
<td>P value</td>
<td>0.9680</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

#### Panel D: US subsample

<table>
<thead>
<tr>
<th>Holding period</th>
<th>HPR return over t trading days before transaction</th>
<th>HPR return over t trading days after transaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
<td>42</td>
</tr>
<tr>
<td>Purchases</td>
<td>1.20%</td>
<td>2.75%</td>
</tr>
<tr>
<td>Sales</td>
<td>-0.31%</td>
<td>0.88%</td>
</tr>
<tr>
<td>Difference</td>
<td>1.51%</td>
<td>1.88%</td>
</tr>
<tr>
<td>P value</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Table 4- Average holding period returns in the trading days prior and subsequent to purchases and sales: fund sample divided into deciles based on number of trades

Table 4 reports average returns over the 21, 42, 63 and 84 trading days prior and subsequent to a purchase or sale by the funds in our sample. Funds are divided into subgroups according to the number of their trades: group 1 comprises the 30% funds that trade the least, group 3 consists of the 30% funds that trade the most, and group 2 comprises the remaining 40%. In this analysis, only trades from Europe, UK, and US are considered. The fund sample is restricted to funds with a minimum of 60 trades over the sample period (i.e. funds that have one trade per month on average). Local currencies are converted into euro. P values are reported corresponding to the percentage of bootstrapped return differences that are smaller than the actual observed return difference in the data set.

Panel A: Decilegroup 1 (0-30%)

<table>
<thead>
<tr>
<th>Holding period</th>
<th>21</th>
<th>42</th>
<th>63</th>
<th>84</th>
<th>21</th>
<th>42</th>
<th>63</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases</td>
<td>2.07%</td>
<td>3.82%</td>
<td>4.99%</td>
<td>6.09%</td>
<td>1.85%</td>
<td>4.75%</td>
<td>6.23%</td>
<td>7.65%</td>
</tr>
<tr>
<td>Sales</td>
<td>2.95%</td>
<td>5.70%</td>
<td>7.00%</td>
<td>8.18%</td>
<td>2.42%</td>
<td>4.39%</td>
<td>5.43%</td>
<td>6.24%</td>
</tr>
<tr>
<td>Difference</td>
<td>-0.88%</td>
<td>-1.88%</td>
<td>-2.01%</td>
<td>-2.09%</td>
<td>-0.58%</td>
<td>0.36%</td>
<td>0.80%</td>
<td>1.41%</td>
</tr>
<tr>
<td>P value</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0390</td>
<td>0.0340</td>
<td>0.9130</td>
<td>0.9970</td>
<td>0.9670</td>
<td>0.9870</td>
</tr>
</tbody>
</table>

Panel B: Decilegroup 2 (30-70%)

<table>
<thead>
<tr>
<th>Holding period</th>
<th>21</th>
<th>42</th>
<th>63</th>
<th>84</th>
<th>21</th>
<th>42</th>
<th>63</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases</td>
<td>1.94%</td>
<td>3.97%</td>
<td>5.74%</td>
<td>7.89%</td>
<td>2.12%</td>
<td>4.35%</td>
<td>6.20%</td>
<td>8.48%</td>
</tr>
<tr>
<td>Sales</td>
<td>1.90%</td>
<td>4.44%</td>
<td>6.60%</td>
<td>8.62%</td>
<td>1.74%</td>
<td>3.95%</td>
<td>5.74%</td>
<td>7.84%</td>
</tr>
<tr>
<td>Difference</td>
<td>0.04%</td>
<td>-0.47%</td>
<td>-0.86%</td>
<td>-0.73%</td>
<td>0.39%</td>
<td>0.40%</td>
<td>0.46%</td>
<td>0.64%</td>
</tr>
<tr>
<td>P value</td>
<td>0.2880</td>
<td>0.0000</td>
<td>0.1250</td>
<td>0.0790</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
### Table 4- Continued

Panel C: Decilegroup 3 (70-100%)

<table>
<thead>
<tr>
<th>Holding period</th>
<th>HPR return over t trading days before transaction</th>
<th>HPR return over t trading days after transaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
<td>42</td>
</tr>
<tr>
<td>Purchases</td>
<td>1.91%</td>
<td>3.89%</td>
</tr>
<tr>
<td>Sales</td>
<td>1.61%</td>
<td>3.79%</td>
</tr>
<tr>
<td>Difference</td>
<td>0.30%</td>
<td>0.10%</td>
</tr>
<tr>
<td>P value</td>
<td>1.0000</td>
<td>0.9900</td>
</tr>
</tbody>
</table>
Table 5- Average holding period returns in the trading days prior and subsequent to purchases and sales: December versus non-December trades

Panel A in Table 5 reports average returns over the subsequent 21, 42, 63 and 84 trading days following a purchase or sale in December. Panel B displays the returns for trades executed during the rest of the year. Returns are obtained from Datastream. In this analysis, only trades from Europe, UK, and US are considered. Local currencies are converted into euro. P values are reported corresponding to the percentage of bootstrapped return differences that are smaller than the actual observed return difference in the data set.

Panel A: December trades

<table>
<thead>
<tr>
<th>Holding period</th>
<th>21</th>
<th>42</th>
<th>63</th>
<th>84</th>
<th>21</th>
<th>42</th>
<th>63</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases</td>
<td>2.21%</td>
<td>5.26%</td>
<td>5.57%</td>
<td>9.05%</td>
<td>3.32%</td>
<td>7.16%</td>
<td>6.66%</td>
<td>9.29%</td>
</tr>
<tr>
<td>Sales</td>
<td>1.99%</td>
<td>5.22%</td>
<td>7.13%</td>
<td>11.05%</td>
<td>3.01%</td>
<td>7.39%</td>
<td>7.18%</td>
<td>9.78%</td>
</tr>
<tr>
<td>Difference</td>
<td>0.23%</td>
<td>0.04%</td>
<td>-1.57%</td>
<td>-2.00%</td>
<td>0.32%</td>
<td>-0.23%</td>
<td>-0.52%</td>
<td>-0.49%</td>
</tr>
<tr>
<td>P value</td>
<td>0.7890</td>
<td>0.0110</td>
<td>0.0000</td>
<td>0.0050</td>
<td>1.0000</td>
<td>0.0840</td>
<td>0.0040</td>
<td>0.7300</td>
</tr>
</tbody>
</table>

Panel B: Non-December trades

<table>
<thead>
<tr>
<th>Holding period</th>
<th>21</th>
<th>42</th>
<th>63</th>
<th>84</th>
<th>21</th>
<th>42</th>
<th>63</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases</td>
<td>1.90%</td>
<td>3.79%</td>
<td>5.57%</td>
<td>7.11%</td>
<td>1.79%</td>
<td>3.53%</td>
<td>5.39%</td>
<td>7.04%</td>
</tr>
<tr>
<td>Sales</td>
<td>1.70%</td>
<td>3.91%</td>
<td>6.07%</td>
<td>7.86%</td>
<td>1.57%</td>
<td>3.42%</td>
<td>5.17%</td>
<td>7.04%</td>
</tr>
<tr>
<td>Difference</td>
<td>0.20%</td>
<td>-0.12%</td>
<td>-0.50%</td>
<td>-0.75%</td>
<td>0.21%</td>
<td>0.10%</td>
<td>0.22%</td>
<td>0.00%</td>
</tr>
<tr>
<td>P value</td>
<td>1.0000</td>
<td>0.9920</td>
<td>0.5900</td>
<td>0.5280</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Table 6 - Calendar-time portfolio returns
Table 6 reports the average returns on the calendar-time portfolios using formation periods of one, two, three, or four months. For each purchase (sale) of a security during the formation period, a position is taken in the calendar-time portfolio. If several funds buy (sell) the same security, the security accounts for more than one observation. Equally-weighted portfolio returns are calculated for the calendar month subsequent to the formation period. Rolling forward the formation period by one month, a time-series of calendar-time portfolio returns for month t+1 is obtained. Returns are obtained from Datastream. Local currencies are converted into euro.

<table>
<thead>
<tr>
<th>Formation period</th>
<th>1 month</th>
<th>2 months</th>
<th>3 months</th>
<th>4 months</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: All transactions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average return buy-portfolio</td>
<td>1.29%</td>
<td>1.25%</td>
<td>1.15%</td>
<td>1.02%</td>
</tr>
<tr>
<td>Average return sell-portfolio</td>
<td>1.31%</td>
<td>1.37%</td>
<td>1.18%</td>
<td>1.05%</td>
</tr>
<tr>
<td>Difference buy-sell portfolio</td>
<td>-0.02%</td>
<td>-0.12%</td>
<td>-0.03%</td>
<td>-0.03%</td>
</tr>
<tr>
<td>T statistic</td>
<td>-1.06</td>
<td>-1.14</td>
<td>-0.58</td>
<td>-0.64</td>
</tr>
<tr>
<td><strong>Panel B: European subsample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average return buy-portfolio</td>
<td>1.49%</td>
<td>1.54%</td>
<td>1.34%</td>
<td>1.21%</td>
</tr>
<tr>
<td>Average return sell-portfolio</td>
<td>1.76%</td>
<td>1.79%</td>
<td>1.41%</td>
<td>1.27%</td>
</tr>
<tr>
<td>Difference buy-sell portfolio</td>
<td>-0.27%</td>
<td>-0.25%</td>
<td>-0.07%</td>
<td>-0.06%</td>
</tr>
<tr>
<td>T statistic</td>
<td>-1.58</td>
<td>-1.55</td>
<td>-0.73</td>
<td>-0.79</td>
</tr>
<tr>
<td><strong>Panel C: UK subsample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average return buy-portfolio</td>
<td>1.41%</td>
<td>1.37%</td>
<td>1.31%</td>
<td>1.21%</td>
</tr>
<tr>
<td>Average return sell-portfolio</td>
<td>1.38%</td>
<td>1.29%</td>
<td>1.25%</td>
<td>1.16%</td>
</tr>
<tr>
<td>Difference buy-sell portfolio</td>
<td>0.03%</td>
<td>0.08%</td>
<td>0.06%</td>
<td>0.05%</td>
</tr>
<tr>
<td>T statistic</td>
<td>0.18</td>
<td>0.67</td>
<td>0.89</td>
<td>0.67</td>
</tr>
<tr>
<td><strong>Panel D: US subsample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average return buy-portfolio</td>
<td>0.84%</td>
<td>0.59%</td>
<td>0.71%</td>
<td>0.59%</td>
</tr>
<tr>
<td>Average return sell-portfolio</td>
<td>0.58%</td>
<td>0.84%</td>
<td>0.89%</td>
<td>0.67%</td>
</tr>
<tr>
<td>Difference buy-sell portfolio</td>
<td>0.26%</td>
<td>-0.25%</td>
<td>-0.18%</td>
<td>-0.08%</td>
</tr>
<tr>
<td>T statistic</td>
<td>0.68</td>
<td>-1.53</td>
<td>-1.40</td>
<td>-0.99</td>
</tr>
</tbody>
</table>
Table 7- Calendar-time portfolio returns: CAPM regressions

Table 7 reports Jensen’s alpha and the market beta for the calendar-time portfolios using formation periods of one, two, three, or four months. Both coefficients are estimated from a CAPM regression of the monthly return difference between the buy and sell portfolio on the market risk premia,

\[ R_{bt} - R_{st} = \alpha + \beta_{EUR} (R_{mt\_EUR} - R_{ft\_EUR}) + \beta_{UK} (R_{mt\_UK} - R_{ft\_EUR}) + \beta_{US} (R_{mt\_US} - R_{ft\_EUR}) + \epsilon_t. \]

The market index for European, UK, and US funds is represented by the MSCI Europe excluding UK, FTSE, and S&P500 index respectively, all expressed in EUR. The risk premia are obtained by subtracting the Euribor 1 month from the benchmarks.

<table>
<thead>
<tr>
<th>Formation period</th>
<th>1 month</th>
<th>2 months</th>
<th>3 months</th>
<th>4 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jensen’s alpha</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>T statistic</td>
<td>-0.98</td>
<td>-0.67</td>
<td>0.23</td>
<td>0.24</td>
</tr>
<tr>
<td>Beta Europe</td>
<td>-0.14</td>
<td>-0.13</td>
<td>-0.08</td>
<td>-0.08</td>
</tr>
<tr>
<td>T statistic</td>
<td>-2.74</td>
<td>-2.81</td>
<td>-3.37</td>
<td>-3.33</td>
</tr>
<tr>
<td>Beta UK</td>
<td>0.18</td>
<td>0.11</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>T statistic</td>
<td>2.55</td>
<td>1.95</td>
<td>1.27</td>
<td>0.78</td>
</tr>
<tr>
<td>Beta US</td>
<td>0.00</td>
<td>0.03</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>T statistic</td>
<td>0.02</td>
<td>0.66</td>
<td>1.85</td>
<td>2.41</td>
</tr>
</tbody>
</table>
Table 8 - PGR and PLR for the equity funds in the data set

Table 8 reports the number of realized gains, paper gains, realized losses and paper losses for the trades executed by the equity oriented funds (i.e. more than 70% equity trades) in our sample over the period August 2002 – April 2007. Gains and losses are defined relative to the average purchase price (i.e. a volume-weighted average of the buy prices preceding the acquisition of the stock). Aggregating paper and realized results cross-sectionally over the equity funds and over time, we calculate the proportion of losses realized (PLR) as the ratio of the realized losses to the sum of the realized losses and paper losses. Analogously, the proportion of gains realized is calculated as the ratio of the realized gains to the sum of the realized gains and paper gains. Under the null, the PLR exceeds or equals the PGR.

<table>
<thead>
<tr>
<th>Equity funds (519 funds)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Realized gains</td>
<td>171955</td>
</tr>
<tr>
<td>Paper gains</td>
<td>6490257</td>
</tr>
<tr>
<td>Realized losses</td>
<td>70582</td>
</tr>
<tr>
<td>Paper losses</td>
<td>2303334</td>
</tr>
<tr>
<td>PLR</td>
<td>0.0297</td>
</tr>
<tr>
<td>PGR</td>
<td>0.0258</td>
</tr>
<tr>
<td>Difference in Proportions</td>
<td>0.0039</td>
</tr>
<tr>
<td>T statistic</td>
<td>30.98</td>
</tr>
</tbody>
</table>

EUR value calculation

<table>
<thead>
<tr>
<th>EUR value calculation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PLR</td>
<td>0.0236</td>
</tr>
<tr>
<td>PGR</td>
<td>0.0212</td>
</tr>
<tr>
<td>Difference in Proportions</td>
<td>0.0025</td>
</tr>
<tr>
<td>T statistic</td>
<td>22.08</td>
</tr>
</tbody>
</table>

Table 9 – Average realized returns

Table 9 displays the average and median returns resulting from the realized gains, paper gains, realized losses, and paper losses reported in table 8. Gains and losses are defined relative to the average purchase price (i.e. a volume-weighted average of the buy prices preceding the acquisition of the stock).

<table>
<thead>
<tr>
<th></th>
<th>Average return</th>
<th>Median return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return on realized gains</td>
<td>0.2761</td>
<td>0.1755</td>
</tr>
<tr>
<td>Return on paper gains</td>
<td>0.2798</td>
<td>0.1748</td>
</tr>
<tr>
<td>Return on realized losses</td>
<td>-0.1120</td>
<td>-0.0752</td>
</tr>
<tr>
<td>Return on paper losses</td>
<td>-0.1222</td>
<td>-0.0790</td>
</tr>
</tbody>
</table>
Table 10- PGR and PLR for geographical subsamples

Table 10 reports the number of realized gains, paper gains, realized losses and paper losses for the trades executed by the equity oriented funds (i.e. more than 70% equity trades) in our sample over the period August 2002 – April 2007. Gains and losses are defined relative to the average purchase price (i.e. a volume-weighted average of the buy prices preceding the acquisition of the stock). We aggregate paper and realized results over time and according to each fund’s geographical orientation (Euro countries, UK, US). Next, we calculate the proportion of losses realized (PLR) as the ratio of the realized losses to the sum of the realized losses and paper losses. Analogously, the proportion of gains realized is calculated as the ratio of the realized gains to the sum of the realized gains and paper gains. Under the null, the PLR exceeds or equals the PGR.

<table>
<thead>
<tr>
<th></th>
<th>Euro countries</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realized gains</td>
<td>16777</td>
<td>56519</td>
<td>17908</td>
</tr>
<tr>
<td>Paper gains</td>
<td>457321</td>
<td>1805849</td>
<td>842965</td>
</tr>
<tr>
<td>Realized losses</td>
<td>4452</td>
<td>17580</td>
<td>10014</td>
</tr>
<tr>
<td>Paper losses</td>
<td>93378</td>
<td>646851</td>
<td>374204</td>
</tr>
<tr>
<td>PLR</td>
<td>0.0455</td>
<td>0.0264</td>
<td>0.0261</td>
</tr>
<tr>
<td>PGR</td>
<td>0.0354</td>
<td>0.0303</td>
<td>0.0208</td>
</tr>
<tr>
<td>Difference in Proportions</td>
<td>0.0101</td>
<td>-0.0039</td>
<td>0.0053</td>
</tr>
<tr>
<td>T statistic</td>
<td>14.08</td>
<td>-16.64</td>
<td>17.56</td>
</tr>
</tbody>
</table>
Table 11 - PGR and PLR for different reference points (equity funds)

Table 11 reports the number of realized gains, paper gains, realized losses and paper losses for the trades executed by the equity oriented funds (i.e. more than 70% equity trades) in our sample over the period August 2002 – April 2007. Gains and losses are defined relative to various reference points, namely the average purchase price (i.e. a volume-weighted average of the buy prices preceding the acquisition of the stock), the highest purchase price, the first purchase price, the most recent purchase price, prior maxima (three months, six months, one year, two years), and the last trading day of December. Aggregating paper and realized results cross-sectionally over equity funds and over time, we calculate the proportion of losses realized (PLR) as the ratio of the realized losses to the sum of the realized losses and paper losses. Analogously, the proportion of gains realized is calculated as the ratio of the realized gains to the sum of the realized gains and paper gains. Under the null, the PLR exceeds or equals the PGR.

<table>
<thead>
<tr>
<th></th>
<th>Average purchase price</th>
<th>Highest purchase price</th>
<th>First purchase price</th>
<th>Most recent purchase price</th>
<th>Prior maximum (3 months)</th>
<th>Prior maximum (6 months)</th>
<th>Prior maximum (1 year)</th>
<th>Prior maximum (2 years)</th>
<th>Last trading day of December</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realized gains</td>
<td>171955</td>
<td>127395</td>
<td>165458</td>
<td>167462</td>
<td>77648</td>
<td>65103</td>
<td>54516</td>
<td>42773</td>
<td>175338</td>
</tr>
<tr>
<td>Paper gains</td>
<td>6490257</td>
<td>4821003</td>
<td>61789428</td>
<td>6307304</td>
<td>2391863</td>
<td>1951078</td>
<td>1648956</td>
<td>1371087</td>
<td>6011897</td>
</tr>
<tr>
<td>Realized losses</td>
<td>70582</td>
<td>115074</td>
<td>77012</td>
<td>75021</td>
<td>193410</td>
<td>205957</td>
<td>210857</td>
<td>179679</td>
<td>89405</td>
</tr>
<tr>
<td>Paper losses</td>
<td>230334</td>
<td>376354</td>
<td>2504413</td>
<td>2429783</td>
<td>6120932</td>
<td>6683978</td>
<td>7044312</td>
<td>6598266</td>
<td>2605649</td>
</tr>
<tr>
<td>PLR</td>
<td>0.0297</td>
<td>0.0299</td>
<td>0.0299</td>
<td>0.0300</td>
<td>0.0306</td>
<td>0.0299</td>
<td>0.0291</td>
<td>0.0265</td>
<td>0.0332</td>
</tr>
<tr>
<td>PGR</td>
<td>0.0258</td>
<td>0.0257</td>
<td>0.0261</td>
<td>0.0258</td>
<td>0.0314</td>
<td>0.0323</td>
<td>0.0320</td>
<td>0.0303</td>
<td>0.0283</td>
</tr>
<tr>
<td>Difference in Proportions</td>
<td>0.0039</td>
<td>0.0041</td>
<td>0.0038</td>
<td>0.0041</td>
<td>-0.0008</td>
<td>-0.0024</td>
<td>-0.0029</td>
<td>-0.0037</td>
<td>0.0049</td>
</tr>
<tr>
<td>T statistic</td>
<td>31.07</td>
<td>36.83</td>
<td>30.44</td>
<td>32.83</td>
<td>-6.22</td>
<td>-17.08</td>
<td>-19.78</td>
<td>-23.88</td>
<td>37.81</td>
</tr>
</tbody>
</table>
Table 12 - PGR and PLR for different reference points: geographical benchmarks

Table 12 reports the number of realized gains, paper gains, realized losses and paper losses for the trades executed by the mutual funds in our sample over the period August 2002 – April 2007. Gains and losses are defined relative to a geographical benchmark index. We aggregate paper and realized results over time and according to each fund’s geographical orientation (Euro countries, UK, US). Next, we calculate the proportion of losses realized (PLR) as the ratio of the realized losses to the sum of the realized losses and paper losses. Analogously, the proportion of gains realized is calculated as the ratio of the realized gains to the sum of the realized gains and paper gains. Under the null, the PLR exceeds or equals the PGR.

<table>
<thead>
<tr>
<th>Benchmark index</th>
<th>Euro countries</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSCI Europe ex UK.</td>
<td>FTSE</td>
<td>S&amp;P500</td>
</tr>
<tr>
<td>Realized gains</td>
<td>14013</td>
<td>38205</td>
<td>13168</td>
</tr>
<tr>
<td>Paper gains</td>
<td>388609</td>
<td>1280507</td>
<td>667512</td>
</tr>
<tr>
<td>Realized losses</td>
<td>7216</td>
<td>35917</td>
<td>14754</td>
</tr>
<tr>
<td>Paper losses</td>
<td>204112</td>
<td>1365098</td>
<td>666754</td>
</tr>
<tr>
<td>PLR</td>
<td>0.0341</td>
<td>0.0256</td>
<td>0.0216</td>
</tr>
<tr>
<td>PGR</td>
<td>0.0348</td>
<td>0.0290</td>
<td>0.0193</td>
</tr>
<tr>
<td>Difference in Proportions</td>
<td>-0.0007</td>
<td>-0.0034</td>
<td>0.0023</td>
</tr>
<tr>
<td>T statistic</td>
<td>-1.35</td>
<td>-16.85</td>
<td>9.48</td>
</tr>
</tbody>
</table>
Table 13 - PGR and PLR for frequent and infrequent traders (equity funds)

Table 13 reports the number of realized gains, paper gains, realized losses and paper losses for the trades executed by the equity oriented funds (i.e. more than 70% equity trades) in our sample over the period August 2002 – April 2007. We split the sample into three groups according to the trading frequency of the funds in the sample. Gains and losses are defined relative to the average purchase price (i.e. a volume-weighted average of the buy prices preceding the acquisition of the stock). Aggregating paper and realized results over time and separately for each group, we calculate the proportion of losses realized (PLR) as the ratio of the realized losses to the sum of the realized losses and paper losses. Analogously, the proportion of gains realized is calculated as the ratio of the realized gains to the sum of the realized gains and paper gains. Under the null, the PLR exceeds or equals the PGR. We assume independent observations.

<table>
<thead>
<tr>
<th></th>
<th>Group 1: infrequent traders</th>
<th>Group 2: moderate traders</th>
<th>Group 3: frequent traders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of funds in this group</td>
<td>424</td>
<td>72</td>
<td>23</td>
</tr>
<tr>
<td>Percentage of sells</td>
<td>31.62%</td>
<td>33.59%</td>
<td>34.79%</td>
</tr>
<tr>
<td>Cumulative percentage</td>
<td>31.62%</td>
<td>65.21%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Average # of trades</td>
<td>180.88</td>
<td>1131.63</td>
<td>3668.13</td>
</tr>
<tr>
<td>Realized gains</td>
<td>54978</td>
<td>58261</td>
<td>58506</td>
</tr>
<tr>
<td>Paper gains</td>
<td>1788614</td>
<td>2305084</td>
<td>2366635</td>
</tr>
<tr>
<td>Realized losses</td>
<td>21715</td>
<td>23216</td>
<td>25861</td>
</tr>
<tr>
<td>Paper losses</td>
<td>599987</td>
<td>774719</td>
<td>929085</td>
</tr>
<tr>
<td>PLR</td>
<td>0.0349</td>
<td>0.0291</td>
<td>0.0271</td>
</tr>
<tr>
<td>PGR</td>
<td>0.0298</td>
<td>0.0247</td>
<td>0.0241</td>
</tr>
<tr>
<td>PLR - PGR</td>
<td>0.0051</td>
<td>0.0044</td>
<td>0.0030</td>
</tr>
<tr>
<td>T statistic</td>
<td>19.32</td>
<td>20.81</td>
<td>15.31</td>
</tr>
</tbody>
</table>
Figure 1 – Average holding period returns in the trading days prior and subsequent to a purchase or sale (European, UK, and US trades)
Figure 2 – Average holding period returns in the trading days prior and subsequent to a purchase or sale: European funds
Figure 3 – Average holding period returns in the trading days prior and subsequent to a purchase or sale: UK funds
Figure 4 – Average holding period returns in the trading days prior and subsequent to a purchase or sale: US funds
Figure 5 – Return index evolution of the S&P500, the FTSE100, and the MSCI Europe ex UK