Plasma tomographic reconstruction from tangentially viewing camera with background subtraction

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Light reflections are one of the main and often underestimated issues of plasma emissivity reconstruction in visible light spectral range. Metallic and other specular components of tokamak wall generate systematic errors in the optical measurements that could lead to wrong interpretation of data. Our analysis is performed at data from the tokamak COMPASS. It is a D-shaped tokamak with specular metallic vessel and possibility of the H-mode plasma. Data from fast visible light camera were used for tomographic reconstruction with background reflections subtraction to study plasma boundary. In this article, we show that despite highly specular tokamak wall, it is possible to obtain a realistic reconstruction. The developed algorithm shows robust results despite of systematic errors in the optical measurements and calibration. The motivation is to obtain an independent estimate of the plasma boundary shape. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4862652]

I. INTRODUCTION

In recent years, fast visible light cameras have become standard equipment of many tokamaks. Usually, cameras are used for observation of plasma-wall interaction, however there are many other applications, e.g., plasma shape, fluctuation, or dust studies.1–5

The plasma edge emits visible radiation mainly because of interaction of neutral particles flux from wall with plasma. Projections along lines of sight (LoS) of this emissivity are afterwards detected. Radiation from cool plasma edge can be used to estimate position of radiating shells.

The cameras are usually installed in tokamaks so that the plasma is observed either in poloidal plane (perpendicular view) or toroidal direction (tangential view). The poloidal view is mostly used for bolometry and soft X-ray pin-hole cameras. It is less suitable for line radiation in the visible spectra because the radiating shell placed near to the plasma edge results in flat projections of the hollow emissivity profile. On the other hand, the tangential configuration is often used to observe the plasma edge and divertor regions.2–4, 6 The lines of sights tangential to the plasma column lead to higher contrast in the projections. Moreover, numerical tests proved that the local plasma emissivity can be reconstructed from one 2D image camera, hence one camera is sufficient for a reconstruction, even though the coverage in the projection space is rather narrow. Using the assumption of toroidal symmetry and field of view covering sufficient part of the plasma, the local emissivity can be inferred using tomographic methods.

The tangential camera geometry setup is more challenging compared to the poloidal and pinhole setups because different effects of the used optical system, e.g., image distortion or camera vibration, need to be deducted. Furthermore, fast plasma fluctuations cannot be any longer considered as toroidally symmetrical and the helical geometry should be accounted for. The toroidal symmetry is valid only if the integration time is long enough to smooth out all rapidly moving helical structures. However, inclusion of the safety factor will “hardwire” the expected plasma shape and position into the reconstruction geometry matrix Eq. (1). This represents an a priori knowledge which, if false, may damage the result, therefore infinite safety factor was used in this work.

Basically, two kinds of information about plasma can be reconstructed: plasma boundary position, shape,1, 8, 9 and fast fluctuations2, 5, 6 – i.e., filaments and blobs. The filaments are expected to be significantly elongated along magnetic field, otherwise the conditions for the tomographic reconstruction would not be fulfilled. Finally, reflections from the vessel components can disturb the reconstruction.9–13 Particularly, in the case of the plasma shape reconstruction, reflections from the vessel wall and in-vessel components are of major importance unlike the case of fast fluctuations where the slowly evolving background can be subtracted. In particular, our data obtained from the tokamak COMPASS14 with a specular steel vessel wall are significantly affected by light reflections.

II. TANGENTIAL TOMOGRAPHY

Plasma tomography is generally an ill-posed task that transforms line integrated emissivity (i.e., plasma projections)
to local radiative flux density (typically to plasma cross-sections). Poloidal plasma tomography is a relatively well investigated problem and the corresponding methods are well established and widely used.\textsuperscript{7,15} At the same time, the tangential tomography is based on rather basic methods; e.g., truncated singular value decomposition (SVD) or linear methods have been used.\textsuperscript{2,5,6,8,9} One of the reasons are the higher computational demands of the tangential tomography.

The main distinction between the tangential camera tomography compared to the poloidal 1D field of detectors is an enormous number of lines of sight while the information is strongly correlated. As a result, although tomographic task can be mathematically overdetermined system, it is still an ill-posed task with lack of information for a direct inversion. Therefore, before the reconstruction, it is recommended to either down-sample data to an optimal size or decompose into low-dimensional orthogonal base vectors, e.g., Bessel or Cormack base in order to increase solving speed and limit the correlations.

Finally, reflections from the tokamak vessel and plasma facing components (PFC) must be subtracted. This is very important because the reflections along with imprecise geometry facing components (PFC) must be subtracted. This is very important because the reflections along with imprecise geometry setting cause systematic errors which can limit reliability of the resulting reconstruction (see Sec. V).

A. Pixel based tomography method with nonlinear constrains

In the pixel-based tomography methods, the unknown two-dimensional (2D) emissivity cross-section \( g(x, y) \) is discretized to \( N \) values \( g_j \) in a finite number of pixels \( j \in 1, \ldots, N \). The unknown reconstruction \( g \) is searched so that it minimizes residuals with the measured projections \( f \), i.e., the integral-based data and reconstructed emissivity projections.

After discretization of the emissivity to the pixel grid, the task corresponds to a set of linear equations:

\[
f_i = \sum_{j=1}^{N} T_{ij} g_j + \xi_i \quad i = 1, \ldots, L,
\]

where \( T_{ij} \) denotes the geometric (contribution) matrix, \( g_j \) is the emissivity in the \( j \)-th pixel, \( \xi_i \sim \mathcal{N}(0, \sigma_i^2) \) represents statistical noise assuming a normal distribution of the projections \( f_i \), and \( L \) is the number of measurements. The read-out and discretization noise intensity can be estimated from dark frames, however it is usually negligible. The main sources of error are reflected light and imperfections in the geometry calibration.

This system of equations is ill-posed and, in the case of fusion diagnostic systems, usually underdetermined.

One of the most common methods that finds a trustworthy solution is the Tikhonov regularization which minimizes the reconstruction residual constrained to a regularization function \( O(g) \):

\[
\min_g \left( \| T g - f \|_2^2 + \lambda O(g) \right), \tag{2}
\]

where \( \| T g - f \|_2^2 \) is quadratic misfit (the residual) and \( \lambda \) is the regularization parameter. The object function \( O(g) \) imposes some realistic \textit{a priori} information, e.g., expected emissivity smoothness, entropy, or probability distribution.

In this contribution, a non-linear regularization function \( O(g) \) similar to the minimum Fisher information method\textsuperscript{16,17} was implemented:\textsuperscript{18}

\[
O(g) = \int \sum_{i,j=1}^{2} \left( \frac{\partial g_i}{\partial x_i} \frac{\partial g_j}{\partial x_j} + \beta \right) \left( \frac{1}{g_i} \right)^{\epsilon} \mathrm{d}x \mathrm{d}y \quad g_i > \epsilon
\]

\[
= \int \sum_{i,j=1}^{2} \left( \frac{1}{g_i} \right)^{\epsilon} \mathrm{d}x \mathrm{d}y \quad g_i \leq \epsilon
\]

where \( \epsilon \) is a small positive number. The proposed \( O(g) \) regularizes Eq. (2) by enforcing sufficient and physically relevant smoothness and positivity to the reconstructed image. Moreover, the \( \alpha, \beta \) terms suppress large flat non-zero artifacts caused by the background reflection (see Sec. III).

The Tikhonov regularization Eq. (2) with a regularization matrix \( \mathbb{H} \) leads to a sparse system that is solved by the Cholesky decomposition:

\[
A g = (T^T T + \lambda \mathbb{H})g = T^T f.
\]

The regularization parameter \( \lambda \) sets the strength of an \textit{a priori} constraint \( O(g) \) with respect to the value of the residual \( \| T g - f \|_2^2 \). Underestimated value of \( \lambda \) results in over-fitting and the other extreme leads to over-smoothing. The optimal value of \( \lambda \) was chosen to minimize the \( \chi^2 \) score of the Pearson distribution:

\[
\chi^2 = \sum_{i=1}^{L} \left( \frac{\sum_{j=1}^{N} T_{ij} g_j - f_i}{\sigma_i^2} \right)^2,
\]

where \( L \) denotes degrees of freedom. Two examples of the \( \chi^2 \) dependence on the smoothness penalty are displayed in Fig. 8.

The proposed regularization term \( O(g) \) Eq. (3) is nonlinear, however it can be solved iteratively using the regularization matrix \( \mathbb{H} \),\textsuperscript{16} which introduces a discretized and linearized regularization term \( O(g) \). \( \mathbb{H} \) is implemented in the form:

\[
\mathbb{H}^{(k)} = \sum_{l} B_l \mathbb{W}^{(k)} B_l + \beta \mathbb{W}^{(k)}.
\]

\( W^{(k)} \) denotes \( k \)-th iteration of the weighting matrix \( \mathbb{W}^{(k)} = \delta_{ij} g_{g}^{\alpha(k-1)} \) and \( B_l \) denotes matrices of the discrete derivatives, corresponding to the difference operator. The constants \( \alpha \) and \( \beta \) should be optimized over many frames to obtain a smooth and artifact suppressed reconstruction.

Consequently, the tomography algorithm consists of two nested cycles. The inner cycle interpolates \( \lambda \) and minimizes the residuals, while the outer cycle optimizes the weighting matrix \( \mathbb{W} \).

III. REFLECTIONS MODEL

Background reflections are one of the important, however often neglected, sources of systematic errors in the case of visible light emissivity measurements. Reflectivity can be described by the bidirectional reflectance distribution function (BRDF) that is defined as the ratio between the incoming irradiance and the outgoing irradiance.\textsuperscript{19} Two widely
used models are the Lambertian and Phong models. The Lambertian model expects a BRDF that is constant and independent of the incidence angle, i.e., matte surface,

\[ f_{r, \text{Lambert}} = c_l, \]

where \( c_l \) is a material constant. The Phong model is used for specular surfaces and it depends on the angle between the ideally reflected ray \( R \) and the incident light vector \( L \) (see Fig. 1),

\[ f_{r, \text{Phong}}(L, R) = c_p \cos^\alpha \theta, \]

where \( c_p, c_s \) denotes another material constants. Constants \( c_l, c_p \) corresponds to the reflectivity of the surface, while \( c_s \) is related to its specularity. Large \( c_s \) results in mirror-like reflection, while \( c_s = 0 \) is the Lambertian model. The reflection vector \( R \) corresponds to

\[ R = 2(\hat{N} \cdot \hat{V})\hat{N} - \hat{V}, \]

where \( \hat{N} \) denotes a unit vector normal to the surface (the hat denotes vectors normalized to unity). The reflectance angle is equal to

\[ (\hat{R} \cdot \hat{V}) = 2(\hat{N} \cdot \hat{L})(\hat{N} \cdot \hat{V}) - (\hat{L} \cdot \hat{V}), \]

where \( \hat{V} \) is the direction from the reflection point to the radiation source. The final form for the Phong reflectance model can be written as the positive part of the \((\hat{R} \cdot \hat{V})\) angle:

\[ f_{r, \text{Phong}}(\hat{L}, \hat{R}, \hat{V}) = c_p [2(\hat{N} \cdot \hat{L})(\hat{N} \cdot \hat{V}) - (\hat{L} \cdot \hat{V})]^+ \cos^\alpha. \]

However, several approximations need to be applied in order to calculate reflectivity of the tokamak vessel.

- Light is emitted from the whole volume of the plasma, therefore the number of the light sources is infinite and even after volume discretization, the number of points is intractable by simple ray-tracing. However, the spatial distribution of the emissivity is usually smooth and hence the difference in incident radiances from similar directions is small.
- Multiple reflections were neglected due to insufficient precision in the determination of the models constants, however the contribution can be non-negligible in certain situations.
- Toroidal symmetry is assumed in order to simplify the reflectivity model. Asymmetrical objects such as diagnostics ports were added to the final model.
- The diffusive model is calculated only as a 2D model in the poloidal plane because of the toroidal symmetry.
- The constants \( c_p, c_l \) depends on the material surface and it is difficult to measure inside of the tokamak vessel, therefore a numerical method for their calculation is proposed.
- The carbon limiter is expected to be a Lambertian object because it has only pure diffuse reflection. Similarly, the diffusive reflection of the metallic vessel was neglected in order to limit the number of searched material constants.

The diffusive model can be calculated under assumption of toroidal symmetry:

\[ I_j^L = \sum_{i} (\hat{L}_i \cdot \hat{N}) f_{r, \text{Lambert}, i} l_i, \]

where \( l_i \) is radiant emissivity from the \( i \)th pixel, it corresponds to \( g_i / ||V_i|| \), where \( V_i \) is the direction from the emissivity pixel to the incident point on the vessel wall.

The Phong model is more complicated to calculate. Because the number of light sources is practically intractable, we have used the Helmholtz reciprocity of the BRDF,\(^{19}\)

\[ f_r(L, V) = f_r(V, L), \]

and viewing cones after the reflection were approximated by a limited number of LoS that were uniformly distributed over the cones of view (Figs. 2 and 3). Therefore, the Phong model

![Fig. 1. An illustration of the direction of the vectors used in the reflective models.](image1)

![Fig. 2. An example of the camera tomography geometry in the poloidal cross-section of the COMPASS tokamak vacuum vessel with poloidal projections of the lines of sight. The reflectivity of the components depends on the reflecting poloidal angle.](image2)

![Fig. 3. Examples of origins of the reflected signal using the diffusion model in log-scale. The reflectivity depends on the angle between the ray incident material and reflecting poloidal and toroidal angle.](image3)
The reflectivity model was updated in every iteration using the emissivity from the previous step while optimal reflectivity parameters are searched. Three iterations are usually sufficient for convergence.

V. RESULTS AND DISCUSSION

The reconstructed plasma emissivity (Fig. 5(a)) should have been precisely a circular shape in the reconstructed

![Image](image_url)

FIG. 4. Calculations of the reflected light using the diffusive and specular reflectivity models.

![Image](image_url)

FIG. 5. Plasma emissivity reconstruction from shot #3487 at time 10 ms. The detached plasma column was reconstructed without any assumption about plasma shape from the magnetic equilibrium code. (a) presents our reconstruction, while (b) and (c) show model including background reflections and the measured data. Finally, (d) highlights differences between data and our model.
FIG. 6. Ratio between total observed illumination and reflected illumination. The radiation reflected from the chamber wall exceeds the plasma radiation in large areas within the field of view.

shot #3487. However, due to minor imperfection in the geometry setting and reflections subtraction (Fig. 5(d)), the shape is slightly deformed. The difference between our model (Fig. 5(c)) and the original image (Fig. 5(b)) shows that shell radiation was slightly overestimated, while the background radiation was underestimated. This can be explained as radiation of weakly ionized plasma out of the main column.

Particularly, near the center of the plasma column, the difference between retrofit and data is around 20%. Other sources of systematic errors are optical calibration imperfections and the simplicity of our reflective model. It was attempted to use a more complex reflectivity model but it led to less stable and unreliable results.

On the other hand, the reflective model Eq. (7) was able to describe relatively well the reflected light including the local structures, e.g., ports and limiters (Fig. 5(c)). The reflected light forms on the average 35% of the observed luminance and locally it can correspond up to 95% (Fig. 6) of the total luminance.

The background subtraction is a crucial issue. This is shown in terms of the reconstructions with and without the subtraction (Fig. 7). Note that although the difference in the reconstructions is significant (Fig. 7), the Pearson $\chi^2$ criterion Eq. (5) reaches a similar score (see Fig. 8). Note that even the best $\chi^2$ exceeds 10 because of the model imperfections. Furthermore, high-complexity models (high smoothness penalty) lead to worse results because of the non-negativity condition. It can be expected that the ideal reconstruction is relatively sparse with low emissivity values in most of the reconstructed space as it is shown in the Fig. 7. In order to describe the sparsity, we have decided to use the Hoyer sparsity as defined as a ratio between the $L_1$ and $L_2$ norm of the data normalized to zero for unitary data and equal to one if all the signal is in one element. The Hoyer sparsity shows significant improvement after the background subtraction and the optimum corresponds to same complexity as the minimal $\chi^2$ value.

VI. CONCLUSION

The proposed background reflection model in combination with some appropriate regularization results in the expected ring shape of the 2D plasma emissivity (Fig. 5(a)). Therefore, the introduced reconstruction scheme including the reflection model is suitable for tangential tomography in tokamaks with metallic vessel and other setup challenges that increase systematic errors.

The regularization Eq. (3) forces emissivity to zero with a non-negative condition. This allows to perform reconstruction despite of systematic errors in input data and iteratively remove background reflections and recover the compact and smooth shape of the plasma emissivity.

In the future, the fast camera in the COMPASS tokamak will be moved to a more suitable position to observe the divertor and a tomographic reconstruction of the plasma divertor legs should be achievable.

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2R. N. van yen, N. Fedorczak, F. Brochard, G. Bonhomme, K. Schneider, M. Farge, P. Monier-Garbet et al., “Tomographic reconstruction of tokamak...


