EDART, a Discrete Algebraic Reconstructing Technique for Experimental Data Obtained with High Resolution Computed Tomography

L. Brabant*, M. Dierick, E. Pauwels, M.N. Boone and L. Van Hoorebeke
Ghent University, Department of Physics and Astronomy
Proeftuinstraat 86, 9000 Gent, Belgium

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Abstract

A novel reconstruction method to improve the image quality of three dimensional (3D) datasets, obtained with high resolution X-ray Computed Tomography (μ CT), for samples consisting of only one material and surrounding air is presented. It combines discrete tomography with iterative reconstruction algorithms, it is applicable for routine μ CT applications and is referred to as the Experimental Discrete Algebraic Reconstruction Technique (EDART). A fast and intuitive method to estimate the attenuation coefficient and segmentation threshold, in case these are unknown, is included. Experimental results illustrate that EDART allows improving the reconstruction quality relatively to standard iterative reconstruction when few projections are available, without significantly increasing the reconstruction time.

keywords: discrete tomography, high resolution CT, iterative reconstruction

1 Introduction

High resolution X-ray computed tomography (μ CT) is a powerful, non-destructive technique which allows for the investigation of the internal structure of objects. An important challenge lies in the correct reconstruction of the projection data obtained with (μ CT). As the datasets obtained with laboratory based μ CT are of substantial size, it is a challenging problem to obtain accurate reconstructions in a reasonable amount of time. To obtain the optimal reconstruction quality, all the information that is available

*Corresponding author. E-mail Loes.Brabant@ugent.be Tel. +32 9264 6611 Fax +32 9264 6697

9 about the sample should be used during the reconstruction. Often tomography
10 is applied on sample consisting of only one or a limited number of materials,
11 which is referred to as discrete tomography [1]. In discrete tomography the
12 purpose is to obtain more accurate reconstructions by using the prior knowl-
13 edge such as the number of materials and the attenuation coefficients of these
14 materials. It is often used to reduce the number of necessary projections. There
15 exist several algorithms for discrete tomography, e.g. [2, 3, 4, 5, 6, 7, 8, 9]. For
16 medical applications, most algorithms assume that the value of the attenuation
17 coefficient is known in advance, because the scanned materials are known in
18 advance. In laboratory based tomography, all kinds of samples are scanned and
19 the attenuation coefficient(s) of the material(s) often are unknown. Addition-
20 ally, most algorithms for discrete tomography are computationally demanding
21 and time consuming, particularly for 3D volumes. A typical dataset obtained
22 with μ CT is of substantial size, so a fast computationally efficient algorithm is
23 required in practical applications.

24 Recently, Batenburg and Sijbers proposed a new reconstruction algorithm for
25 discrete tomography, called DART (Discrete Algebraic Reconstruction Tech-
26 nique; [10]) which had several practical applications [11, 12, 13]. DART com-
27 bines discrete reconstruction with iterative reconstruction algorithms. Iterative
28 reconstruction algorithms [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25] are becom-
29 ing a more widely used alternative for analytical reconstruction algorithms, such
30 as the algorithm of Feldkamp, David and Kress (FDK) [26], which are based
31 on filtered back projections. These algorithms can be used to reduce artifacts
32 such as cone beam artifacts [27, 28], metal artifacts [29, 30] and beam harden-
33 ing artifacts [31, 32, 33]. Moreover, they can provide better noise handling and
34 reconstructions of higher quality when only a limited number of projections are
35 available [34, 35, 36, 37]. With DART it is possible to obtain more accurate
36 reconstructions for samples consisting of a number of discrete materials, even
37 when the number of projections is limited.

38 Although DART can produce reconstructions of much better quality in com-
39 parison with regular iterative reconstruction algorithms, the DART algorithm
40 requires accurate knowledge about the attenuation coefficient(s) and the thresh-
41 old(s) in order to produce accurate results. Several methods for the determi-
42 nation of the attenuation coefficient and threshold are available [38, 39, 40].
43 However, most algorithms are very time consuming or are not applicable for
44 every type of sample, which makes them not always practically feasible for rou-
45 tine μ CT applications. Even when the attenuation coefficient is known, the
46 reconstruction times of DART can increase because several discretization and
47 reconstruction steps are needed. The reconstruction times reported in [40] indi-
48 cate that the computational time for DART is significantly larger in comparison
49 with the reconstruction time of classical iterative reconstruction, especially for
50 datasets where the attenuation coefficient has to be estimated.

51 The question arises whether it would be possible to obtain a better result in
52 comparison with classical iterative reconstruction algorithms while keeping the
53 reconstruction time in the same order of magnitude. In this paper an algorithm
54 inspired by the DART algorithm will be presented: the Experimental Discrete

Algebraic Reconstruction Technique (EDART). With this algorithm it is not our goal to obtain the best possible result, but to obtain a better result in comparison with standard iterative reconstruction techniques without significantly increasing the reconstruction time. The EDART algorithm will be explained for reconstruction of 3D datasets consisting of only one material and surrounding air, but it can be extended towards datasets consisting out of more materials. Additionally, a fast and intuitive way to estimate the attenuation coefficient and threshold in the dataset consisting of one material and surrounding air is presented.

2 Materials and methods

In this section, first the basic concepts of iterative reconstruction are reviewed. The EDART algorithm will be presented, as well as a fast and intuitive way to determine the attenuation coefficient of the material of the sample in case this is not known in advance. Subsequently, the samples and parameters and set-up used for scanning them are described and finally the methods for evaluations of the results are discussed.

2.1 Iterative Reconstruction

Iterative reconstruction algorithms consider the reconstruction process as the optimization of a discrete representation of the object $f(x, y, z)$ in order to satisfy a system of equations that describes the imaging modality. The object function is represented on a 3D grid of N voxels (3D equivalent of pixels) corresponding to function values f_j with $j = 1, 2, \dots, N$. The line integral corresponding to a detector pixel can be written in a discrete form, p_i , which is called a ray sum. Algebraic reconstruction algorithms try to find a solution to the linear equation system:

$$\mathbf{W} \cdot \mathbf{f} = \mathbf{p} . \quad (1)$$

$\mathbf{f}$ is the $(N \times 1)$ column vector representing the function values f_j and $\mathbf{p}$ is a $(M \times 1)$ column vector containing the ray sums p_i , with $M = RM_p$ the total number of measured ray sums in all the projection images; R represents the number of projections and M_p the number of detector pixels. $\mathbf{W}$ is the $(M \times N)$ matrix of the weights w_{ij} , which represent the contribution of the j -th voxel to the i -th ray sum.. In this paper the SART algorithm [15] is used, but both DART and EDART can be used with different algebraic reconstruction techniques. With SART the update process is described by [15]:

$$f_j^{(k+1)} = f_j^k + \lambda \frac{\sum_{p_i \in P_\phi} \left(\frac{p_i - \sum_{n=0}^N w_{in} f_n^k}{\sum_{n=0}^N w_{in}} \right) w_{ij}}{\sum_{p_i \in P_\phi} w_{ij}} . \quad (2)$$

Where f_j^k represents the linear attenuation coefficient of voxel j after the k -th iteration, λ is a relaxation parameter and P_ϕ is the projection with projection angle ϕ . One SART iteration uses every projection once.

91 2.2 EDART

92 The Experimental Discrete Algebraic Reconstruction Technique (EDART) was
 93 developed because we needed an algorithm which improves the reconstruction
 94 quality by taking the discrete nature of the sample into account, without sig-
 95 nificantly increasing reconstruction time. With EDART we will try to improve
 96 the reconstruction quality by using only one discretization step. These are the
 97 different steps in the EDART algorithm in case the attenuation coefficient μ of
 98 the material of the sample and a realistic threshold T are known in advance, in
 99 comparison with a SART reconstruction with $2 \times N_{it}$ iterations:

- 100 1. Perform a SART reconstruction with N_{it} iterations.
- 101 2. Apply a smoothing algorithm to reduce the noise (we use a median filter,
 102 but other types of filter can be used as well).
- 103 3. Segment the volume. Every voxel with a value lower than T is set to zero
 104 and the others are set to μ .
- 105 4. The segmented volume is used as input for a new SART reconstruction
 106 with N_{it} iterations.
- 107 5. The result is smoothed and segmented.

108 In this way the total number of iterations remains the same as in the SART
 109 algorithm. Note that each SART iteration uses every projection once. The only
 110 overhead which is introduced by using EDART is applying the smoothing and
 111 the discretization step.

112 The necessary number of iterations N_{it} will of course depend on the available
 113 number of projections. When only a limited number of projections are available
 114 the reconstruction quality can be improved by increasing the number of itera-
 115 tions. However, increasing N_{it} will also increase the reconstruction time, so the
 116 number of iterations should only be increased when it has an effect on the recon-
 117 struction quality. To determine if the reconstruction is affected by changing the
 118 number of iterations we evaluated whether the histogram changed significantly
 119 for a single slice reconstruction when the number of iterations was changed.
 120 Several single slice reconstructions with increasing number of iterations were
 121 performed and Otsu's threshold [41] was determined for each reconstruction. If
 122 the determined threshold did not vary significantly when the number of itera-
 123 tions was increased, the current number of iterations was set as N_{it} .

124 2.3 Determining the attenuation coefficient and threshold

125 In the previous section we assumed that the attenuation coefficient and thresh-
 126 old of the sample are known in advance. In case of non-medical experimental
 127 datasets this is often not the case. Moreover, when using the experimental data
 128 artifacts can occur, the projection images contain noise and the samples are
 129 often not completely discrete. As a result, there will be a variation in the re-
 130 constructed attenuation coefficients of voxels belonging to the same material.

131 In this case a *virtual* attenuation coefficient μ for the material which results
 132 in the best reconstruction quality needs to be determined. In this section a
 133 method to estimate μ and T is presented. When the EDART algorithm is used
 134 in combination with this method we will refer to this as EDART(calc), while
 135 the EDART algorithm in combination with known μ and T is referred to as
 136 EDART(known).

137 In the case of a discrete sample which consists only of one material, a single slice
 138 of the sample can be used to estimate the attenuation coefficient and threshold.
 139 We select an initial value μ^a and perform the steps listed below to determine a
 140 new value for the attenuation coefficient, μ^b . To determine the thresholds T^a
 141 and T^b we used Otsu's method [41] but other algorithms can be used as well.

- 142 a) Perform a SART reconstruction with N_{it} iterations and apply a smoothing
 143 operation.
- 144 b) Determine a threshold T^a to segment the volume; every voxel with a value
 145 lower than T^a is set to zero and the others are set to μ^a
- 146 c) Use the segmented slice as input for a new SART reconstruction with N_{it}
 147 iterations and apply a smoothing operation.
- 148 d) Determine a new threshold for the sample, T^b .
- 149 e) Determine a new value for the attenuation coefficient of the sample, μ^b .
 150 This is done by selecting the histogram bin with the highest peak value in
 151 the range $[T, h_{max}]$, with h_{max} the highest value in the histogram range.

152 During an iterative reconstruction, the difference between the measured and
 153 the calculated ray sum is first multiplied with a relaxation parameter and then
 154 backprojected. This difference should be minimal. If the initial solution is not
 155 good, the difference will be rather large and the initial solution will be corrected
 156 during this backprojection. As a result, if the initial value μ^a is too small, the
 157 new attenuation coefficient μ^b will be larger than μ^a . If we start with a μ^a which
 158 is too large, the resulting μ^b will be smaller. If the value for μ^a is close to the
 159 real value for μ the difference between μ^a and μ^b will be minimal. Therefore we
 160 propose a new algorithm for determining μ and T , of which the pseudo code is
 161 listed below.

162 This algorithm starts with an initial guess value which is set as central
 163 value μ_c . n is an integer value (typically $n \in [2, 4]$) which is chosen to allow
 164 a variation around μ_c and μ_{step} is a relatively large step value (typically such
 165 that $\frac{\mu_c}{\mu_{step}} \in [5, 10]$) for stepwise changing μ_i , with i the index of the calculation.
 166 The value of μ_c is dependent of the sample, if the initial value of μ_c turns out
 167 to be too small or too large it is changed in line 16 or line 29 of the algorithm
 168 respectively. ϵ_{stop} is a chosen stop value (typically $\epsilon_{stop} \in [0.001, 0.020]$); a
 169 smaller value of ϵ_{stop} will increase the accuracy of the resulting μ and T . Lines
 170 4 until 7 ensure that μ_0^a is positive. In lines 8 until 11 $2n + 1$ values of μ^a and μ^b
 171 are determined. If the conditions in line 14 and 20 are fulfilled the stop criterion

Algorithm 1 Determine μ and T

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1: Initialize  $n, \mu_c, \epsilon_{stop}$  and  $\mu_{step}$ 
2: while Stop criterion is not met do
3:    $\mu_0^a \leftarrow \mu_c - n\mu_{step}$ 
4:   while  $\mu_0^a < 0$  do
5:      $\mu_{step} \leftarrow \frac{\mu_{step}}{2}$ 
6:      $\mu_0^a \leftarrow \mu_c - n\mu_{step}$ 
7:   end while
8:   for  $j \leftarrow 0, 2n$  do
9:     Calculate  $\mu_j^b$  ▷ Using steps a) until e) mentioned above
10:     $\mu_{j+1}^a \leftarrow \mu_j^a + \mu_{step}$ 
11:  end for
12:   $i \leftarrow 0$ 
13:  while (Stop criterion is not met) AND ( $\mu_c$  remains unchanged) do
14:    if ( $\mu_i^b < \mu_i^a$ ) AND ( $\mu_j^b < \mu_j^a$  for each  $j > i$ ) then
15:      if  $i = 0$  then
16:         $\mu_c \leftarrow \mu_0^b$  ▷  $\mu_c$  is changed
17:        if  $\mu_0^b - \mu_0^a < n * \frac{\mu_{step}}{2}$  then
18:           $\mu_{step} \leftarrow \frac{\mu_{step}}{2}$ 
19:        end if
20:      else if  $|\mu_{i-1}^b - \mu_{i-1}^a| < \epsilon_{stop}$  then
21:        stop criterion is met
22:         $T \leftarrow T_{i-1}$ 
23:         $\mu \leftarrow \mu_{i-1}^b$ 
24:      else
25:         $\mu_c \leftarrow \mu_{i-1}^b$  ▷  $\mu_c$  is changed
26:         $\mu_{step} \leftarrow \frac{\mu_{step}}{2}$ 
27:      end if
28:    else if  $i = 2n$  then
29:       $\mu_c \leftarrow \mu_{2n}^b$  ▷  $\mu_c$  is changed
30:      if  $\mu_i^b - \mu_i^a < n * \frac{\mu_{step}}{2}$  then
31:         $\mu_{step} \leftarrow \frac{\mu_{step}}{2}$ 
32:      end if
33:    else
34:       $i \leftarrow i + 1$ 
35:    end if
36:  end while
37: end while

```

is met and the values of μ and T are returned. Every time a new reconstruction is executed the result can be visualized so the user can also visually evaluate the results.

When using this approach, two parameters μ and T are determined based on the evolution of one parameter. This is why we used the evolution around a central value μ_c . Indeed, it can happen that μ_i^a generates a lower μ_i^b only because the threshold is changed to compensate for the difference between the measured and the calculated ray sum and not because the optimal μ is smaller than μ^a . In this case a *local* decrease will occur and in one of the next steps ($j = i + 1, i + 2, ..$) there will be an increase: $\mu_j^b > \mu_j^a$. Thus it is very important to ensure that the selected i for which a decrease occurs indicates a *global* decrease, this means that for each $j > i : \mu_j^b < \mu_j^a$, which is the second condition in line 14.

The EDART method was presented for the reconstruction of samples consisting out of one material and surrounding air, however it can be extended towards multi-material samples. If the attenuation coefficient and threshold are known in advance, the extension is trivial, if this is not the case the method for calculating μ and T needs to be extended as well. For the segmentation a multi-level thresholding technique should be used, for example the one mentioned in [42]. Then each $\mu(i)$ can be calculated by applying the method mentioned above in the interval $[T(i - 1), T(i)]$.

2.4 Samples

EDART and SART reconstructions of three samples with different structures are compared: a toy sample, an aluminum foam sample and a sample of four cookies. Micro-CT scanning was performed using the micro-CT scanner setup at UGCT [43]. This flexible setup allows selecting the optimal scanning parameters (X-ray source, detector, filters...) to maximize resolution and image quality depending on the sample. For all the samples the Feinfocus FXE 160 X-ray tube was used. For the toy and the aluminum foam sample this was combined with a Varian PaxScan 2520V flat panel detector and for the sample of four cookies with a PerkinElmer 1620 CN3 CS flat panel detector. A 3 mm aluminum filter was used for the aluminum foam sample and a 1 mm aluminum filter for the other samples. The X-ray tube was operated at 80 kV for the sample of cookies, 100 kV for the aluminum foam sample and at 120 kV for the toy sample. The projection images were cropped and rebinned, resulting in 1000 projections of 910 x 725 pixels for the toy sample, 1500 projections of 940 x 748 pixels for the aluminum foam sample and 1200 projections of 1019 x 519 pixels for the sample of cookies. The resulting voxel pitch was 34 μm for the toy sample, 67 μm for the aluminum foam sample and 56 μm for the sample of cookies.

A virtual phantom of the toy sample was created as well: the toy sample was reconstructed with the SART algorithm and binarized using a global thresholding algorithm. A virtual attenuation coefficient of 1.0 cm^{-1} was assigned to the material and the 512 projections were simulated with the in house developed Projection Simulator [28]. The advantage of using a simulation is that the

exact solution, the ground truth, is known in advance, which makes it easier to compare different algorithms. Simulations do not suffer from artifacts (for example arising from the polychromaticity of the beam which is not taken into account during the reconstruction). Another advantage is that the attenuation coefficient is known, which also allows testing the above mentioned algorithm for determining the attenuation coefficient.

2.5 Evaluation

The results can be visually evaluated, or in case a ground truth is available, the accuracy of the reconstructions can be quantified using the relative *Number of Misclassified voxels (rNMV)*, which is defined as:

$$rNMV = \frac{MV}{VoI}, \quad (3)$$

with MV the total number of *Misclassified Voxels*, i.e. the total number of voxels that are classified in as being part of a different phase than in ground truth volume and VoI the *Voxels of Interest*, i.e. the total number of voxels belonging to the phase of interest (either the material of the sample or the air) in the ground truth volume.

3 Results and Discussion

In this section the reconstruction results using EDART and SART with different number of projections will be described for three samples: the toy sample, the aluminum foam sample and the cookie sample. At first the reconstruction of the virtual toy phantom will be discussed. In this case the exact solution is known. For the experimental datasets the exact solution is not known, in these cases a SART reconstruction using a large number of projections is binarized with a global thresholding method using Otsu's threshold. This binary solution is considered to be the ground truth.

3.1 Virtual Toy phantom

For the virtual toy phantom we are most interested in the cavity inside the sample, as this is invisible for the outside observer. In Figure 1 the cavity in the phantom is visualized with *VGStudio Max 2.0*. The sample was reconstructed with SART and EDART(known) using 8, 16, 32 and 64 projections with $\mu = 1.0 \text{ cm}^{-1}$ and $T = 0.5 \text{ cm}^{-1}$. In each case the projections were evenly distributed over 360° . Additionally the EDART(calc) algorithm was used to estimate μ and T . In Table 1 the resulting values for the estimated μ and T are shown for the reconstructions using different number of projections N_p . For 8 and 16 projections $N_{it} = 20$ was used and for $N_p = 32$ and $N_p = 64$, $N_{it} = 10$ was used for the EDART reconstructions. To compare reconstructions with the same amount of iterations, $2 \times N_{it}$ iterations were used for the SART reconstructions. The SART reconstructions were segmented by using Otsu's threshold, in

Figure 1: Cavity in the virtual toy phantom. The scale bar indicates the scale of the real sample of which this phantom was created.

Table 1: Calculated μ and T for the virtual toy phantom in the case of different number of projections. The scale bar indicates the scale of the real sample of which this phantom was created.

N_p	μ (cm^{-1})	T (cm^{-1})
8	0.904	0.434
16	0.991	0.475
32	1.007	0.480
64	1.000	0.493

253 order to be able to compare the discrete images. In Figure 2 the $rNMV$ for the
 254 different reconstructions of the cavity is shown; to determine the cavity region,
 255 the same region as in Figure 1 was extracted from the different reconstructions
 256 and the voxels belonging to the air phase were visualized. Figure 3 shows the
 257 EDART(calc) and SART reconstructions of the cavity using 8, 16 and 32 pro-
 258 jections, visualized with *VGStudio Max 2.0*. From Figure 2 it can be observed
 259 that the $rNMV$ is higher for the SART than for the EDART reconstruction.
 260 Especially for the lower number of projections it is clear that EDART performs
 261 better than SART, when more projections are available the difference decreases.
 262 The $rNMV$ of the EDART(calc) reconstructions is slightly higher than that of
 263 the EDART(known) reconstructions, but still lower than the SART reconstruc-
 264 tions. From Figure 3 it is clear that a larger part of the cavity can be retrieved
 265 with the EDART algorithm than with the SART algorithm. Especially more of
 266 the pore voxels at the top of the sample can be identified with EDART. Note
 267 that these pore voxels are the most difficult to identify because they are fewer
 268 in number and surrounded by more material-voxels than the pore voxels at the
 269 bottom of the sample. Additionally, more details can be distinguished with
 270 the EDART algorithm: the inscription of the number 20 at the bottom of the
 271 sample can already be distinguished with the EDART algorithm when only 8
 272 projections are available; for the SART algorithm 32 projections are necessary,
 273 and even then the number is better defined in the EDART reconstruction.

Figure 2: The $rNMV$ of the cavity in the toy phantom for different reconstruction techniques with various number of projections: SART, EDART with known μ and T (EDART(known)), and EDART with the calculated values for μ and T (EDART(calc)), as mentioned in Table 1.

Figure 3: Reconstructed cavity (same region as in Figure 1) in the virtual toy sample in case of SART with $N_p = 8$ (a), $N_p = 16$ (b) and $N_p = 32$ (c), and EDART(calc) with $N_p = 8$ (d), $N_p = 16$ (e) and $N_p = 32$ (f). The scale bar indicates the scale of the real sample of which this phantom was created.

Figure 4: 3D rendering visualized with *VGStudio Max 2.0* (a) and a reconstructed slice (b) of the aluminum foam sample

3.2 Experimental samples

The EDART algorithm was tested on three experimental datasets with different structures. Each sample consists of one material and surrounding air: an aluminum foam sample, a sample of four cookies and a toy sample (of which a phantom was created in the previous section). As a ground truth for these samples a SART reconstruction with 2 iterations using 1000 projections in case of the toy sample, 1500 projections for the aluminum foam sample and 1200 projections for the sample of the four cookies was used. This reconstruction was first smoothed and then binarized with a global threshold algorithm using Otsu's threshold. In Figure 4(a) and 4(b) a 3D rendering visualized with *VGStudio Max 2.0* and a reconstructed slice of the binarized aluminum foam are shown, respectively. Figure 5(a) and 5(b) show a 3D rendering visualized with *VGStudio Max 2.0* and a reconstructed slice of the sample of cookies, respectively; the rectangle indicates a porous region in one of the cookies. For both of these samples the phase of interest is the material phase so this is visualized. For the toy sample we are again most interested in the cavity, the 3D rendering of this cavity is not shown as it is similar to the one shown in Figure 1.

For these samples, μ and T are not known, so the EDART(calc) algorithm is used for the reconstructions. The samples were reconstructed using different number of projections N_p . In Table 2 the number of iterations N_{it} and the resulting μ and T are shown for the different samples reconstructed with different N_p . In order to compare reconstructions with the same number of iterations $2 \times N_{it}$ iterations were used for the SART reconstructions. The SART reconstructions were binarized with Otsu's threshold as global threshold equal to 0.327 cm^{-1} , 0.134 cm^{-1} and 0.097 cm^{-1} for the foam, cookie and toy sample, respectively. The SART reconstruction time for using all available projections was approximately 30 minutes for the sample of cookies and the toy sample and 45 minutes for the aluminum foam sample (exact reconstruction times are not given because they depend on the implementation of the code and the used computational hardware). Reducing the number of projections reduces these reconstruction

Figure 5: 3D rendering visualized with *VGStudio Max 2.0* (a) and a reconstructed slice (b) of the sample of four cookies. The rectangle indicates a porous region in one of the cookies

times as well. The EDART reconstructions resulted in a constant increase of reconstruction time (independent of the number of projections), which varied between 5 and 7 minutes, in comparison with the SART reconstructions, including the time to calculate μ and T . In Figure 6(a), 6(b) and 6(c) the $rNMV$ for the SART and EDART(calc) reconstructions is shown for the aluminum foam sample, the sample of cookies and the cavity in the toy sample, respectively.

A trend can be observed in Table 2: as the number of projections decreases, the calculated values for μ and T decrease as well. This trend is most significant for the sample of cookies. The cookies have a very fine structure with many boundaries, and as the number of projection decreases, more information which is necessary for an accurate reconstruction is missing. As a result, the boundaries of the reconstructed sample will be less well-defined and the boundaries will be smeared out over multiple voxels because of the partial volume effect, which will cause the attenuation coefficient to vary for different voxels. The attenuation coefficient is no longer discrete and the calculated attenuation coefficient will be smaller because of the partial volume effect. Because this sample has many boundaries, this effect is significant.

From Figure 6 it can be observed that the $rNMV$ of the EDART(calc) reconstructions is lower than that of the SART reconstructions in case there are few projections available. If the number of projections increases, the difference in $rNMV$ decreases. For the sample of cookies the SART reconstruction using 600 projections has even a slightly lower $rNMV$ than the EDART(calc) reconstructions. The better results for the EDART(calc) reconstructions in case the number of available projections is limited can also be visually observed. Figure 7 shows the same reconstructed cross section of the aluminum foam sample as that in Figure 4(b) in case of SART using $N_p = 50$ (a), $N_p = 150$ (b) and EDART(calc) using $N_p = 50$ (c) and $N_p = 150$ (d). It can be observed that the material phase is significantly overestimated in case of the SART reconstructions. Figure 8 shows the same reconstructed cross section of the sample of

Table 2: N_{it} , calculated μ and calculated T for the aluminum foam sample, the sample of four cookies and the toy sample in the case of different number of projections N_p

Sample	N_p	N_{it}	μ (cm^{-1})	T (cm^{-1})
Aluminum foam	20	10	0.527	0.257
	50	5	0.623	0.310
	100	5	0.637	0.336
	150	5	0.647	0.349
	250	5	0.636	0.341
	500	3	0.637	0.329
Cookies	10	10	0.151	0.098
	20	10	0.187	0.100
	50	5	0.189	0.098
	100	5	0.211	0.114
	200	5	0.232	0.121
	400	3	0.241	0.133
Toy	600	2	0.243	0.134
	5	10	0.186	0.083
	10	5	0.189	0.089
	20	5	0.196	0.093
	50	5	0.198	0.099
	100	5	0.199	0.099

cookies as that in Figure 5(b) in case of SART using $N_p = 20$ (a), $N_p = 50$ (b) and EDART(calc) using $N_p = 20$ (c) and $N_p = 50$ (d). Again the material phase is overestimated in case of the SART reconstruction. When comparing reconstructions obtained using the same number of projections, the porous regions, for example within the rectangle, are better defined in case of the EDART(calc) reconstruction.

4 Conclusions

In this paper a method for reconstructing experimental 3D datasets obtained with high resolution X-ray tomography, for samples consisting only of one material and surrounding (and enclosed) air was presented. This method is referred to as Experimental Discrete Algebraic Reconstruction Technique (EDART) and focuses on improving reconstruction quality in case few projections are present, without significantly increasing the reconstruction time. Additionally, a fast method to estimate the attenuation coefficient and threshold of the material in case these are unknown was presented. The results of the application of EDART on experimental samples were presented, these illustrate that EDART allows for the improvement of the reconstruction quality in case of limited projections in comparison with iterative reconstruction algorithms such as SART and that the

Figure 6: The $rNMV$ of the aluminum foam sample (a), the sample of cookies (b) and cavity in the toy sample (c) for SART and EDART(calc) reconstructions (μ and T as mentioned in Table 2) using various number of projections.

351 increase in reconstruction time is limited.

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