A Fixed-Angle Heat Spreading Model for Dynamic Thermal Characterization of Rear-Cooled Substrates

Bjorn Vermeersch
Seminar 'Physical Electronics'

● Outline

- ▶ **Introduction**
- ▶ Model definition
- ▶ Exact calculations
- ▶ Results
- ▶ Anisotropic substrates
- ▶ Conclusions

● Introduction

Problem formulation

? Dynamic behaviour

Introduction

Thermal engineering

Introduction

Steady state fixed-angle models

Introduction

How does a fixed-angle model look like?

Introduction

Literature overview

REFERENCE	SPREADING ANGLE	HEAT SOURCE	SUBSTRATE LAYERS
Balents '69	45°	□	single
Cook '76	45°	□ multi	multi
David '77	45°, 32.5°	□	single
Holway '77	45°	○	multi
Zimmer '81	32.5°	□	single
Van Nie '83	geo	▭	single
Dean '85	45°	multi ▭	single
Masana '96	geo	○ ▭ assym. pos.	limited in lateral dir.
Guenin '03	45°	□	single

“The 45° heat spreading angle: an urban legend?”

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Model definition

Fixed-angle heat spreading

Model definition

Electrothermal analogy

Model definition

Equivalent distributed network

Using phasor representation:

$$\begin{cases} -\frac{dQ}{dz} = j\omega c(z) \cdot T(z) \\ -\frac{dT}{dz} = r(z) \cdot Q(z) \end{cases}$$

$$r(z) = \frac{1}{k \cdot A(z)} \quad , \quad c(z) = C_v \cdot A(z)$$

$$A(z) = (a + 2z \cdot \tan \phi)^2$$

Model definition

Differential equation

$$\begin{cases} -\frac{dQ}{dz} = j\omega c(z) \cdot T(z) \\ -\frac{dT}{dz} = r(z) \cdot Q(z) \end{cases} \rightarrow \boxed{\frac{d^2 T}{dz^2} + \frac{2\alpha}{1 + \alpha z} \frac{dT}{dz} - \gamma T(z) = 0} \quad \begin{aligned} \alpha &= \frac{2 \tan \phi}{a} \\ \gamma &= \frac{j\omega C_v}{k} \end{aligned}$$

Substitution: $T(z) = \frac{\Psi(z)}{1 + \alpha z} \longrightarrow \frac{d^2 \Psi}{dz^2} - \gamma \cdot \Psi(z) = 0$

$$\boxed{T(z) = C_1 \frac{\cosh(\sqrt{\gamma} z)}{1 + \alpha z} + C_2 \frac{\sinh(\sqrt{\gamma} z)}{1 + \alpha z}}$$

Boundary conditions: $Q(0) = -\frac{1}{r(0)} \frac{dT}{dz} \Big|_{z=0} = P \quad , \quad T(t_s) = 0$

Model definition

Thermal impedance

$$T(z) = C_1 \frac{\cosh(\sqrt{\gamma}z)}{1 + \alpha z} + C_2 \frac{\sinh(\sqrt{\gamma}z)}{1 + \alpha z} \quad \gamma = \frac{j\omega C_v}{k} \quad , \quad \alpha = \frac{2 \tan \phi}{a}$$

- boundary conditions
- **division by dissipated power P**
- normalization (→ model applicable for any material!)

$$\tilde{Z}_{th}(\omega) = \frac{1}{2\lambda \tan \phi + \sqrt{j\omega} \cdot \coth(\sqrt{j\omega})} \quad Z_0 = \frac{t_s}{ka^2} \quad , \quad \omega_0 = \frac{k}{C_v t_s^2}$$

$$\lambda = \frac{t_s}{a}$$

fitting parameter

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Exact calculations

Fundamental solution for 3D space

Green's function:

$$G(\vec{r}|\vec{r}') = \frac{1}{4\pi k R} \exp\left(-\sqrt{\frac{j\omega C_v}{k}} R\right)$$

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

For distributed heat source:
superposition

$$T(x, y, z) = \iiint_{\text{source}} p(\vec{r}') G(\vec{r}|\vec{r}') d\vec{r}'$$

Exact calculations

Multiple reflection technique

$$T_{\text{source}}(x, y) = 2 \frac{P}{a^2} \int_{-a/2}^{a/2} \int_{-a/2}^{a/2} H(x, x', y, y') dx' dy'$$

average value $\rightarrow Z_{\text{th}}$

$$H = G(z - z' = 0) + 2 \sum_{n=1}^{\infty} (-1)^n G(z - z' = 2nt_s)$$

G = Green's function

Exact calculations

Model validation

- ▶ calculation of sufficient number (N) of impedance points over wide frequency range (logarithmic distribution, 10 points per decade)

- ▶ error function:

$$e(\phi) = \frac{1}{N} \sum_{i=1}^N \frac{|\tilde{Z}_{\text{exact}}^{(i)} - \tilde{Z}_{\text{model}}^{(i)}(\phi)|}{|\tilde{Z}_{\text{exact}}^{(i)}|}$$

(i) = evaluated at i-th frequency

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Results

Model validation: error curves

Results

Optimal spreading angle

$e(\phi_{\text{opt}})$ ranges between 0.8% and 5.4%

Results

Thermal impedance plots (1)

Results

Thermal impedance plots (2)

■ exact calculation
 — heat spreading model with $\phi = \phi_{opt}(\lambda)$

Results

Step response

$$P(f) = \frac{P_0}{2} \delta(f) + \frac{P_0}{j2\pi f}$$

$$T(t) = \int_{-\infty}^{\infty} Z_{th}(f) P(f) \exp(j2\pi f t) df$$

$$\tilde{T}(\tilde{t}) = \frac{1}{2} + \frac{1 + 2\lambda \tan \phi}{\pi} \int_0^{\infty} \text{Im} \left[\frac{\exp(j2\pi \tilde{f} \tilde{t})}{\tilde{f} \left[2\lambda \tan \phi + \sqrt{j\tilde{f}} \coth \left(\sqrt{j\tilde{f}} \right) \right]} \right] d\tilde{f}$$

$$T_0 = \frac{P_0 Z_0}{1 + 2\lambda \tan \phi}$$

$$t_0 = \frac{2\pi C_v t_s^2}{k}$$

Results

Transient heating curves

Results

Thermal rise time

Results

Thermal rise time - denormalized

Results

Global recipe & case study

HEAT SOURCE $300\mu\text{m} \times 300\mu\text{m}$

	Si	GaAs	Al ₂ O ₃	Cu
k [W/mK]	160	50	22	380
C_v [10^6 J/m ³ K]	1.78	1.86	2.98	3.47
k / C_v [mm ² /s]	90	27	7	110

t_s [μm]	rise time [ms]			
50	0.020	0.068	0.246	0.017
100	0.068	0.226	0.821	0.055
150	0.130	0.433	1.575	0.106
300	0.352	1.175	4.277	0.288
500	0.669	2.231	8.125	0.548
1000	1.420	4.77	17.248	1.163

t_s [μm]	thermal resistance [K/W]			
50	2.9	9.3	21.2	1.2
100	4.8	15.3	34.9	2.0
150	6.0	19.2	43.6	2.5
300	7.7	24.7	56.0	3.2
500	8.5	27.2	61.9	3.6
1000	9.2	29.4	66.7	3.9

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Anisotropic substrates

Why?

- Inherent property of certain materials
- Thermal engineering: preferential heat flow

Anisotropic substrates

Practical example

carbon fibres
embedded in
Al or Cu matrix

In plane vs
Z
direction

PHYSICAL & MECHANICAL PROPERTIES, (UNITS)	Al filled composite	Cu filled composite	NOTES
Thermal Conductivity, (W/mK)	260 - 300	300 - 340	In-plane values
Thermal Conductivity, (W/mK)	180 - 200	220 - 250	Through-thickness values

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Anisotropic substrates

Mathematical treatment (1)

- ▶ 3D isotropic heat equation (phasor notation):

$$k\nabla^2 T(x, y, z) - j\omega C_v T(x, y, z) = 0$$

- ▶ if $k_{xy} \neq k_z$:

$$k_{xy} \frac{\partial^2 T}{\partial x^2} + k_{xy} \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} - j\omega C_v T = 0$$

- ▶ coordinate transformation: $\boxed{z' = \beta z}$

$$k_{xy} \frac{\partial^2 T}{\partial x^2} + k_{xy} \frac{\partial^2 T}{\partial y^2} + \beta^2 k_z \frac{\partial^2 T}{\partial z'^2} - j\omega C_v T = 0$$

- ▶ Now choose β such that $\beta^2 k_z = k_{xy}$

Anisotropic substrates

Mathematical treatment (2)

► $\beta = \sqrt{\frac{k_{xy}}{k_z}}$ “anisotropy factor”

$$k_{xy} \nabla^2 T(x, y, z') - j\omega C_v T(x, y, z') = 0$$

► transformation of boundary conditions:

$$\begin{aligned} -k_z \frac{\partial T}{\partial z} \Big|_0 &= \frac{P}{a^2} \quad \Leftrightarrow \quad -k_z \beta \frac{\partial T}{\partial z'} \Big|_0 = \frac{P}{a^2} \\ \Leftrightarrow \quad -\frac{k_{xy}}{\beta^2} \beta \frac{\partial T}{\partial z'} \Big|_0 &= \frac{P}{a^2} \quad \Leftrightarrow \quad -k_{xy} \frac{\partial T}{\partial z'} \Big|_0 = \beta \frac{P}{a^2} \end{aligned}$$

Multiply dissipated power (and hence temperatures) with β

$$\begin{aligned} T(z = t_s) &= 0 \\ \Leftrightarrow \quad T(z' = \beta t_s) &= 0 \end{aligned}$$

Relate temperatures to appropriate location

Anisotropic substrates

Model modification

- ▶ temperature distribution in an anisotropic (k_{xy}, k_z) substrate with thickness t_s
=
 β times the temperature in an isotropic (k_{xy}) substrate with thickness βt_s .
- ▶ (!!!) keep factor β in Z_{th} : related to **original** source

- ▶ Hence:
$$\tilde{Z}'_{th}(\lambda, \phi) = \beta \cdot \tilde{Z}_{th}(\beta\lambda, \phi') \quad \text{with} \quad \phi' = \phi_{opt}(\beta\lambda)$$

$$\tilde{t}'_{rise}(\lambda) = \tilde{t}_{rise}(\beta\lambda)$$

- ▶ (!!!) Don't forget to reinterpret normalization values

Anisotropic substrates

Impact on steady state

$$\blacktriangleright R_{th} = \frac{r_0 t_s}{1 + \alpha t_s} = \frac{t_s}{k a^2} \cdot \frac{1}{1 + 2\lambda \tan \phi} \quad \mathbf{k = k_{xy} = \text{constant}}$$

$$\longrightarrow \boxed{\frac{R'_{th}}{R_{th}} = \frac{R}{R_{iso}} = \beta^2 \cdot \frac{1 + 2\lambda \tan(\phi_{opt}(\lambda))}{1 + 2\beta\lambda \tan(\phi_{opt}(\beta\lambda))}}$$

intuitive expectations (based on 1-D viewpoint)

“ k_z is dominant,
pretend isotropic”

$$R_{th} \div \frac{1}{k} \longrightarrow \boxed{\frac{R}{R_{iso}} = \frac{1/k_z}{1/k_{xy}} = \beta^2}$$

“anisotropic: β times temp.
in isotropic substrate
 k_{xy} but β times thicker”

$$R_{th} \div t_s \longrightarrow \boxed{\frac{R}{R_{iso}} = \beta^2}$$

Anisotropic substrates

R_{th} @ constant k_{xy} : results

Anisotropic substrates

R_{th} @ constant k_{xy} : analysis

THIN SUBSTRATES

β^2 reasonable for R/R_{iso}

bar shaped body with
approx. 1-D heat flow

- small contribution to R_{th} (**GEOMETRY**)
- limited heat spreading (**SMALL ϕ**)

$$\frac{R}{R_{iso}} \approx \frac{\frac{t_s}{k_z a^2}}{\frac{t_s}{k_{xy} a^2}} = \frac{k_{xy}}{k_z} = \beta^2$$

THICK SUBSTRATES

$R/R_{iso} \approx \beta$

"large" surface ($\gg$ source)

$$R = \beta \cdot R_{\text{scaled pyr}} \approx \beta \cdot R_{iso}$$

Anisotropic substrates

Impact on transient behaviour

- ▶ temperature distribution in an anisotropic (k_{xy}, k_z) substrate with thickness t_s

=

β times the temperature in an isotropic (k_{xy}) substrate with thickness βt_s .

no influence on trise: time scale unaltered

$$\longrightarrow \tilde{t}'_{\text{rise}}(\lambda) = \tilde{t}_{\text{rise}}(\beta\lambda) \quad t_0 = \frac{2\pi C_v t_s^2}{k}$$

$$\frac{t'_{\text{rise}}}{t_{\text{rise}}} = \frac{\Delta}{\Delta_{\text{iso}}} = \beta^2 \cdot \frac{0.069 - 0.071 \cdot \frac{1 - 0.687(\beta\lambda)^{-1.086}}{1 + 0.687(\beta\lambda)^{-1.086}}}{0.069 - 0.071 \cdot \frac{1 - 0.687 \cdot \lambda^{-1.086}}{1 + 0.687 \cdot \lambda^{-1.086}}}$$

**$k = k_{xy} =$
constant**

Anisotropic substrates

t_{rise} @ constant k_{xy} : results

Anisotropic substrates

t_{rise} @ constant k_{xy} : analysis (1)

THIN SUBSTRATES: β^2 reasonable for $\Delta/\Delta_{\text{iso}}$

step response for 1-D heat flow

$$\tilde{T} = 1 - \exp(-t / \tau_{\text{equiv}})$$

$$\tau_{\text{equiv}} = R_{\text{equiv}} C_{\text{equiv}} \approx 0.369 \frac{C_v t_s^2}{k}$$

$$t_{\text{rise}} \propto \tau_{\text{equiv}}$$

$$\frac{t'_{\text{rise}}}{t_{\text{rise}}} = \frac{0.369 \frac{C_v t_s^2}{k_z}}{0.369 \frac{C_v t_s^2}{k_{xy}}} = \frac{k_{xy}}{k_z} = \beta^2$$

Anisotropic substrates

t_{rise} @ constant k_{xy} : analysis (2)

- ▶ **OTHER CASES:** physical explanation not straightforward at all
- ▶ thermal diffusion characterized by whole spectrum of time constants (even for 1-D heat flow)
- ▶ trouble:
 - fitting with single exponential impossible if no dominant time constant
 - C_{equiv} not directly proportional to material volume
 - ...

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Conclusions

- ▶ dynamic fixed-angle heat spreading model
- ▶ extends numerous works for steady state
- ▶ frequency domain representation (Z_{th}) & step response
- ▶ error < 5% if appropriate spreading angle chosen
- ▶ simple expressions for ϕ_{opt} & t_{rise}
- ▶ valid for wide range of thicknesses ($\lambda = 0.1 \dots 10$)
- ▶ can be applied for anisotropic materials
- ▶ effect of anisotropy quantified by β + physically explained
- ▶ allows quick yet accurate estimation of time-dependent thermal behaviour

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