Weak heat transfer coefficient dependency of thermal spreading resistance in convectively cooled substrates

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Outline

- Introduction
- Exact calculations
- Approximate model
- Discussion
- Conclusions
Introduction

Problem formulation

- calculation of thermal resistance (maximum temperature used)

\[ R_{th} = \frac{T_{\text{source}}}{P} \]
Introduction

Earlier works in literature (1)


Fig. 3  Bottom-side convection. Nondimensional maximum temperature as a function of substrate thickness $t^*$ and bottom-side Biot number $Bi_1$.

Fig. 4. Spreading resistance for $\Delta x/a = 0.25$, $\Delta y/a = 0.25$. Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates
Introduction

Earlier works in literature (2)

\[ \theta_{\text{max}} = \left\{ \frac{\left( t^* e^{-\lambda t^*}(Bi_1 - \lambda) + (1 + Bi_1 t^*) (e^{-\lambda t^*} - 1) - \frac{\delta}{\phi} \right)}{2 \alpha \beta} \right\} \times \frac{1}{I_0(m) - \frac{m}{M} I_1(m) \frac{I_0(M) K_1(Mb^*) + I_1(Mb^*) K_0(M)}{I_1(M) K_1(Mb^*) - I_1(Mb^*) K_1(M)}} + \frac{\delta}{\phi} \times \left( 1 - \frac{Bi_1}{\alpha} \right) + \frac{1 + Bi_1 t^*}{\alpha} \] (8)

Table 2  Terms appearing in the approximate solution

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>(e^{-\lambda t^<em>}(Bi_1 - \lambda) + \lambda(1 + Bi_1 t^</em>))</td>
</tr>
<tr>
<td>(\beta)</td>
<td>(t^* - t^* \lambda Bi_1/(2 \alpha) + Bi_1(e^{-\lambda t^*} - 1)/\lambda)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>(1 - Bi_1 e^{-\lambda t^<em>}(1 + \lambda t^</em>)/\alpha)</td>
</tr>
<tr>
<td>(\phi)</td>
<td>(Bi_1[1 - Bi_1(\lambda t^* + e^{-\lambda t^*})/\alpha])</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>(\frac{Bi_1 + Bi_2 + Bi_1 Bi_2 t^<em>}{Bi_2 + \lambda(1 - e^{-\lambda t^</em>}) + Bi_1 t^<em>(Bi_2 + \lambda) + Bi_1 e^{-\lambda t^</em>}})</td>
</tr>
<tr>
<td>(\eta)</td>
<td>(t^* + t^* \lambda (Bi_2 - \epsilon(Bi_2 + \lambda))/2 + \epsilon(e^{-\lambda t^*} - 1)/\lambda)</td>
</tr>
<tr>
<td>(\mu)</td>
<td>(Bi_1{1 + t^<em>[Bi_2 - \epsilon(Bi_2 + \lambda)] - \epsilon e^{-\lambda t^</em>}} + Bi_2(1 - \epsilon)/\epsilon)</td>
</tr>
<tr>
<td>(m)</td>
<td>((\phi/\beta)^{1/2})</td>
</tr>
<tr>
<td>(M)</td>
<td>((\mu/\eta)^{1/2})</td>
</tr>
</tbody>
</table>

Introduction

Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates

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**New approach**

- $R_{th}$ vs. normalized **substrate thickness** with Biot number ($Bi = \frac{hd}{k}$) as a parameter

- **approximate solution** accurate but still very **complicated**, with lot of variables

- **thickness not a real design parameter** but determined by technology (e.g. Si: 300µm)

- $R_{th}$ vs. **heat transfer coefficient**

- **simple model** to provide insight
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**Exact calculations**

**Infinite series solution**

- Lee, Song & Moran (ASME conference 1995)

\[
T(r, z) = \frac{p a}{k} \mathcal{E} \left( \frac{1}{\text{Bi}} + \zeta \right) + 2 \sum_{n=1}^{\infty} \frac{J_1(\lambda_n \varepsilon)}{\lambda_n^2 \tau_0(\lambda_n)} \cosh(\lambda_n \zeta) \cdot \frac{\tanh(\lambda_n \zeta) + \lambda_n}{\text{Bi} \tan(\lambda_n \tau)}
\]

where \(\varepsilon = a/b\), \(\tau = t/b\), \(\zeta = z/b\), \(\gamma = r/b\), \(\text{Bi} = \frac{h b}{k}\)


\[
G(r|r'; t) = \frac{\exp\left(-\frac{c_v(x-x')^2+(y-y')^2}{4\pi k t}\right)}{4\pi k t} \sum_{n=1}^{\infty} \left[ \alpha_n \cos(\alpha_n z) + \frac{h}{k} \sin(\alpha_n z) \right] \left[ \alpha_n \cos(\alpha_n d) + \frac{h}{k} \sin(\alpha_n d) \right] \exp\left(-\frac{\alpha_n k t}{k'}\right)
\]

\[
G_{dc}(r|r'; t = 0) = \int_0^\infty G(r|r'; t) dt
\]

steady state:

\[
= \sum_{n=1}^{\infty} C_n K_0\left(\alpha_n \sqrt{(x-x')^2 + (y-y')^2}\right)
\]

\[
T(x, y, z) = \int \int G_{dc}(r|r') dx' dy'
\]

Weak h-dependency of spreading \(R_{th}\) in convectively cooled substrates

\[
b = 10^5 a
\]
Exact calculations

Method comparison

Al$_2$O$_3$ substrate ($k = 22$ W/mK)

Weak $h$-dependency of spreading $R_{th}$ in convectively cooled substrates
Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates

Exact calculations

LTCC [4 W/mK]

heat source
10mm x 10mm

$d = 500\mu m$

$d = 1\text{mm}$

$h [W/m^2K]$

$R_{th} [K/W]$
Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates

Exact calculations

$\text{Al}_2\text{O}_3$ [22 W/mK]

heat source

10mm x 10mm

$d = 600\mu\text{m}$

$d = 2\text{mm}$

$h [\text{W/m}^2\text{K}]$
Exact calculations

Si [160 W/mK]

heat source
10mm x 10mm

Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates
Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates

$Cu \ [380 \ W/mK]$
Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates

**Exact calculations**

**General tendency**

$$R_{th} = A \ln(h) + B$$
Outline

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- **Approximate model**
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Approximate model

Weak \( h \)-dependency of spreading \( R_{th} \) in convectively cooled substrates
Approximate model

Calculations (1)

- **Heat equation**
  \[ \nabla^2 T(\rho) - \frac{h}{kd} T(\rho) = 0 \quad \rho \geq R \]

- **Boundary conditions**
  \[ - (2\pi Rd) \cdot k \frac{dT}{d\rho} \bigg|_{\rho=R} = P \quad , \quad T(\rho \to \infty) \text{ is finite} \]

- **Solution**
  \[ T(\rho) = \frac{P \cdot L}{2\pi k \cdot R \cdot d} \cdot \frac{K_0(\rho / L)}{K_1(\rho / L)} \]

\[ R_{th} = \frac{T(R)}{P} = \frac{1}{2\pi kd} \cdot \frac{K_0\left(\frac{R}{L}\right)}{\frac{R}{L} K_1\left(\frac{R}{L}\right)} \]
Approximate model

Calculations (2)

\[
\frac{R}{L} \ll 1
\]

\[
L = \sqrt{\frac{kd}{h}} \begin{cases} 
R \text{ small} & d \text{ large} \\
\ k \text{ large} & h \text{ small} 
\end{cases}
\]

For small arguments:

\[
K_0(x) \approx \ln(2) - \ln(x) - \gamma + O(x^2)
\]

\[
x \cdot K_1(x) \approx 1 + O(x^2)
\]

\[
R_{th} \approx -\frac{1}{4\pi kd} \ln(h) + \frac{\ln(2) - \ln\left(\frac{R}{\sqrt{kd}}\right) - \gamma}{2\pi kd}
\]

\[
R_{th} \approx A \ln(h) + B
\]
Approximate model

Results (1)

Weak $h$-dependency of spreading $R_{th}$ in convectively cooled substrates

- LTCC - $d = 1\text{mm}$
- AI2O3 - $d = 600\mu\text{m}$
Approximate model

Results (2)

Weak h-dependency of spreading $R_{\text{th}}$ in convectively cooled substrates
Approximate model

Results (3)

Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates
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Model discrepancy

- correct slope but **underestimates** $R_{th}$
- reason: heat source is on top surface, not in substrate
Discussion

Rth vs. h relation

- $R_{th} = f(\ln(h))$: **weak** dependency

**Al2O3, d = 2mm**

*Normalized temperature profiles at bottom*

- Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates
Discussion

A = \frac{1}{4\pi kd} \quad \text{independent of source dimension}

$\text{Si, } d = 300\mu m$

Exact results

Weak h-dependency of spreading $R_{th}$ in convectively cooled substrates
Outline

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Conclusions

- surface heat source on substrate with bottom-side convective cooling
- $R_{th}$ vs. $h$
- for wide variety of $k$ and $d$: $R_{th} \approx A \cdot \ln(h) + B$
  in range of at least 3 decades ($h = 1 – 1000 \text{ W/m}^2\text{K}$)
- explanation with simple model
- weak ln($h$) dependency due to compensation
  (less cooling = more spreading)
Acknowledgements

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