Prediction of the spatial occurrence of fire induced spalling in concrete slabs using random fields

R. Van Coile, P. Criel, R. Caspeele, L. Taerwe

Ghent University, Department of Structural Engineering, Magnel Laboratory for Concrete Research, Ghent, Belgium

Keywords
Fire, concrete slab, tensile strength, uncertainty, probability, random fields

Abstract
As the loss of concrete cover can significantly influence the reliability of concrete elements during fire, spalling should be taken into account when performing reliability calculations. However, the occurrence and spatial variation of spalling are highly uncertain. A first step towards a probabilistic analysis of spalling is made by combining existing deterministic models with a stochastic representation of the concrete tensile strength and by using random fields to model the tensile strength spatial variation.

1. Introduction
During a fire, some types of concrete are known to be susceptible to a sudden detachment of the outer layer if a specific combination of influencing factors occurs. This phenomenon is generally known as explosive spalling. However, for a given reinforced concrete slab configuration, the occurrence of spalling is highly uncertain. It is unclear which percentage of slabs can be expected to suffer from spalling and what may be the spatial extent of the loss of concrete cover. While deterministic calculation tools have been proposed to calculate the occurrence of spalling, these models have not yet been linked to existing probabilistic frameworks (to the best of the authors knowledge). However, this would allow to assess the probability of occurrence and predict spatial variations. The simplified approach proposed in this paper can be considered as a first step towards a probabilistic assessment of fire-induced spalling.

2. Necessity of a probabilistic assessment of spalling
Since performance based design codes and regulations are being implemented by an increasing number of countries, as e.g. the United Kingdom [1] and New Zealand [2], the safety level of structural elements during fire should be assessed to allow for comparing design alternatives. Using a cross-section calculation tool, Van Coile et al. [3] quantified the influence of the concrete cover on the structural reliability $\beta_{\text{fr}}$ during exposure to the ISO 834 standard fire (Fig.1). The differences between the curves can be related to the accelerated heating of the reinforcement in case of a smaller concrete cover. When assuming that the concrete cover is spalling immediately at the beginning of the fire, which is a conservative assumption, the reliability is strongly reduced for very short fire durations (Fig.1).

Clearly, spalling can have a significant impact on the reliability and should be taken into account in a performance-based design. However, assuming the immediate occurrence of
spalling is overly conservative. Therefore, the probability and time of occurrence should be assessed, as well as the spatial variation.

![Figure 1: Reliability $\beta_{f_{i,t}}$ as a function of the ISO 834 duration, for different values of the concrete cover c](image)

### 3. Deterministic model proposed in literature

Spalling is thought to be caused by the combined effect of the buildup of pore pressure during heating due to water evaporation [4] and tensile stresses caused by the temperature-gradient across the concrete cross-section [5]. Hertz performed a comprehensive literature study of different available theories that explain the spalling phenomenon and concluded that the formation of vapour inside the concrete is the most important factor [6].

Bazant and Thonguthai developed a hydrothermal model which calculates the pore pressure and temperature in heated concrete walls taking into account the water released into the pores by dehydration at elevated temperatures [7]. Dwaikat and Kodur proposed a similar model for predicting fire-induced spalling in concrete structures by assuming that spalling occurs when the effective pore pressure in the concrete exceeds the temperature dependent tensile strength of the concrete [8]:

$$nP_v \geq f_{ct,\theta} = k_{f_{ct,\theta}} f_{ct,20^\circ C}$$

with $n$ the concrete porosity, $P_v$ the pore pressure, $k_{f_{ct,\theta}}$ the temperature dependent reduction factor for the concrete tensile strength, and $f_{ct,20^\circ C}$ the 20°C concrete tensile strength.

The simplified model proposed by Dwaikat and Kodur can be used to predict the time of occurrence of spalling. Results are given in Fig.2 for the slab configuration of Tab.1, for different 20°C concrete tensile strength values $f_{ct,20^\circ C}$. It is assumed that spalling is limited to the depth of the concrete cover due to the restraining effect of the reinforcement.

Clearly, the 20°C concrete tensile strength $f_{ct,20^\circ C}$ has a very large influence on the spalling behavior predicted by the model. If the stochastic representation of the tensile strength (see next section) is linked to the results of Fig.2, a first assessment of the probability of spalling can be made.
Figure 2: Spalling behavior for different classes of $f_{ct,20°C}$ as a function of the fire exposure time

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Dimension</th>
<th>Nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>total slab thickness</td>
<td>mm</td>
<td>150</td>
</tr>
<tr>
<td>l</td>
<td>span of the slab</td>
<td>m</td>
<td>6</td>
</tr>
<tr>
<td>b</td>
<td>width of the slab</td>
<td>m</td>
<td>6</td>
</tr>
<tr>
<td>$f_{c,20°C}$</td>
<td>20°C concrete compressive strength</td>
<td>MPa</td>
<td>82.7</td>
</tr>
<tr>
<td></td>
<td>$f_{ct}(20°C) = 60$ MPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>concrete cover</td>
<td>mm</td>
<td>20</td>
</tr>
<tr>
<td>$k_{fc,θ}$</td>
<td>concrete compressive strength reduction factor at temperature θ</td>
<td></td>
<td>temperature dependent [8]</td>
</tr>
<tr>
<td>$ρ_c$</td>
<td>cement content per m³ of concrete</td>
<td>kg/m³</td>
<td>415</td>
</tr>
<tr>
<td>RH</td>
<td>initial relative humidity inside the concrete pores</td>
<td></td>
<td>0.9</td>
</tr>
<tr>
<td>$m_{l0}$</td>
<td>mass of water for saturation of the concrete at 20°C per unit volume of concrete</td>
<td>kg/m³</td>
<td>75</td>
</tr>
<tr>
<td>k</td>
<td>initial intrinsic concrete permeability at 20°C</td>
<td>m²</td>
<td>10-18</td>
</tr>
</tbody>
</table>

Table 1: Characteristics of the investigated slab configuration

4. Assessing the probability of spalling

According to JCSS [9], the 20°C concrete tensile strength $f_{ct,20°C}$ is related to the 20°C compressive strength $f_{c,20°C}$:

$$f_{ct,20°C} = 0.3 \left( f_{c,20°C} \right)^{2/3} \eta_{fct} \tag{2}$$

with $\eta_{fct}$ a stochastic multiplication factor with mean 1 and coefficient of variation 0.3. The compressive strength itself is modeled by a lognormal distribution with mean $f_{ck} + 2\sigma$ and coefficient of variation 0.15 [10].

Using 10000 Monte Carlo simulations, and subdividing $f_{ct,20°C}$ in 8 classes in accordance with Fig.2, the probability of the different curves of Fig.2 is assessed (Tab.2).
<table>
<thead>
<tr>
<th>Class [MPa]</th>
<th>Class number [-]</th>
<th>Class representative value [MPa]</th>
<th>Number of realizations</th>
<th>Percentage of realizations [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0; 2.2]</td>
<td>Class 1</td>
<td>2.1</td>
<td>2</td>
<td>0.02</td>
</tr>
<tr>
<td>[2.2; 2.4]</td>
<td>Class 2</td>
<td>2.3</td>
<td>8</td>
<td>0.08</td>
</tr>
<tr>
<td>[2.4; 2.6]</td>
<td>Class 3</td>
<td>2.5</td>
<td>18</td>
<td>0.18</td>
</tr>
<tr>
<td>[2.6; 2.8]</td>
<td>Class 4</td>
<td>2.7</td>
<td>28</td>
<td>0.28</td>
</tr>
<tr>
<td>[2.8; 3]</td>
<td>Class 5</td>
<td>2.9</td>
<td>49</td>
<td>0.49</td>
</tr>
<tr>
<td>[3; 3.2]</td>
<td>Class 6</td>
<td>3.1</td>
<td>75</td>
<td>0.75</td>
</tr>
<tr>
<td>[3.2; 3.4]</td>
<td>Class 7</td>
<td>3.3</td>
<td>91</td>
<td>0.91</td>
</tr>
<tr>
<td>[3.4; ∞]</td>
<td>Class 8</td>
<td>-</td>
<td>9729</td>
<td>97.29</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>-</td>
<td>10000</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2: Subdivision of realization for $f_{ct,20°C}$ in 8 classes

Based on these considerations, a probability of spalling of 2.7% is determined. As the calculations are based on many simplifying assumptions, this result should be considered with caution. However, it may be useful to compare the probability predicted by the model for different design alternatives. Furthermore, the results of Tab.2 can be used for reliability calculations.

However, when using the results of Tab.2 as input for the cross-sectional reliability tool developed by Van Coile et al. [3], spalling is assumed to occur across the entire exposed surface and the bending capacity of the slab will be underestimated. A first step towards a more realistic consideration of the spatial occurrence of spalling can be made by applying random fields to model the spatial variability of the concrete tensile strength.

5. Assessing the spatial occurrence of spalling using random fields

In concrete slabs, mechanical properties may deviate considerably across the length and width of the element. This spatial variation of mechanical properties can be modeled through random fields. In this paper a random field with an exponential decay of the covariance function is assumed:

$$COV(A, B) = \sigma^2 \exp \left(-\frac{\tau(A, B)}{b}\right)$$  \hspace{1cm} (3)

with A and B two points on the slab, $COV(A, B)$ the covariation between these points, $\tau(A, B)$ the distance between the points and $b$ the correlation length.

In order to avoid unrealistic small fluctuations of the random field, the five first eigenvalues of the correlation matrix are used to define the actual random field.

For every simulated slab the mean concrete tensile strength is determined according to the classification of Tab.2. Subsequently, a Gaussian random field of $f_{ct,20°C}$ is generated assuming a coefficient of variation 0.1, a correlation length of 2 m, and using a discretization of the slab width and length in square elements with size 10 cm².

The following graphs Fig.3 and Fig.4 have been generated for class 6, i.e. for a mean 20°C concrete tensile strength of 3.1 MPa. If this mean value is assumed to be representative for the entire slab, spalling is assumed to occur across the entire slab surface after approximately 56
minutes, up to a depth of 18 mm (Fig.2). However, if the spatial variation of the tensile strength is taken into account, some areas exhibit spalling earlier, while other areas exhibit no spalling at all. Clearly, this will significantly affect the calculated structural behavior.

Figure 3: Example 1 of the spatial occurrence of spalling for a concrete slab with mean $f_{ct,20^\circ C}$ of class 6 (3.1 MPa) for different ISO 834 durations – first realization of the random field

Figure 4: Example 2 of the spatial occurrence of spalling for a concrete slab with mean $f_{ct,20^\circ C}$ of class 6 (3.1 MPa) for different ISO 834 durations – second realization of the random field
Clearly, taking into account the spatial variation of spalling will have a large influence on the subsequent calculations of strength and deflection during fire exposure. Furthermore, it is very interesting to note that every random field realization will result in a different spatial distribution of spalling, e.g. Fig.3 and Fig.4. Consequently, a large number of simulations would be necessary in order to determine the expected area which spalls.

While the proposed method needs to be elaborated further before it can be applied for actual performance-based design, the use of random fields seems promising. If the stochastic variation of crucial parameters as the concrete tensile strength are taken into account, a better estimation can be made of the reliability of concrete elements subject to spalling, and different design alternatives can be compared.

6. Conclusion

- Combining existing deterministic models for spalling with a stochastic representation of the input variables, a first estimation can be made of the probability of spalling and the spalled surface area.
- As an example, the spalling model proposed by Dwaikat and Kodur (2009) is combined with a stochastic representation of the 20°C concrete tensile strength and the probability of spalling is estimated. However, this method assumes the tensile strength to be uniform for the entire slab.
- Subsequently, random fields are used to model the spatial variation of the tensile strength. This allows to visualize random realizations of the spatial occurrence of spalling. The use of random fields appears to be promising.

7. References