(a)

(b)

(c)

Figure 4 Comparison of the characteristics of the aperture-coupled microstrip antenna: (a) return loss; (b) H-plane; (c) E-plane

ment in the radiation patterns of the aperture-coupled microstrip antenna under test was achieved by employing the GA/FSS ground plane.

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ON THE EVALUATION OF SELF-PATCH INTEGRALS IN THE METHOD OF MOMENTS

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Received 8 April 2005

ABSTRACT: Starting with the general expression for the m -dimensional scalar Green's function or its singular part, we transform the $2n$ -dimensional moment integral into an n -dimensional integral in the Fourier domain. This allows us to perform a powerful singularity extraction and to replace the singular Green's kernel by a nonsingular truncated kernel, for which the moment integrals can be evaluated by means of numerical-quadrature methods. Additionally, when utilizing a Galerkin pulse basis, a double Gauss theorem permits us to replace the $2n$ -dimensional moment integral with a $2(n - 1)$ -dimensional boundary integral, again offering dimensionality and computational gain. © 2005 Wiley Periodicals, Inc. *Microwave Opt Technol Lett* 47: 22–26, 2005; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.21070

Key words: Moment Methods; Green's function; Fourier transforms; integration; singular systems

1. INTRODUCTION

The numerical evaluation of multiple integrals with singular kernels arises quite naturally in the method of moments (MoM) solution [1] of electromagnetic-scattering problems. One of the most important of these integrals is the so-called self-term element or self-patch integral, which extracts the near-field interaction of the excitations and sources in the same cell or patch. Most of the methods currently in use perform preliminary calculations to obtain analytic or quasi-analytic expressions for some Green's function-based potentials [2–5], most often by invoking a judicious Gauss theorem [6], while afterwards employing numerical-quadrature techniques to calculate the remaining scalar products [7, 8].

Here we follow another approach. Starting with the general expression for the m -dimensional scalar Green's function or its singular part, we transform the $2n$ -dimensional moment integral into an n -dimensional integral in the Fourier domain. This allows us to perform a powerful singularity extraction and to replace the singular Green's kernel by a nonsingular truncated kernel, for which the moment integrals can be evaluated by means of numerical quadrature methods [9].

As an additional technique, when utilizing a Galerkin pulse basis, a double Gauss theorem permits us to replace the $2n$ -dimensional moment integral with a $2(n - 1)$ -dimensional boundary integral, again offering dimensionality and computational gain. Applied to some pertinent examples, the methods prove highly accurate.

2. METHOD OF MOMENTS

Consider the m -dimensional Green's function $g_m(r; k_0)$ obtained as the fundamental solution of the Helmholtz equation in R^m as follows:

$$\nabla^2 g_m + k_0^2 g_m = \delta(\mathbf{x}) \quad k_0 = \frac{2\pi}{\lambda} \geq 0, \quad (1)$$

with outgoing spherical wave condition, assuming an $e^{-ik_0 c t}$ harmonic time dependence, with c the vacuum speed of light. It is well-known that

$$g_m(r; k_0) = \frac{i}{4} \left(\frac{k_0}{2\pi r} \right)^{(m-2)/2} H_{(m-2)/2}^{(1)}(k_0 r), \quad (2)$$

where $r = \|\mathbf{x}\|$, the Euclidean norm of $\mathbf{x}$, and $H_{(m-2)/2}^{(1)}(\cdot)$ is the Hankel function of the first kind of order $(m-2)/2$. Note that

$$g_1(r; k_0) = \frac{i}{2k_0} e^{ik_0 r} \quad k_0 > 0 \quad (3)$$

which is nonsingular, while $g_2(r; k_0)$ exhibits a logarithmic singularity

$$g_2(r; k_0) \approx -\frac{1}{2\pi} \ln r \quad r \rightarrow 0+ \quad (4)$$

and

$$g_m(r; k_0) \approx \frac{\Gamma(-1+m/2)}{4\pi^{m/2} r^{m-2}} \quad r \rightarrow 0+ \quad m \geq 3. \quad (5)$$

In order to focus on the MoM, we consider the following canonical problem. In the presence of a wavenumber discontinuity with simply connected compact support $\Xi \subset R^n$, with k_0 outside Ξ and k_1 inside Ξ , one needs to solve the Lippmann-Schwinger integral equation:

$$\Phi(\mathbf{x}) = \Phi_0(\mathbf{x}) + (k_0^2 - k_1^2) \int_{\Xi} g_n(\|\mathbf{x} - \mathbf{x}'\|; k_0) \Phi(\mathbf{x}') d\mu(\mathbf{x}') \quad \mathbf{x} \in \Xi \quad (6)$$

Here, $\Phi_0(\mathbf{x})$ is the incoming wave and $\mu(\mathbf{x})$ is the Lebesgue measure in R^n . A typical moment-method [1] procedure with basis functions $\{b_k(\mathbf{x})\}$ and test functions $\{t_k(\mathbf{x})\}$ is to write the unknown $\Phi(\mathbf{x})$ as $\sum_{k=1}^N c_k b_k(\mathbf{x})$ and then test the resulting equations with $t_1(\mathbf{x}), \dots, t_N(\mathbf{x})$, leading to the matrix equation

$$Dc = u + (k_0^2 - k_1^2) Ec, \quad (7)$$

where, for $l, k = 1, \dots, N$,

$$D_{lk} = \int t_l(\mathbf{x}) b_k(\mathbf{x}) d\mu(\mathbf{x}), \quad (8)$$

$$u_l = \int t_l(\mathbf{x}) \Phi_0(\mathbf{x}) d\mu(\mathbf{x}), \quad (9)$$

$$E_{lk} = \iint t_l(\mathbf{x}) g_n(\|\mathbf{x} - \mathbf{x}'\|; k_0) b_k(\mathbf{x}') d\mu(\mathbf{x}) d\mu(\mathbf{x}'). \quad (10)$$

When the supports of $t_l(\mathbf{x})$ and $b_k(\mathbf{x})$ do not overlap, the double integral in the right-hand side of Eq. (10) can be evaluated using standard numerical quadrature procedures [9]. On the other hand, when the supports of $t_l(\mathbf{x})$ and $b_k(\mathbf{x})$ do overlap, the singularity of the Green's function comes into play, and we need to evaluate the double integral differently.

3. SELF-PATCH CALCULATIONS

For the sake of generality, we will take on singular kernels $K(t) = g_m(t; k_0)$, with $m \geq 2$, or their singular parts as defined in Eqs. (4) and (5). In general, we need to evaluate integrals of the kind

$$\mathcal{J}(t, b) = \iint t(\mathbf{x}) K(\|\mathbf{x} - \mathbf{x}'\|) b(\mathbf{x}') d\mu(\mathbf{x}) d\mu(\mathbf{x}'), \quad (11)$$

when $\sup_{\mathbf{x}} |t(\mathbf{x}) b(\mathbf{x})| > 0$. Taking $\rho > 0$, we can write $\mathcal{J}(t, b)$ as

$$\mathcal{J}(t, b) = \mathcal{J}_\rho(t, b) + \mathcal{J}_{out}(t, b), \quad (12)$$

where

$$\mathcal{J}_{out}(t, b) = \iint_{\|\mathbf{x} - \mathbf{x}'\| > \rho} t(\mathbf{x}) K(\|\mathbf{x} - \mathbf{x}'\|) b(\mathbf{x}') d\mu(\mathbf{x}) d\mu(\mathbf{x}') \quad (13)$$

and

$$\mathcal{J}_\rho(t, b) = \iint_{\|\mathbf{x} - \mathbf{x}'\| \leq \rho} t(\mathbf{x}) K(\|\mathbf{x} - \mathbf{x}'\|) b(\mathbf{x}') d\mu(\mathbf{x}) d\mu(\mathbf{x}'). \quad (14)$$

The contribution $\mathcal{J}_{out}(t, b)$ can be calculated using standard numerical quadrature procedures [9], but see also subsection 3.1. The term $\mathcal{J}_\rho(t, b)$ represents the self-patch contribution. It can be written as

$$\mathcal{J}_\rho(t, b) = \int_0^\rho K(p) d \iint t(\mathbf{x}) Y(p - \|\mathbf{x} - \mathbf{x}'\|) b(\mathbf{x}') d\mu(\mathbf{x}) d\mu(\mathbf{x}'), \quad (15)$$

where $Y(t)$ is the Heaviside unit step. We therefore need to evaluate

$$\mathcal{Q}(p, t, b) = \iint t(\mathbf{x}) Y(p - \|\mathbf{x} - \mathbf{x}'\|) b(\mathbf{x}') d\mu(\mathbf{x}) d\mu(\mathbf{x}'). \quad (16)$$

Introducing the Fourier transforms,

$$\tilde{t}(\mathbf{k}) = \int e^{-i\mathbf{k} \cdot \mathbf{x}} t(\mathbf{x}) d\mu(\mathbf{x}), \quad (17)$$

and similarly for $\tilde{b}(\mathbf{k})$, we can write

$$\mathcal{Q}(p, t, b) = (2\pi)^{-2n} \iiint e^{ik \cdot x + ik' \cdot x'} Y(p - \|x - x'\|) \tilde{t}(k) \tilde{b}(k') d\mu(x) d\mu(x') d\mu(k) d\mu(k'). \quad (18)$$

Putting $x' = x + v$ and integrating with respect to x and k' yields

$$\mathcal{Q}(p, t, b) = (2\pi)^{-n} \iint e^{-ik \cdot v} Y(p - \|v\|) \tilde{t}(k) \tilde{b}(-k) d\mu(v) d\mu(k) \quad (19)$$

Now the multiple integral with respect to v in Eq. (19) is known [10, p. 587], that is, we have

$$\begin{aligned} \int_{\|v\| \leq p} e^{-ik \cdot v} Y(p - \|v\|) d\mu(v) &= \int_{\|v\| \leq p} e^{-ik \cdot v} d\mu(v) \\ &= \left(\frac{2\pi p}{\|k\|} \right)^{n/2} J_{n/2}(p\|k\|), \end{aligned} \quad (20)$$

where $J_{n/2}(\cdot)$ is the Bessel function of order $n/2$. In other words, we obtain

$$\mathcal{Q}(p, t, b) = \int \left(\frac{p}{2\pi\|k\|} \right)^{n/2} J_{n/2}(p\|k\|) \tilde{t}(k) \tilde{b}(-k) d\mu(k), \quad (21)$$

and of course we keep in mind that

$$\mathcal{F}_p(t, b) = \int_0^p K(p) d\mathcal{Q}(p, t, b). \quad (22)$$

For $p \rightarrow 0+$ we have, by Parseval's identity and the series development of $J_{n/2}(z)$ in the vicinity of $z = 0$:

$$\mathcal{Q}(p, t, b) \rightarrow V_n(p) \int t(x) b(x) d\mu(x), \quad (23)$$

where $V_n(p)$ is the volume of the n -dimensional Euclidean ball, that is,

$$V_n(p) = \int_{\|x\| \leq p} d\mu(x) = \frac{\pi^{n/2} p^n}{\Gamma(1 + n/2)}. \quad (24)$$

Hence the self-patch contribution is approximately

$$\mathcal{F}_p(t, b) \approx \int t(x) b(x) d\mu(x) \int_0^p K(p) dV_n(p) \quad p \rightarrow 0+. \quad (25)$$

Note that, for expression (25) to exist, we need

$$\lim_{p \rightarrow 0+} K(p) p^n = 0 \quad (26)$$

and also $|\int t(x) b(x) d\mu(x)| < \infty$. Since we tacitly assumed $t(x)$, $b(x)$ in $L_1(R^n)$ —otherwise, the Fourier transform (17) would not exist for sure—by Hölder's inequality it is sufficient that one of $t(x)$, $b(x)$ is in $L_\infty(R^n)$.

3.1. Truncated Kernels

Putting $\langle t|b \rangle = \int t(x) b(x) d\mu(x)$, and for ρ small enough, we have the approximate formula

$$\mathcal{F}(t, b) \approx \mathcal{F}_{out}(t, b) + \langle t|b \rangle \int_0^\rho K(p) dV_n(p). \quad (27)$$

Now $\mathcal{F}_{out}(t, b)$ can be written as

$$\mathcal{F}_{out}(t, b) = \iint t(x) K_\rho(\|x - x'\|) b(x') d\mu(x) d\mu(x'), \quad (28)$$

where

$$K_\rho(t) = \begin{cases} K(t) & t > \rho \\ 0 & t \leq \rho \end{cases}, \quad (29)$$

which is a discontinuous kernel. Since numerical-quadrature methods often rely on the continuity of the integrand, one may expect that the evaluation of $\mathcal{F}_{out}(t, b)$ by means of standard numerical procedures may not be as straightforward as it looks. A better kernel is the truncated kernel

$$K_\rho^1(t) = K(\max(t, \rho)), \quad (30)$$

which is continuous. Defining

$$\mathcal{F}_{out}^1(t, b) = \iint t(x) K_\rho^1(\|x - x'\|) b(x') d\mu(x) d\mu(x'), \quad (31)$$

it is readily seen that

$$\begin{aligned} \mathcal{F}_{out}(t, b) &= \mathcal{F}_{out}^1(t, b) - K(\rho) \mathcal{Q}(\rho, t, b) \approx \mathcal{F}_{out}^1(t, b) \\ &\quad - \langle t|b \rangle K(\rho) V_n(\rho), \end{aligned} \quad (32)$$

and hence a more suitable approximation is

$$\mathcal{F}(t, b) \approx \mathcal{F}_{out}^1(t, b) + \langle t|b \rangle Z(\rho), \quad (33)$$

where $Z(\rho)$ is given by

$$Z(\rho) = \int_0^\rho K(p) dV_n(p) - K(\rho) V_n(\rho). \quad (34)$$

We now calculate $Z(\rho)$ in some important cases. Firstly, take $m = 2$ and take $K(p)$ as in Eq. (4), the singular part of $g_2(p; k_0)$. It is easily shown that

$$Z(\rho) = \frac{\pi^{-1+n/2} \rho^n}{2n\Gamma(1 + n/2)}. \quad (35)$$

Secondly, take $m \geq 3$ and take $K(p)$ as in Eq. (5), the singular part of $g_m(p; k_0)$. We have

$$\begin{aligned} Z(\rho) &= \pi^{(n-m)/2} \rho^{n-m+2} \frac{\Gamma(-1 + m/2)}{2\Gamma(n/2)} \left[\frac{1}{n - m + 2} - \frac{1}{n} \right] \\ &\quad m < n + 2. \end{aligned} \quad (36)$$

Note that $Z(\rho)$ is a monomial in both cases, and that (35) can be obtained from (36) by means of the limit procedure $m \rightarrow 2$.

4. A GALERKIN PULSE BASIS GAUSS THEOREM

Consider the Galerkin pulse basis case $t(\mathbf{x}) = b(\mathbf{x}) = 1$ with compact support $\Omega \subset R^n$ with piecewise smooth boundary $\partial\Omega$. The self-patch integral is

$$\mathcal{G}(\Omega) = \int_{\Omega} \int_{\Omega} K(\|\mathbf{x} - \mathbf{x}'\|) d\mu(\mathbf{x}) d\mu(\mathbf{x}') \quad (37)$$

and $\langle t|b \rangle = \mu(\Omega)$. The $2n$ -dimensional integral (37) can be reduced to a $2(n-1)$ -dimensional integral by means of the following Gauss theorem procedure, borrowed from [6]. Suppose we have two kernels $\check{K}(\|\mathbf{x} - \mathbf{x}'\|)$ and $\hat{K}(\|\mathbf{x} - \mathbf{x}'\|)$ such that

$$\nabla \cdot \{(\mathbf{x} - \mathbf{x}')\check{K}(\|\mathbf{x} - \mathbf{x}'\|)\} = K(\|\mathbf{x} - \mathbf{x}'\|), \quad (38)$$

$$\nabla \hat{K}(\|\mathbf{x} - \mathbf{x}'\|) = (\mathbf{x} - \mathbf{x}')\check{K}(\|\mathbf{x} - \mathbf{x}'\|). \quad (39)$$

Then

$$\mathcal{G}(\Omega) = - \int_{\partial\Omega} \int_{\partial\Omega} \mathbf{n}_{\mathbf{x}} \cdot \mathbf{n}_{\mathbf{x}'} \hat{K}(\|\mathbf{x} - \mathbf{x}'\|) d\nu(\mathbf{x}) d\nu(\mathbf{x}'), \quad (40)$$

where $\mathbf{n}_{\mathbf{x}}$ and $\nu(\mathbf{x})$ are the outward unit normal and Lebesgue measure on $\partial\Omega$, respectively. This follows from applying the Gauss theorem for divergences with respect to $\mathbf{x}$ and afterwards the Gauss theorem for gradients with respect to $\mathbf{x}'$ to the integrand of (37). The kernels $\check{K}(t)$ and $\hat{K}(t)$ are given by [6]:

$$\check{K}(t) = \int_0^1 K(tu) u^{n-1} du, \quad (41)$$

$$\hat{K}(t) = \int^t \check{K}(u) u du. \quad (42)$$

Note that (41) is a definite integral, while (42) is an indefinite integral—one can always add a constant to it. It is easily seen that $\hat{K}(t)$ satisfies the differential equation

$$\frac{d^2 \hat{K}}{dt^2} + \frac{n-1}{t} \frac{d\hat{K}}{dt} = K(t). \quad (43)$$

When $K(t) = t^{\beta}$, $\beta + n > 0$, we obtain

$$\hat{K}(t) = \begin{cases} \frac{t^{\beta+2}}{(\beta+2)(\beta+n)} & \beta \neq -2 \\ \frac{\ln t}{n-2} & \beta = -2 \end{cases}, \quad (44)$$

and when $K(t) = \ln t$, we obtain

$$\hat{K}(t) = \frac{t^2}{4n^2} [-n-2 + 2n \ln t]. \quad (45)$$

Note that $\hat{K}(t)$ is not singular when $\beta > -2$. As a last result we calculate $\widehat{K}_{\rho}^1(t)$, given $\hat{K}(t)$. It is obvious that we can take

Figure 1 The approximations $\mathcal{G}_{app}(\rho)$ and $\mathcal{G}_{out}^1(\Omega)$ as compared to the exact value $3/8\pi$

$$\widehat{K}_{\rho}^1(t) = \frac{K(\rho)t^2}{2n} \quad t \leq \rho. \quad (46)$$

Also, from the differential equation (43), it is seen that

$$\widehat{K}_{\rho}^1(t) = A + Bt^{2-n} + \hat{K}(t) \quad t \geq \rho, \quad (47)$$

where A, B are coefficients to be determined in order that $\widehat{K}_{\rho}^1(t)$ and its first derivative are continuous at $t = \rho$. Note that for $n = 2$, the term t^{2-n} in (47) has to be replaced by $\ln t$. From (46) it follows that, since

$$|\widehat{K}_{\rho}^1(t)| \leq \frac{|K(\rho)|\rho^2}{2n} \quad t \leq \rho, \quad (48)$$

when $\lim_{t \rightarrow 0+} K(t)t^2 = 0$, we do not need even to bother about the truncated $\widehat{K}_{\rho}^1(t)$ procedure; $K(t)$ and $\hat{K}(t)$ alone suffice to calculate the self-patch integral by means of the Gauss theorem procedure.

5. NUMERICAL SIMULATIONS

As a first numerical experiment, we consider $m = 2$, $n = 1$ with logarithmic kernel, $t(x) = x$ and $b(x') = 1$ over the unit interval $\Omega = [0, 1]$. The exact value is

$$\mathcal{G}(\Omega) = -\frac{1}{2\pi} \int_0^1 \int_0^1 x \ln|x-x'| dx dx' = \frac{3}{8\pi}. \quad (49)$$

Since $\langle t|b \rangle = 1/2$, for ρ small enough, we have the approximation [see (35) with $n = 1$]:

$$\mathcal{G}(\Omega) \approx \mathcal{G}_{app}(\rho) = \mathcal{G}_{out}^1(\Omega) + \frac{\rho}{2\pi}. \quad (50)$$

In Figure 1, the approximations $\mathcal{G}_{app}(\rho)$ and $\mathcal{G}_{out}^1(\Omega)$ are compared with the exact value $\mathcal{G}(\Omega)$ for $0 < \rho \leq 0.02$. The numerical calculations were done by means of the adaptive MATLAB® routine `dblquad`. It is seen that the self-patch term $\rho/2\pi$ is really necessary in order to achieve sufficient accuracy.

As a second numerical example, we consider $n = 2$, $m = 3$ with Coulomb kernel and Galerkin pulse basis over the unit square $\Omega = [0, 1]^2$. The exact value is

$$\mathcal{J}(\Omega) = \frac{1}{4\pi} \int_{\Omega} \int_{\Omega} \frac{d\mu(\mathbf{x}) d\mu(\mathbf{x}')}{\|\mathbf{x} - \mathbf{x}'\|} = \frac{1 - \sqrt{2} + 3 \ln(1 + \sqrt{2})}{3\pi} \quad (51)$$

By the formulas in (44), we have $\hat{K}(t) = t/4\pi$, and hence by result (40), we have

$$\mathcal{J}(\Omega) = -\frac{1}{4\pi} \int_{\partial\Omega} \int_{\partial\Omega} \mathbf{n}_{\mathbf{x}} \cdot \mathbf{n}_{\mathbf{x}'} \|\mathbf{x} - \mathbf{x}'\| d\nu(\mathbf{x}) d\nu(\mathbf{x}'). \quad (52)$$

This can be written as

$$\mathcal{J}(\Omega) = \frac{1}{\pi} \int_0^1 \int_0^1 [\sqrt{1 + (x - x')^2} - |x - x'|] dx dx'. \quad (53)$$

Evaluating (53) numerically with `dblquad`, we have obtained $\mathcal{J}(\Omega) = 0.236600507\dots$ as compared to the exact $\mathcal{J}(\Omega) = 0.236600502\dots$

As a third numerical simulation, we consider $n = m = 2$ with a logarithmic kernel and Galerkin pulse basis over the unit square $\Omega = [0, 1]^2$. The exact value is

$$\begin{aligned} \mathcal{J}(\Omega) &= -\frac{1}{2\pi} \int_{\Omega} \int_{\Omega} \ln(\|\mathbf{x} - \mathbf{x}'\|) d\mu(\mathbf{x}) d\mu(\mathbf{x}') \\ &= -\frac{1}{24\pi} [4\pi - 25 + 4 \ln 2]. \end{aligned} \quad (54)$$

By formula (45), we have

$$\hat{K}(t) = \frac{t^2}{8\pi} [1 - \ln t], \quad (55)$$

and hence by result (40), we can calculate the integral on a straightforward basis. We have, with $\ell(t) = t^2(1 - \ln t)$,

$$\mathcal{J}(\Omega) = \frac{1}{2\pi} \int_0^1 \int_0^1 [\ell(\sqrt{1 + (x - x')^2}) - \ell(|x - x'|)] dx dx'. \quad (56)$$

Evaluating (56) numerically with `dblquad`, we have obtained $\mathcal{J}(\Omega) = 0.12813349\dots$ as compared to the exact $\mathcal{J}(\Omega) = 0.12813353\dots$

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CHARACTERIZATION AND APPLICATIONS OF A COMPACT CPW DEFECTED GROUND STRUCTURE

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ABSTRACT: A compact coplanar waveguide (CPW) defected ground structure is investigated. Its frequency characteristics are modeled by full-wave EM simulations. Some applications are investigated, including two electronic bandgap (EBG) devices which are constructed based on a periodical structure of the proposed defected ground without and with loading of lumped-element capacitors. Also, a varactor-tuned defected ground structure of this type has been demonstrated. The theoretical, simulated, and experimental results are presented. © 2005 Wiley Periodicals, Inc. *Microwave Opt Technol Lett* 47: 26–31, 2005; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.21071

Key words: coplanar waveguide (CPW); electronic bandgap (EBG) structures; resonant elements; bandstop filter

1. INTRODUCTION

The rapid growth in the field of wireless communications or RF/microwave communications demands new microwave frequency-selective devices or filters to meet the requirements of small size, light weight, low cost, and tunability. To this end, there is an increasing interest in developing microwave components using coplanar waveguide structures. The main advantages of CPW technology are: (i) a required single metal level, which eases the shunt connections to ground and allows further flexibility (as compared to microstrip technology) for the design of EBG-CPW structures; (ii) integration with active devices. Since coplanar waveguides are also capable for technologies such as microwave monolithic integrated circuits (MMIC), micro-electro-mechanical systems (MEMS), it would be desirable to develop CPW filters [1–6].