Modeling of the Resistive and Inductive Behaviour of Layered and Coated Conductors

Thomas Demeester and Daniel De Zutter (Fellow IEEE)

Dept. of Information Technology

Ghent University, Belgium

Email: thomas.demeester@intec.ugent.be, daniel.dezutter@intec.ugent.be

Abstract

On-chip conductors have a finite thickness and are often composed of different metals. An efficient method to model the broadband resistive and inductive behaviour of such conductors is derived and applied to a few illustrative configurations.

I. INTRODUCTION

In modern high-frequency interconnect structures, the broadband transmission line characteristics are very important for signal integrity. The frequency-dependent resistive and inductive behaviour crucially depends on the material properties and the shape of the conductors as a consequence of current crowding effects at the higher frequencies. The capacitive behaviour is determined by the shape of the conductors and the properties of the dielectric they are embedded in. In this contribution the surface admittance concept as introduced in [1] is used to model thick conductors composed of homogeneous parts with different conductivity. Both coated and layered conductors are considered. A particular advantage of the proposed Dirichlet to Neumann (DtN) technique is the combination of a boundary integral equation and a surface admittance description which exactly accounts for the presence of the finite conductors, yielding correct results up to frequencies at which the skin depth in the conductor is much smaller than its cross-sectional dimensions. This is computationally more difficult to achieve with a volume discretization as e.g. in [2].

II. INDUCTANCE MODEL

The considered configurations are invariant in the longitudinal z -direction and non-magnetic. They consist of a number of composite conductors embedded in a dielectric substrate. A $e^{-j\beta z + j\omega t}$ dependence is assumed for the fields, which are governed by the quasi-TM equations (see e.g. [2] and [3]). For each fundamental mode the transmission line characteristics are described by the telegrapher equations

$$j\beta \mathbf{V} = j\omega \tilde{\mathbf{L}} \mathbf{I} = (\mathbf{R} + j\omega \mathbf{L}) \mathbf{I} \quad \text{and} \quad j\beta \mathbf{I} = j\omega \tilde{\mathbf{C}} \mathbf{V} = (\mathbf{G} + j\omega \mathbf{C}) \mathbf{V}, \quad (1)$$

in which $\mathbf{V}$ and $\mathbf{I}$ denote the circuit voltages and currents, and with $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{C}}$ the complex inductance and capacitance matrices. For each composite conductor p , the electric potential ϕ is a constant V_p on its outer boundary and satisfies the diffusion equation

$$\nabla_{xy}^2 \phi = j\omega \mu_0 \sigma_i \phi \quad (2)$$

inside each homogeneous subregion S_i with conductivity σ_i . On the inner boundaries between different metals, ϕ is therefore not equal to V_p . Hence, the theory of [1] cannot readily be extended to composite conductors. For a rigorous description, an equivalent problem (denoted with the subscript 'eq') needs to be formulated, replacing all materials by volume sources in free space. The dielectric substrate of the conductors has no influence on the inductance problem and is therefore no longer considered. We first express the longitudinal electric field E_z in terms of the scalar potential ϕ_{eq} and the vector potential $\mathbf{A}_{eq}$ of the equivalent problem as

$$E_z = j\beta \phi_{eq} - j\omega A_{z,eq}. \quad (3)$$

A careful analysis, the details of which will not be presented here, shows that all fields in the equivalent free space formulation are identical to those in the original problem (as should be). This is also the case for the scalar and vector potential *outside* the conductors. However, *inside* each conductor ϕ_{eq} satisfies Laplace's equation and hence

$\phi_{eq} = V_p$ throughout the cross-section of conductor p . This is in particular the case at each metal-metal boundary inside conductor p .

In the same way as in [1], we now introduce an equivalent surface current J_{si} on the boundary c_i of each homogeneous subregion S_i of conductor p , such that

$$J_{si}(\mathbf{r}) = \mathcal{Y}_i E_z(\mathbf{r}) = \frac{1}{j\omega\mu_0} [\mathcal{D} - \mathcal{D}_0]_i \cdot E_z(\mathbf{r}), \quad \mathbf{r} \in c_i, \quad (4)$$

with $\mathcal{Y}_i$ the differential surface admittance operator as in [1] for the considered area S_i . The DtN operator $\mathcal{D}$ maps the boundary value $E_z(\mathbf{r} \in c_i)$ to the actual normal derivative of E_z at the boundary c_i , whereas $\mathcal{D}_0$ maps the same boundary value to the normal derivative that would be obtained if the conducting material were replaced by free space. The, as yet unknown, surface currents J_{si} exactly describe the behaviour of the conductors over the complete frequency range and radiate in free space. By expressing $A_{z,eq}$ via the introduced surface current sources, in terms of E_z , (3) leads to the following integral equation on each boundary c_i

$$J_{si}(\mathbf{r}) = j\beta\mathcal{Y}_i V_p + j\omega\mathcal{Y}_i \sum_{\forall c_j} \oint_{c_j} G(\mathbf{r}|\hat{\mathbf{r}}) J_{sj}(\hat{\mathbf{r}}) d\hat{\mathbf{c}}, \quad \mathbf{r} \in c_i. \quad (5)$$

$G(\mathbf{r}|\hat{\mathbf{r}}) = \mu_0/(2\pi) \ln |\mathbf{r} - \hat{\mathbf{r}}|$ and the summation in (5) runs over the complete set of boundaries of all the conductors in the configuration. Eqn. (5) pertains to conductor p . A similar integral equation applies to the other conductors. Applying the Method of Moments, discretization of (5) leads to the numerical values for $J_{si}/j\beta$. Although β is unknown, it will soon become clear that this does not present a problem.

Next, we turn to the determination of the inductance matrix $\tilde{\mathbf{L}}$. Suppose that the potential at the surface of conductor p is V_p while all other conductors are at zero potential. Solution of the appropriate set of integral equations (5) yields the corresponding $J_{si}/j\beta$ values for conductor p as well as for any other conductor q . In a similar way as in [1], integrating the diffusion equation

$$\nabla_{xy}^2 E_z(\mathbf{r}) = j\omega\mu_0\sigma_i E_z(\mathbf{r}) \quad (6)$$

over each part of a composite conductor while taking into account (4), shows that the total volume current I_p flowing through conductor p is equal to $\sum_{\forall c_i} (\oint_{c_i} J_{si} dc)$, i.e. the sum of all equivalent surface currents introduced for that particular conductor. The same reasoning of course applies to I_q . Finally, from the first Telegrapher equation in (1), we immediately deduce that the elements of the p 'th column of $\tilde{\mathbf{L}}^{-1}$ are given by

$$[\tilde{\mathbf{L}}^{-1}]_{qp} = \frac{j\omega}{j\beta V_p} I_p \quad (7)$$

through which the complex $\tilde{\mathbf{L}}^{-1}$ -matrix can be determined and hence the resistance and inductance matrices $\mathbf{R}$ and $\mathbf{L}$. Note that in the above equation only the knowledge of I_p/β is required and hence knowledge of all J_{si}/β suffices.

To complete the transmission line analysis, the capacitance matrix must be found. As we here restrict ourselves to dielectric substrates, the capacitance matrix is readily obtained from a classical MoM-formulation for the surface charges on the constant potential surfaces of the conductors.

III. RESULTS

A. Example 1 : layered conductors

As a first application the differential microstrip pair shown in Fig. 1 is modeled. Both conductors consist of three horizontal layers, of which the middle one (chromium) has a lower conductivity than the bottom and top layer (copper). The copper layers have a thickness of $2\mu\text{m}$, whereas the chromium layer is only $1\mu\text{m}$ thick. The distance between the lines is chosen equal to its width, i.e. $5\mu\text{m}$. The conductors are placed on top of an infinite $8\mu\text{m}$ thick dielectric substrate ($\epsilon_r = 10.2$) residing on a perfectly electrically conducting ground plane. In Fig. 2, the elements of the resistance and inductance matrix are shown, as well as the effective relative permittivity $\epsilon_{r_{eff}}$ and the attenuation factor α for both the odd and the even mode ($\epsilon_{r_{eff}}$ and α are defined by $\beta = k_0 \sqrt{\epsilon_{r_{eff}}} - j\alpha$, with k_0 the free space wave number). The results for the configuration of Fig. 1 are compared to those obtained for homogeneous copper conductors.

Fig. 1. Symmetrical pair with layered conductors (copper - chromium - copper). All dimensions are in micrometres.

Fig. 2. Inductance (nH/m) and resistance (Ω/m) matrix elements for the configuration of Fig. 1 are shown in the left graph. The corresponding effective relative permittivity (ϵ_{eff}) and attenuation (dB/mm) for the odd and the even propagation mode are shown on the right. The simulation results for the layered conductor ('Cu - Cr - Cu') are shown together with those for homogeneous conductors ('homog. Cu')

B. Example 2 : coated conductors

Consider the transmission line structure shown in Fig. 3, taken from [4]. It consists of three signal conductors (numbered 1 to 3) that are placed between two larger reference conductors. Each of the signal conductors (with dimensions of $238 \text{ nm} \times 500 \text{ nm}$) consists of copper, with a 10 nm thick chromium coating covering all sides except for the top side. In Fig. 4, some elements of the inductance matrix L and the resistance matrix R are shown. The simulation results for the coated signal lines (denoted as 'coated Cr/Cu') are compared to two situations with homogeneous signal conductors, on the one hand full copper signal lines (denoted with 'homog. Cu'), and on the other hand full chromium signal lines ('homog. Cr'). The reference conductors remain unchanged (i.e. copper). The inductance matrix L is entirely determined by L_{11} , L_{12} , L_{13} and L_{22} , because of reciprocity and the symmetry of the configuration. From those elements, only L_{11} is not shown, as it is quite similar to L_{22} . The corresponding resistance results are shown on the right of Fig. 4. For the highest shown frequencies (approaching the limit of validity of the quasi-TM approximation), the skin depth in the coated lines becomes very small ($\delta_{Cu} = 6.6 \text{ nm}$, $\delta_{Cr} = 18.1 \text{ nm}$ at 100 THz), and a large fraction of the current flows through the less conducting coating. For an increasing frequency, the resistance R_{22} increases therefore more rapidly for the coated conductors than for the homogeneous copper conductors. Note that R_{13} for 'homog. Cr' and 'coated Cr/Cu' becomes negative from a certain frequency onwards, and is from there onwards not shown on these logarithmic curves. The total resistance matrix however remains positive-definite, as required for a lossy structure.

Fig. 3. Transmission line system with two copper reference conductors and three coated signal conductors (inside: Cu with $\sigma_{Cu} = 58.8$ MS; coating: Cr with $\sigma_{Cr} = 7.75$ MS). The dimensions are: $w = s = 238$, $t = t' = 500$, $h = 450$, $w' = 3238$, $\delta = 10$, all in nanometres.

Fig. 4. Inductance (left) in nH/m and resistance (right) in Ω/m for the configuration of Fig. 3, denoted with 'coated Cr/Cu'. Results for analogue configurations 'homog. Cu' and 'homog. Cr' for resp. homogeneous copper and chromium signal lines are shown as well.

ACKNOWLEDGMENT

This work was supported by a grant from the Research Foundation - Flanders (FWO).

REFERENCES

- [1] D. De Zutter and L. Knockaert, "Skin effect modeling based on a differential surface admittance operator," *IEEE Trans. Microw. Theory Tech.*, vol. 53, no. 8, pp. 2526–2538, 2005.
- [2] G. Plaza, R. Marques, and F. Medina, "Quasi-TM MoL/MoM approach for computing the transmission-line parameters of lossy lines," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 1, pp. 198–209, 2006.
- [3] J. Aguilera, R. Marques, and M. Horno, "Improved quasi-static spectral domain analysis of microstrip lines on high-conductivity insulator-semiconductor substrates," *IEEE Microwave and Guided Wave Letters*, vol. 9, no. 2, pp. 57–59, 1999.
- [4] "Compact modeling of on-chip passive structures at high frequencies," IST-2001-34058 project.

The papers in this book comprise the digest of the meeting mentioned on the cover and title page. They reflect the authors' opinions and are published as presented and without change, in the interest of timely dissemination. Their inclusion in this publication does not necessarily constitute endorsement by the editors, the Institute of Electrical and Electronics Engineers, Inc.

Copyright and Reprint Permissions: Abstracting is permitted with credit to the source. Libraries are permitted to photocopy beyond the limits of U. S. copyright law for private use of patrons those articles in this volume that carry a code at the bottom of the first page, provided the per-copy fee indicated in the code is paid through the Copyright Clearance Center, 27 Congress Street, Salem, MA 01970. Instructors are permitted to photocopy isolated articles for noncommercial classroom use without fee. For other copying, reprint or republication permission, write to IEEE, Copyrights Manager, IEEE Service Center, 445 Hoes Lane, P. O. Box 1331, Piscataway, NJ 08855-1331. All rights reserved. Copyright by The Institute of Electrical and Electronics Engineers. Inc.

IEEE Catalog Number: 07TH8937C

ISBN: 0-4244-0883-0

Library of Congress: 2006939756

IEEE
16th Topical Meeting
on
**Electrical Performance
of Electronic Packaging**

October 29–31, 2007

Renaissance Atlanta Downtown Hotel • Atlanta, Georgia

Sponsored by:

The IEEE Microwave Theory and Techniques Society

and

**The IEEE Components, Packaging and Manufacturing
Technology Society**

IEEE #07TH8937