GLOBAL RESISTANCE FACTOR FOR CONCRETE SLABS EXPOSED TO FIRE

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Abstract. In this paper a safety format is proposed to be applied when using Finite Element Models (FEM) to calculate the structural response of concrete slabs during fire. It is suggested to divide the structural resistance obtained with a FEM analysis by a global partial factor. While maintaining a high level of simplicity, the proposed method takes into account the consequence class of the structure and the fire incidence rate, and allows for the incorporation of other fire protection measures.

1 INTRODUCTION

Non-linear FEM analyses have become increasingly important for the assessment of new and existing concrete structures. Since this type of analysis takes into account the structural interaction between different sections and possible stress redistributions, a more economic design can be obtained. However, there are no clear prescriptions in the current codes on which safety format should be used for a design based on non-linear FEM analysis. Especially for concrete structures exposed to fire a non-linear FEM analysis can be of great value and an adequate safety format should be proposed.

2 A SAFETY FORMAT FOR NON-LINEAR FINITE ELEMENT ANALYSIS

Different possible safety formats for non-linear analysis are listed by Cervenka [1]. It is concluded that the standard partial safety factor method (PSFM) cannot be applied to non-linear FEM calculations since the use of the extremely low design values for material properties may alter the structural response calculated by the non-linear FEM analysis. Furthermore, the variability of the concrete cover has an important influence on the obtained safety level of concrete elements exposed to fire due to its effect on the reinforcement temperature [2]. These effects cannot be accounted for directly with the PSFM.

It is concluded that the global resistance factor (GRF) is the most promising safety format to be used for non-linear fire design of concrete structures. In this approach a single non-linear analysis is performed using mean values for the material characteristics and geometrical properties. Subsequently the calculated resistance $\mu_R$ is divided by a global resistance factor $\gamma_R$ to derive the design value for the structural resistance $R_d$ (1), [4].
This paper derives a global resistance factor $\gamma_R$ to be used when calculating the bending moment capacity of concrete slabs exposed to fire.

3 THE GLOBAL RESISTANCE FACTOR

In accordance with EN 1990 [7], the design value for the structural resistance is defined by (2), with $\Phi$ the standardized cumulative normal distribution, $\alpha_R$ the sensitivity factor of the resistance and $\beta$ the reliability index.

$$P[R \leq R_d] = \Phi(-\alpha_R \beta)$$

Assuming a lognormal distribution for the structural resistance $R$, (3) is derived from (2), with $\nu_R$ the coefficient of variation of the resistance.

$$R_d = \mu_R \exp(-\alpha_R \nu_R \beta)$$

If the resistance follows a normal distribution, $R_d$ is defined by (4).

$$R_d = \mu_R (1 - \alpha_R \nu_R \beta)$$

Combining equations (1) and (3) yields (5). This equation was used by Holický to demonstrate the large variability of $\gamma_R$ for concrete elements at normal temperatures [5].

$$\gamma_R = \exp(\alpha_R \nu_R \beta)$$

If the resistance $R$ follows a normal distribution, equation (6) should be used.

$$\gamma_R = \frac{1}{1 - \alpha_R \nu_R \beta}$$

It is clear from these equations that $\gamma_R$ depends on the target value of the reliability index $\beta$ and the coefficient of variation $\nu_R$ of the resistance effect.

4 THE TARGET RELIABILITY INDEX $\beta$ IN CASE OF FIRE

For the reliability index $\beta$ in equations (5) and (6), the target reliability index $\beta_{t,EN1990}$ for structural elements exposed to fire (considering a reference period of 1 year) is defined by (7) with $P_{f,EN1990}$ the maximum allowable probability of structural failure during fire (i.e. at elevated temperatures), $P_{f,EN1990}$ the annual probability of structural failure during normal design conditions (i.e. at 20°C), $p_f$ the annual probability that the structure is exposed to a fully developed fire that threatens structural integrity and $\beta_{t,EN1990}$ the target reliability index for structures in normal conditions for a one year reference period [9].

$$P_{f,\beta} = \frac{P_{f,EN1990}}{p_f} = \frac{\Phi(-\beta_{t,EN1990})}{p_f} = \Phi(-\beta_{t,EN1990})$$

This safety concept was developed by Weilert and Albrecht [9] and is now incorporated in the German code [10]. An English summary of the concepts and calculations is provided in [11].

Although it is demonstrated by Holický and Retief that the tabulated values for $\beta_{t,EN1990}$ are often a crude simplification and the actual target reliability index for a construction should be calculated based...
on the design working life, an appropriate discount rate and the consequences of a structural failure [8], the values prescribed by EN 1990 will be accepted in this paper in order to maintain full compatibility with the Eurocodes.

The annual probability $p_{fi}$ that the structure is exposed to a fire that threatens structural integrity is calculated by equation (8), taking into account the annual probability of fire initiation $p_1$, the probability that the fire is not extinguished by the users of the structure $p_2$, nor by the fire brigade $p_3$, and the probability of failure of the sprinkler system $p_4$. A table with values for $p_1$ is suggested in [12]. Other fire mitigation measures can easily be included in the calculation.

$$p_{fi} = p_1 p_2 p_3 p_4$$  \hspace{1cm} (8)

The calculation procedure is illustrated by Table 1 for different types of buildings and for different consequence classes RC [7]. For a warehouse the value of $p_{fi}$ is largely dependent on the compartmentation. In Table 1, a warehouse compartment of 1000 m² is assumed. The calculations are performed for buildings where no sprinkler system is available (i.e. $p_4 = 1$).

Table 1. Calculation of $\beta_{t,fi}$ for different building with different use and consequence class

<table>
<thead>
<tr>
<th>Example building</th>
<th>Consequence Class</th>
<th>$\beta_{\text{EN1990}}$ [-]</th>
<th>$p_{fi}$ [-]</th>
<th>$\beta_{t,fi}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hospital</td>
<td>RC3</td>
<td>5.2</td>
<td>1.12·10⁻⁴</td>
<td>3.12</td>
</tr>
<tr>
<td>Office building</td>
<td>RC2</td>
<td>4.7</td>
<td>4.2·10⁻⁵</td>
<td>1.87</td>
</tr>
<tr>
<td>Residential building</td>
<td>RC2</td>
<td>4.7</td>
<td>9.4·10⁻⁵</td>
<td>2.20</td>
</tr>
<tr>
<td>Production hall</td>
<td>RC2</td>
<td>4.7</td>
<td>1.28·10⁻⁴</td>
<td>2.32</td>
</tr>
<tr>
<td>Warehouse</td>
<td>RC1</td>
<td>4.2</td>
<td>1.4·10⁻³</td>
<td>2.34</td>
</tr>
</tbody>
</table>

For practical use $\beta_{t,fi}$ is illustrated in Figure 1 as a function of the consequence class and the probability of a fully developed fire $p_{fi}$.

![Figure 1](image-url)
5 CALCULATION OF THE GLOBAL RESISTANCE FACTOR IN CASE OF CONCRETE SLABS

The sensitivity factor $\alpha_R$ for the resistance effect can generally be approximated by $0.8$ [13]. The evaluation of the global resistance factor $\gamma_R$ through equations (5) and (6) requires an assessment of the coefficient of variation $V_R$ of the bending moment capacity of concrete slabs during fire. The computational effort of calculating the mean and standard deviation of $R$ through a probabilistic finite element analysis (PFEA) makes PFEA inefficient for many practical design situations. One can however assess the mean $\mu_R$ and standard deviation $V_R$ of the slab configuration through Monte Carlo simulations using a simplified full-probabilistic model, adapted from [14].

The mechanical strain $\varepsilon_{\sigma,\theta}$ at temperature $\theta$ is calculated by (9), adapted from [15], with $\varepsilon_{0,\theta}$, the free thermal elongation and $\varepsilon_{tot,\theta}$, the total cross section deformation. For slabs the influence of transient strains can be neglected [16].

$$\varepsilon_{\sigma,\theta} = \varepsilon_{tot,\theta} - \varepsilon_{0,\theta} \quad (9)$$

Mechanical material properties of concrete and reinforcement are applied in accordance with EN 1992-1-2 [17], but taking into account an additional uncertainty with respect to the reduction of the mechanical properties at elevated temperatures as explained in [14]. The actual calculation of the bending moment capacity is based on the same assumptions as made by Kodur and Dwaikat [18]:

1. Plane sections remain plane (Euler-Bernoulli hypothesis)
2. Bond slip between concrete and reinforcement is neglected
3. Spalling is neglected

No model uncertainty was taken into account, as it isn’t yet clear which model uncertainty would be appropriate for non-linear FEM analysis for concrete elements exposed to fire and how these would need to be incorporated.

For a concrete slab with nominal properties according to Table 2, the calculated evolution and scatter of the bending moment capacity during fire are visualized in Figure 2.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Dimension</th>
<th>Nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>thickness</td>
<td>mm</td>
<td>200</td>
</tr>
<tr>
<td>$f_{ck}(20^\circ C)$</td>
<td>20°C characteristic concrete compressive strength</td>
<td>MPa</td>
<td>20</td>
</tr>
<tr>
<td>$f_{yk}(20^\circ C)$</td>
<td>20°C characteristic steel yield strength</td>
<td>MPa</td>
<td>500</td>
</tr>
<tr>
<td>$E_c(20^\circ C)$</td>
<td>20°C concrete modulus of elasticity</td>
<td>GPa</td>
<td>28.8</td>
</tr>
<tr>
<td>$E_s(20^\circ C)$</td>
<td>20°C steel modulus of elasticity</td>
<td>GPa</td>
<td>200</td>
</tr>
<tr>
<td>$c_{nominal}$</td>
<td>concrete cover</td>
<td>mm</td>
<td>35</td>
</tr>
<tr>
<td>$\phi$</td>
<td>reinforcement diameter</td>
<td>mm</td>
<td>10</td>
</tr>
<tr>
<td>$s$</td>
<td>bar spacing bottom reinforcement</td>
<td>mm</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 2. Calculated bending moment capacity $M_{R,fi,t}$ and coefficient of variation $V_R$ as a function of the time of fire exposure.

It is clear that after 90 minutes of exposure to the standard fire, both the mean value $\mu_R$ and the standard deviation of the bending moment capacity decrease, while the coefficient of variation $V_R$ increases.

The Monte Carlo simulations allow the visualization of the evolution of the distribution of the bending moment capacity during fire. Figure 3 shows a comparison between the observed histogram ‘A’ based on the simulations and the lognormal approximation ‘LN’ at different durations of exposure to the ISO 834 standard fire curve.

The lognormal approximation is found to result in slightly higher values compared to the simulations. However, after a long fire exposure time, a normal approximation would result in overly conservative values for the global resistance factor. Therefore, equation (5) – based on a lognormal assumption – is used for the evaluation of $\gamma_R$. Results for the buildings according to Table 1 are given in Table 3.
Table 3. Global resistance factor $\gamma_R$ for different types of buildings, as a function of the duration of the ISO 834 standard fire.

<table>
<thead>
<tr>
<th>Building type</th>
<th>0 min</th>
<th>30 min</th>
<th>60 min</th>
<th>90 min</th>
<th>120 min</th>
<th>150 min</th>
<th>180 min</th>
<th>210 min</th>
<th>240 min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hospital</td>
<td>1.21</td>
<td>1.22</td>
<td>1.23</td>
<td>1.26</td>
<td>1.36</td>
<td>1.52</td>
<td>1.58</td>
<td>1.69</td>
<td>1.71</td>
</tr>
<tr>
<td>Office building</td>
<td>1.12</td>
<td>1.12</td>
<td>1.13</td>
<td>1.15</td>
<td>1.20</td>
<td>1.28</td>
<td>1.31</td>
<td>1.37</td>
<td>1.38</td>
</tr>
<tr>
<td>Residential</td>
<td>1.15</td>
<td>1.15</td>
<td>1.16</td>
<td>1.17</td>
<td>1.24</td>
<td>1.34</td>
<td>1.38</td>
<td>1.45</td>
<td>1.46</td>
</tr>
<tr>
<td>Production hall</td>
<td>1.16</td>
<td>1.16</td>
<td>1.17</td>
<td>1.19</td>
<td>1.26</td>
<td>1.36</td>
<td>1.40</td>
<td>1.48</td>
<td>1.49</td>
</tr>
<tr>
<td>Warehouse</td>
<td>1.16</td>
<td>1.16</td>
<td>1.17</td>
<td>1.19</td>
<td>1.26</td>
<td>1.37</td>
<td>1.41</td>
<td>1.48</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 3 indicates that as soon as the bending moment capacity starts decreasing after 90 minutes of exposure (Figure 1), the global resistance factor increases significantly. This stresses the necessity of calculating a time-dependent value of $\gamma_R$ for concrete elements exposed to fire. However, the above mentioned results are related to the specific evolution of $V_R$ for the slab configuration of Table 2. Research is ongoing to generalize the results of Table 3.

Figure 4 shows the evolution of the global resistance factor as a function of the fire exposure time for different target values for the reliability index during fire $\beta_{t,fi}$ and assuming a lognormal distribution of $M_{R,fi,t}$. For practical applications, it is possible to interpolate between the different curves.

![Figure 4. Global resistance factor $\gamma_R$ for different target reliability indices $\beta_{t,fi}$, as a function of the time of exposure to the ISO 834 standard fire curve.](image)

The important influence of the reinforcement temperature on the bending moment capacity of concrete slabs exposed to fire is well known. Therefore, the concrete cover and the presence of insulation significantly affect the structural fire resistance for a given slab configuration. While the global resistance factors in Figure 4 can be considered a good approximation for conditions with a standard concrete cover and no insulation, $\gamma_R$ in case of a specific design situation can also be expressed as a function of the nominal reinforcement temperature (Figure 5).
Figure 5. Global resistance factor $\gamma_R$ for different target reliability indices $\beta_{t,fi}$, as a function of the bottom reinforcement temperature.

If the fire resistance should be assessed with respect to a natural fire or hydrocarbon fire, a designer can use the reinforcement temperature calculated in the single FEM analysis and the target reliability index $\beta_{t,fi}$ calculated by (7) to find the appropriate $\gamma_R$ in Figure 5.

6 APPLICATION EXAMPLE

In order to validate the before mentioned global partial factor approach, a 2D FEM analysis of a slab with nominal values according to Table 2 is performed with the FEM program Atena [19]. For all design variables the mean values are used (e.g. Table 4).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Dimension</th>
<th>Nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{c}(20^\circ\text{C})$</td>
<td>20°C mean concrete compressive strength</td>
<td>MPa</td>
<td>25.4</td>
</tr>
<tr>
<td>$f_{y}(20^\circ\text{C})$</td>
<td>20°C mean steel yield strength</td>
<td>MPa</td>
<td>581.4</td>
</tr>
</tbody>
</table>

The calculation of the bending moment capacity of the slab is carried out in three steps. First the response to the self-weight of the slab is calculated. Subsequently, the slab is exposed to the ISO 834 standard fire curve for a required duration. In this example, calculations are performed for multiples of 30 minutes, up to 240 minutes of fire exposure. Finally, a load displacement analysis is performed with a point load in the middle of the slab in order to determine the ultimate load capacity and calculate the corresponding bending moment capacity of the slab after the specified time of fire exposure (e.g. 30 minutes, 60 minutes...). Figure 6 shows the evolution of the calculated $\mu_R$ and gives an overview of the different values of $M_{Rd,fi,t}$ as a function of the target reliability index $\beta_{t,fi}$ calculated based on the proposed global partial factors.
Based on the single FEM simulation, \( M_{Rd,fi,t} \) can be calculated once the target reliability index \( \beta_{t,fi} \) and the corresponding global resistance factor \( \gamma_R \) are chosen. The resulting value for \( M_{Rd,fi,t} \) should be compared to the design value of the bending moment induced by the design loads \( M_{Ed,fi,t} \) in order to assess whether the required fire resistance time is achieved. In accordance with EN 1992-1-2 [17], \( M_{Ed,fi,t} \) can be assumed constant during the fire and can be approximated by 0.7 times \( M_{Ed} \) the design value calculated for normal conditions.

### 7 CONCLUSION

- A global partial factor format for non-linear FEM analysis of concrete slabs exposed to fire is proposed.

- The necessary computational efforts are minimized by requiring only a single FEM analysis to derive the appropriate design value of the bending moment resistance.

- While taking into account the consequence class of the building, the fire incidence rate and the probability of the fire growing to a fully developed fire, the proposed method remains easy to use due to the availability of some illustrating graphs.

### 8 CONCLUSION

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Figure 6. Design value of the bending moment capacity of the analysed slab configuration for different target reliability indices \( \beta_{t,fi} \) as a function of the time of exposure to the ISO 834 standard fire curve.
REFERENCES


