High-Frequency Issues Using Rotating Voltage Injections Intended For Position Self-Sensing

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Abstract—The rotor position is required in many control schemes in electrical drives. Replacing position sensors by machine self-sensing estimators increases reliability and reduces cost. Solutions based on tracking magnetic anisotropies through the monitoring of the incremental inductance variations are efficient at low-speed and standstill operations. This inductance can be estimated by measuring the response to the injection of high-frequency signals. In general, however, the selection of the optimal frequency is not addressed thoroughly. In this paper, we propose discrete-time operations based on a rotating voltage injection at frequencies up to one third of the sampling frequency used by the digital controller. The impact on the rotation-drive, the computational requirement, the robustness and the effect of the resistance on the position estimation are analyzed regarding the signal frequency.

Index Terms—AC motor drives, Sensorless control, High-frequency signal-injection, Permanent-magnet (PM) machine

NOMENCLATURE

$\alpha \beta$ axes of the stationary frame;
$qd$ axes of the synchronous frame;
$xy$ axes of the anisotropy frame;
$\varphi_x$ angle of the anisotropy frame with respect to $\alpha \beta$;
$x$ denotes a space vector, that can be the current $i$, the voltage $v$, the flux $\varphi$ or the back-emf $\epsilon_{PM}$;
$\alpha$ denotes an anisotropic parameter, that can be the resistance $r$ or the incremental self-inductance $L_i$;
$\alpha_s$, $\alpha$, positive and negative parameters, corresponding to $\alpha$;
$x(t)$ continuous-time value;
$\nu_s$, $T_s$ sampling frequency and sampling period; $\nu_s = 1/T_s$;
$x(t[k])$ samples of the value at sampling instants $t[k]$;
$\bar{x}[k]$ average of two consecutive samples;
$\delta x[k]$ backward difference between two samples;
$\Xi_c$ contribution of the normal rotation-drive operations;
$\Xi_i$ contribution of signals injected for the self-sensing;
$\{\omega_k\}$ frequencies (in radian $\omega = 2\pi \nu$) related to $\Xi_c$;
$\{\omega_l\}$ frequencies related to $\Xi_i$;
$X(z)$ $z$-transform of the discrete-time value $x$;
$X(e^{j\omega T_s})$ Fourier-transform of the discrete-time value $x$;

I. INTRODUCTION

Many closed-loop control schemes used in electrical drives, such as most vector-control schemes, require the knowledge of the rotor position [1], [2]. This position can be measured by external dedicated sensors, such as encoders, resolvers and hall-effect sensors. However, more and more, these sensors are removed [3] in order to 1) increase the reliability of the drive by reducing the risk of failure, 2) reduce the cost of these external sensors or 3) to save space. This strategy is often referred to as position/motion-sensorless or self-sensing control. In this paper, the latter terminology is preferred since it reflects the principle: electromechanical phenomenons in the machine itself, that vary with the rotor position, are used to estimate the rotor position. These phenomenons can be observed and tracked from measurable electrical variables, such as currents and voltages [1], [2], [4], [5].

Here, we consider only self-sensing methods based on the current samples used for the digital current control, and on the knowledge of the supplied voltage. Methods using additional sensors can be very efficient, but these sensors introduce additional costs and processing. Among them, we find those using current-slope measurements in order to detect current variations in response to a pulse injection [6], to detect the current ripples due to the pulse-width modulation (PWM) [7] or during the zero-sequence of the PWM [8]; those using very high-frequency digital sampling instead of current slope sensors [9]; those using current samples at specific instants during the PWM [10], [11]; and those using zero-sequence voltage measurements [12]. They are not further discussed in this document.

At high speeds, the back-emf is a reliable source to estimate the rotor position without much effort [13], [14]. Its signal-to-noise quality however decreases with the rotation speed. Note that some methods do not involve directly the back-emf, but estimate the linking magnetic flux. They are then sometimes referred to as fundamental-model-based methods [15].
or simply stator-flux-based methods using MRAS observer [16] or extended Kalman filters [17]. At low rotation speed and standstill, an estimation of the rotor position can be obtained from anisotropic properties linked to the rotor position [8], [12], [18]–[36], assuming some feasibility considerations [37]–[39]. These anisotropies can be due to variations in the rotor geometry or to magnetic saturation effects in the iron [19] and are revealed through anisotropic parameters, such as the incremental inductance. Special attention can be given to the machine design in order to increase their anisotropy [38], [40]. It is also possible to increase the anisotropy of existing machines by adding a copper turn wound around the poles [41], [42]. The comparison between the different self-sensing methods are largely discussed in [15]. Some back-emf-based methods also take the anisotropy into account in the model, even if the anisotropy is not used to estimate the position [43]. Solutions that combine the back-emf-based method at low-speed and the anisotropy-based method at higher-speed have been implemented in [9], [44]. Hybrid solutions using both methods simultaneously in order to compensate for their respective error sources are proposed in [13], [45]–[47]. This paper focuses on anisotropy-based position self-sensing methods intended for the vector control of permanent-magnet (PM) machines, that naturally present some anisotropic properties.

Misalignment between the anisotropy and the real rotor position can be due to significant stator currents [22], [23], [25], [33], [36], [39], [47]–[49], to the so-called secondary and multiple saliencies [23], [25], [48], or to more general spatial (slots) harmonics related to the conductor distributions and the nonsinusoidal magnetic-field [24], [25], [50]. These issues are largely addressed in the literature.

In the vast majority of anisotropy-based strategies without extra sensors, the anisotropy is tracked performing high-frequency signal injection in addition to the rotation-drive operating signals (also sometimes referred to as “fundamental” operating signals). In some specific situations, the variations of the rotation-drive operating signals can be large enough to perform the estimation without signal injection. E.g. [51] and [28] propose solutions based on the signal variations in a direct-torque controller. These specific cases are not studied here. Many different types of signal injection can be used: test-pulse trains [6], [10], [11], [28], [29], pulse-width modulation (PMW) modifications [9], carrier-based pulsating and rotating-signals injection [11], [12], [23], [25]–[27], [30], [32], [44]. In some pulse-train injection strategies, it is required to interrupt the rotation-drive operations during short periods [6], leading to some distortions on the drive. An improved pulse-train strategy is proposed in [29] using test-pulse signals without affecting the rotation-drive operations. Comparisons between several signal injections are proposed by [20], [52]. This paper focuses on the rotating-signal injection that yields good performances without initial knowledge of the parameters. The impact of the rotating-signal injection on the rotation-drive operation is discussed in this paper.

Besides [27], [30], [32], [35], [53], the resistance impact is often neglected in signal-injection operations, assuming an ideal inductive machine. The eddy currents however increase with the frequency and may significantly affect the apparent resistance value [27], [32], [34], [54], [55], leading to position-estimation errors [23], [27]. This issue is considered in this paper.

The frequency of the injected signals is often selected between 400 Hz and 2 KHz [19], [56] and many papers introduce self-sensing using continuous-time operations. We propose here to study discrete-time operations up to one third of the sampling frequency used by the current controller, which is the maximum possible frequency defining rotating signals. The benefits of the proposed method are analyzed regarding disturbing interactions between the rotation-drive and the self-sensing operations, regarding the filtering and the computational requirement, the robustness and the impact of the apparent resistance.

Section II describes the discrete-time model of the permanent-magnet machine and states the expressions between voltages and currents; Section III addresses the principles of the signal injection, proposes the discrete-time operations and analyzes the benefits using higher signal frequencies; Section IV discusses the resistance impact with experimental cases; Section V shows experimental results using a signal injected at one third of the sampling frequency and Section VI concludes.

II. MODEL OF THE PERMANENT-MAGNET MACHINE

A. Continuous-time Circuit Model

The machine model is described using the concept of space vectors in a complex frame [57]. The space vector \( \mathbf{z} \) refers to the supply voltage applied to the stator circuit terminals and \( i \) to the electrical current flowing through the terminals. The magnetic flux \( \psi \) linked by the stator circuit can be divided in two main contributions: 1) the contribution of the currents \( \mathbf{z} \) written \( \psi_{\mathbf{z}} = l_i \) where \( l \) is the stator inductance and 2) the contribution of the PM written \( \psi_{\text{PM}} = \mathbf{z}_{\text{PM}} \). Since the stator-circuit model uses the derivative of the flux, we also define the incremental inductance \( l_t \). The back-emf-based force (back-emf): \( \mathbf{z}_{\text{PM}} = \frac{d\psi_{\text{PM}}}{dt} \). In the \( \alpha/\beta \) stationary reference-frame, the continuous-time relation between the electromagnetic values of the machine stator-circuit is: \( \mathbf{z} = \frac{d\psi}{dt} + r_i \mathbf{z} \), where \( r \) is the resistance. Replacing all the values yields:

\[
\mathbf{z} \frac{d\mathbf{z}}{dt} + r_i \mathbf{z} = \mathbf{z} - \mathbf{z}_{\text{PM}}
\]

(1)

B. Discrete-time Circuit Model

We assume that the different operations are performed by digital controllers. A discretized model of (1) is therefore required. The current measurements are sampled with a frequency \( \nu_s \) at instants \( t^{[k]} = kT_s \), where \( T_s = 1/\nu_s \) is the sampling period. For convenience, the pulse-width modulated signal (PWM) driving the voltage-source inverter (VSI) is synchronized with the sampling times. Let us defined the mean value of \( \mathbf{z} \) between two sampling times as follows:

\[
\mathbf{z}^{[k]} \triangleq \frac{1}{T_s} \int_{t^{[k]}}^{t^{[k+1]}} \mathbf{z}(t)dt
\]

(2)
Fig. 1. Illustration of $\delta \psi_s$ related to $\delta i$ along the $x$-axis and along the $y$-axis. The blue dashed lines represent the path drawn by the space vectors when $\delta i$ rotates. $\delta \psi_s$ describes an ellipse.

Fig. 2. Illustration of $\delta \psi_s$ related to $\delta i$ along an arbitrary direction and modeled by the sum of the positive and negative contributions. The blue dashed lines represent the circle drawn by the two contributions when $\delta i$ rotates.

Assuming that the inverter nonlinearities are compensated [4], [31], [36], [52], the mean voltage supplied by the VSI should be equal to the command voltage sent to the PWM-VSI. The backward current difference is:

$$\delta i[k] = \frac{1}{2} (i[k] - i[k-1])$$

Due to the PWM, the current exhibits ripples between sampling instants and the exact computation of its mean value is not straightforward. We however assume the approximation of an equivalent piecewise-linear mean current computed as the average of two consecutive samples:

$$\bar{i}[k] \approx \frac{i[k] + i[k-1]}{2}$$

Applying (2)-(4) on (1) and assuming constant parameters during the sampling periods, the discrete-time stator-circuit model yields:

$$T_s \frac{\delta i[k]}{\delta i[k]} + r_s^2[k] = \bar{i}[k] - \bar{\psi} \delta \psi_s$$

C. Anisotropic Machine Model

We assume a constant magnetic state of the machine, i.e. constant saturation level of the iron. This is valid if we consider small estimation periods during which the rotor position does not significantly change, and if we consider small current variations. The magnetic anisotropy is revealed by the variations of the incremental inductance $l_t$ linking $\delta i$ to $\delta \psi_s$ as a function of their orientation. As illustrated in Fig. 1, the $xy$ frame is defined such that the axes $x$ and $y$ are respectively along the directions that correspond to the maximum $l_{tx}$ and minimum $l_{ty}$ of the incremental inductance:

$$\begin{pmatrix} \delta \psi_{s,x} \\ \delta \psi_{s,y} \end{pmatrix} = \begin{pmatrix} l_{tx} & 0 \\ 0 & l_{ty} \end{pmatrix} \begin{pmatrix} \delta i_x \\ \delta i_y \end{pmatrix}$$

The angle of the $x$-axis with respect to the $\alpha$-axis is called the anisotropy angle and is noted $\varphi_x$. As explained in [4], [24] and as illustrated in Fig. 2, the relation between $\delta i$ and $\delta \psi_s$ along any direction can be modeled as the contribution of two components:

$$\delta \psi_s = l_t \delta i = l_{tx} \delta i_x + l_{ty} \delta i_y e^{j2\varphi_x}$$

where $l_{tx}$ is called the positive incremental inductance and $l_{ty}$ is the negative incremental inductance. The second component contains the anisotropy angle through a rotation of the complex conjugate $\delta i^*$. They are linked to the maximum and minimum values by:

$$l_{tx} = \frac{l_{tx} + l_{ty}}{2} \quad \& \quad l_{ty} = \frac{l_{tx} - l_{ty}}{2}$$

Note that these values are affected by significant variations of the stator currents [25], [33], [36].

The anisotropy angle $\varphi_x$ is linked to the total magnetic field $\psi$ [45], that is partly produced by the PM. The anisotropy $xy$ frame is therefore used as an indicator of the synchronous $qdl$ frame, defined by the PM orientation. Misalignment must however be corrected if the contribution of the stator currents to the magnetic field becomes significant [49]. Another correction must be performed if $\psi_{PM}$ is not oriented along the $d$-axis. This is generally due to a nonsinusoidal shape of the magnetic field in the air-gap and to a nonsinusoidal distribution of the stator windings [24], [36]. The misalignment between the $xy$ frame and the $qdl$ frame is sometimes referred to as an estimation error. It is however an error only for the purpose of the position self-sensing estimation. Note that if we use the $qdl$ frame to define the relation between $\delta i$ and $\delta \psi_s$, a coupling between the $q$ and $d$-axis appears due to the misalignment [22]. This is referred to as magnetic cross-coupling [25], [36], [39], [58]. This issue is not further discussed here since we strictly focus on the anisotropy angle estimations, and not on the relation with the PM location.

Assuming that the resistance is also possibly anisotropic [27], [35] and defined similarly to (7), the anisotropic model (5) yields:

$$\left( \begin{array}{c} l_{lx} \delta i_x[k] + r_s^2 \bar{i}^{(*)} \delta \psi_s \end{array} \right) + \left( \begin{array}{c} l_{lx} \delta i_x[k] + r_s^2 \bar{i}^{(*)} \delta \psi_s \end{array} \right) e^{j2\varphi_x} = \bar{i}^{(*)}$$

where we defined $\bar{i}^{(*)} = \bar{i} - \bar{i}_{PM}$.

D. The $z$-Transform Of The Anisotropic Machine Model

The operations can be described using the $z$-transform of the discrete-time anisotropic-model relation linking the current difference $\delta i$ to the mean voltage $\bar{u}$. Let us first introduce the transfer function $D(z)$ linking the $z$-transform of the mean current $\bar{I}(z) = \mathbb{Z}\{I\}$ to the $z$-transform of the current difference $\delta I(z) = \mathbb{Z}\{\delta i\}$. Using (4) and (3) yields:

$$\delta I(z) = D(z) \bar{I}(z) \Rightarrow D(z) = \frac{2(1 - z^{-1})}{1 + z^{-1}}$$

Secondly, note that the $z$-transform of a conjugate value $z^*$ is $\mathbb{Z}^*(z^*) = \mathbb{Z}\{z^*\}$ [59]. Using (10) and assuming time-invariant anisotropy angle and parameters, the $z$-transform of the anisotropic model (9) can be written as the contribution of
two transfer-functions \( Z_+(z) \) and \( Z_-(z) \), that we respectively call positive and negative integral-impedances:

\[
Z_+(z)\delta I^+ + Z_-(z)e^{j2\varphi_s}\delta I^-(z*) = \tilde{U}(z)
\]

(11)

where it is found:

\[
\begin{align*}
Z_+(z) &= \frac{l_s}{T_s} + r_s/D(z) \\
Z_-(z) &= \frac{l_i}{T_s} + r_s/D(z)
\end{align*}
\]

(12)

In most of the drives, the voltage is the commanded input signal and the current is the measured output signal. It is therefore required to reverse the relation (11). As explained in the annexes, the reversed relation can be written as the contribution of two transfer-functions \( Y_+(z) \) and \( Y_-(z) \), that we respectively call positive and negative derivative-admittances:

\[
Y_+(z)\tilde{U}(z) + Y_-(z)e^{j2\varphi_s}\tilde{I}^-(z*) = \delta I(z)
\]

(13)

\[E \text{. The Fourier-Transform Of The Anisotropic Machine Model}\]

The signal injection is a strategy based on a repetitive voltage sequence, generally at fixed frequencies, as described in the next section. It is therefore convenient to use the discrete-time Fourier-transform (DTFT) that is found replacing \( z \) by a unitary complex value \( e^{j\omega T_s} \) in the relations, where \( \omega \leq \omega_s/2 = \pi/T_s \). As demonstrated in the annexes, the transfer function \( D(z) \) linking the mean current to the current difference (10) yields:

\[
D(e^{j\omega T_s}) = j\omega T_s \quad \text{where} \quad \omega \triangleq \tan(\omega T_s/2)/(T_s/2)
\]

(14)

Note that \( \omega \) tends to \( \omega \) when \( \omega \ll \omega_s \). Introducing successively (14) in (12), then in (33), the DTFT of positive and negative derivative-admittances are:

\[
\begin{align*}
Y_+(e^{j\omega T_s}) &= T_s \left( \frac{l_{ix} - jr_x/\omega}{l_{ix} - jr_x/\omega} \right) \left( \frac{l_{iy} - jr_y/\omega}{l_{iy} - jr_y/\omega} \right) \\
Y_-(e^{j\omega T_s}) &= -T_s \left( \frac{l_{ix} - jr_x/\omega}{l_{ix} - jr_x/\omega} \right) \left( \frac{l_{iy} - jr_y/\omega}{l_{iy} - jr_y/\omega} \right)
\end{align*}
\]

(15)

This result is not convenient to use in self-sensing operations. By consequence, the resistance is often neglected and these derivative-admittances become very simple as the imaginary and frequency dependent factors vanish:

\[
\begin{align*}
r_x \ll \omega l_{ix} & \Rightarrow Y_+ = T_s \frac{l_{ix}}{l_{ix}l_{iy}} \\
r_y \ll \omega l_{iy} & \Rightarrow Y_- = -T_s \frac{l_{iy}}{l_{ix}l_{iy}}
\end{align*}
\]

(16)

\[III \text{. Signal Injection Strategy}\]

\[A. \text{ Principle and Assumptions}\]

The principle is illustrated in Fig. 3. It consists in the injection of a high-frequency voltage \( \tilde{u} \) computed by the self-sensing operations in addition to the low-frequency voltage \( \tilde{u}_c \) computed by the rotation-drive operations: \( \tilde{u} = \tilde{u}_c + \tilde{u}_i \). As a consequence, a high-frequency current response \( \tilde{i}_i \) is added to the low-frequency current response \( \tilde{i}_c \) controlled by the rotation-drive operations: \( \tilde{i} = \tilde{i}_c + \tilde{i}_i \).

In order to prevent or reduce disturbing interactions, the signal-injection operations and the rotation-drive operations should produce signals \( \tilde{x}_i \) and \( \tilde{x}_c \) covering separated frequency ranges \( \{ \omega_i \} \) and \( \{ \omega_c \} \) respectively. In terms of DTFT \( \tilde{X} = \mathcal{F}\{ x \} \), the condition is:

\[
\left\{ \begin{array}{ll}
X_c(e^{j\omega T_s}) & \ll X_i(e^{j\omega T_s}) & \text{for} \ \omega \in \{ \omega_i \} \\
X_i(e^{j\omega T_s}) & \gg X_c(e^{j\omega T_s}) & \text{for} \ \omega \in \{ \omega_c \}
\end{array} \right.
\]

(17)

Note that the frequency content of the PWM is not considered in discrete-time operations and is, by consequence, excluded from the condition (17).

The high-frequency signals inevitably produce a high-frequency torque leading to high-frequency vibrations (that are audible under 20kHz) and, by consequence, to a high-frequency back-emf. The mechanical damping effects (due to the inertia plus the frictions of the machine and the coupled load) tend however to increase with the frequency, reducing the high-frequency back-emf to a negligible value. In terms of DTFT of the back-emf \( \tilde{E}_{PM} = \mathcal{F}\{ E_{PM} \} \) and of the voltage \( \tilde{V} = \mathcal{F}\{ \tilde{v} \} \), this leads to:

\[
\left| \tilde{E}_{PM}(e^{j\omega T_s}) \right| \ll \left| \tilde{V}(e^{j\omega T_s}) \right| \quad \text{for} \ \omega \in \{ \omega_i \}
\]

(18)

The back-emf is therefore neglected in self-sensing operations.

\[B. \text{ High-Frequency Anisotropic Model}\]

Introducing the conditions (17) and (18) in the anisotropic relation (13) yields:

\[
Y_+(e^{j\omega T_s})\tilde{U}(e^{j\omega T_s}) + Y_-(e^{j\omega T_s})e^{j2\varphi_s}\tilde{I}^-(e^{j\omega T_s}) = \delta I(e^{j\omega T_s})
\]

(19)

Note that (19) is valid for any type of high-frequency signal.

In most of the papers dealing with rotating voltage injection, the self-sensing operations are based on the current samples instead of the current-differences. Our choice of the current-differences is however justified by the frequency. Using (10) and (14), it is found:

\[
\left| \delta I(e^{j\omega T_s}) \right| = |\omega T_s| \left| \tilde{I}(e^{j\omega T_s}) \right|
\]

(20)

Assuming a fixed signal injection amplitude, from (4) and (3), it can be shown that the signal-to-noise ratio of the current-differences becomes favorable above \( \omega T_s \geq 2 \). Using (14), it corresponds to: \( \omega T_s \geq \pi/2 \).
\[ Y_{\omega_i}(e^{-j\omega_i T_s}) = \delta I_{\omega_i}(e^{-j\omega_i T_s}) \]  

\[ Y_{\omega_i}(e^{-j\omega_i T_s}) = \delta I_{\omega_i}(e^{-j\omega_i T_s})/\omega_{\omega_i} \]  

\[ \delta I_{\omega_i}(e^{-j\omega_i T_s}) := \text{LPF} \left( \delta I_{\omega_i}[k] e^{j\omega_i k T_s} \right) \]  

\[ \delta I_{\omega_i}(e^{-j\omega_i T_s}) := \text{LPF} \left( \delta I_{\omega_i}[k] e^{j\omega_i k T_s} \right) \]  

\[ \text{LPF}^{[k]}(\omega) := \frac{1}{N} \sum_{n=0}^{N-1} e^{-j[k-n]} \]  

Its characteristic for the negative frequencies is illustrated in Fig. 4 with dashed lines for the case \( N = 3 \) and \( N = 12 \).

Assume that the high-frequency \( \omega_i \) of the injected signals is an integer fraction \( N_1 \geq 3 \) of the sampling frequency \( \omega_s \):

\[ \omega_i = \omega_s/N_1 \Rightarrow \omega_s T_n = 2\pi/N_1 \]  

The moving average (25) can then be used as a LPF for the operation (24), selecting an integer multiple \( N \) of \( N_1 \). Considering the case \( N_1 = 3 \), the characteristic of the moving averages shifted around \( -\omega_i = -\omega_s/3 \) is illustrated in Fig. 4 with plain lines for \( N = N_1 = 3 \) and \( N = 4 \times N_1 = 12 \). The choice of \( N \) depends on the expance of the low-frequency range \( \omega_c \) to be removed. However, assuming that \( \omega_s \) is much higher than \( \{\omega_c\} \), a higher \( N \) does not strongly improve the attenuation characteristic at low frequencies, while it requires more computational power.

Until now, we assumed a constant anisotropy angle \( \varphi_x \) during the operations. In practice however, this angle \( \omega_x = d\varphi_x/dt \) varies due to the machine rotation and to possible harmonics in the anisotropy variations, introducing a rotating term applied to the current. This leads to a spectrum shift of \( 2\omega_x \) since the DTFT becomes:

\[ F \left\{ e^{j2\omega_s t \omega_x[k]} \right\} = \delta I^{*}(e^{-j(\omega_s-2\omega_s)T_n}) \]  

It is negligible if \( 2\omega_s \ll |\omega_s| \), but this condition must be checked. Other spectrum dispersions are due to possible variations in the parameter values. The choice of \( N \) also depends on this spectrum dispersions around \( -\omega_i \). Higher \( N \), higher the risk to filter beside the high-frequency current response. We propose therefore to use to lowest \( N = N_1 \) for the operations (24).

Apart from this, the moving average can also be used to remove the high-frequency current component in the samples \( \tilde{y}(t[k]) \) for the rotation-drive operations:

\[ \text{LPF}^{[k]}(\omega) = 0 \Rightarrow \text{LPF}^{[k]}(\omega) = \text{LPF}^{[k]}(\omega) \]  

If the controller bandwidth is much smaller than \( \omega_s \) however, the filtering becomes unnecessary [56].

In many papers, operations are based on infinite impulse response (IIR) filters [11], [12], [33], [44], [45], [52], [56], [61]–[63]. The comparison between IIR and FIR filters would require further analysis, but apriori, the moving average provides a simple solution with good filtering characteristics and with good phase linearity. Moreover, the stabilization time of the FIR is not greater than \( N \) sampling periods, while it can be much longer with IIR filters for the same bandwidth.

\[ E. \text{ Issue Regarding The PWM-VSI} \]

We assume that the voltages are not measured but the command voltage is used instead. Dead-times in the pulse-width modulation (PWM) and voltage drops at the semiconductors.
of the voltage-source inverter (VSI) are common nonlinearities that must be managed. They can generally be linearized and compensated assuming fixed phase current flows [4], [31], [36], [44]. Moreover, their frequency content is mainly present in the low-frequencies.

When a phase current crosses zero however, non-compensable dead-times and voltage drops discontinuities occur, referred to as zero-crossing clamping phenomena. Even small, they may lead to significant estimation errors [36], [52], [64]. The smaller \( N_t \) however, the smaller the number of estimations affected by the zero-crossing nonlinearity and better is the robustness of the self-sensing regarding the inverter nonlinearities. This is valid also regarding any other interruption in the measurements or in the signal injection.

\[ F. \; \text{Position Extraction} \]

As it is widely assumed in the literature, if the resistance impact is negligible compared to the inductance, the positive \( Y_+ \) and negative \( Y_- \) derivative-admittances are strictly real values (16). Using (24), the angle \( e^{j\omega t} \) is then easily extracted by:

\[
\hat{\phi}_x = \frac{\angle (-\text{LPF}(\delta_i/\bar{E}_i))}{2} \tag{29}
\]

where \( \angle \) denotes the complex argument. Note that there is an ambiguity of \( \pi \) on \( \hat{\phi}_x \) that is inherent to this method. This ambiguity can be initially removed by tracking differences of the incremental inductance along opposite magnetizing directions (positive and negative values along the \( d \)-axis), injecting current oscillations of large amplitudes and assuming that the inductance differences is larger than the noise [4], [5], [61], [62], [65]. The ambiguity may also be removed at higher speed using back-emf observation [47].

In practice however, the resistance is not always negligible, and \( Y_+ \) is not strictly real, leading to angle estimation errors. Using (29), this error is the half complex argument of \( Y_+ \) given in (15):

\[
\hat{\phi}_x = \hat{\phi}_x - \frac{\angle (-Y_+(e^{-j\omega T_s}))}{2} = \frac{\left(\angle (l_+ + j r_+ / \hat{\omega}_i) - \angle (l_- + j r_- / \hat{\omega}_i)\right)}{2} \tag{30}
\]

G. Discussion On The Injected Amplitude And Frequency

The discussion is based on a fixed high-frequency current amplitude. The selection of the amplitude is a compromise: on one hand, it should be the smallest possible in order to reduce the resistive loss, the vibrations and to limit the problems of possible zero-crossing nonlinearities. On the other hand, the amplitude must be high enough in order to satisfy the condition (17) and in order to provide a good signal-to-noise ratio.

The higher the signal injection frequency, the higher the required injected voltage amplitude, assuming a fixed current response amplitude. This reduces the range of voltage allowable for the rotation-drive operations. Moreover, the audible nuisance increases with the frequency. Assuming that these issues are managed, using higher frequencies however present significant benefits. According to the discussed aspects, they are:

1) reduction of frequency interactions (18) and decrease of the back-emf influence (17);
2) low computation requirements, since the computation steps of the moving average (25) is proportional to \( N_t \);
3) low settling time at initialization and restart, due the stabilization time of only \( N_t \) sampling periods;
4) robustness regarding inverter nonlinearities and other interruptions.

It is also beneficial regarding the resistance impact (30), assuming that \( r_\pm / \hat{\omega}_i \) decreases with the frequency \( \omega_0 \). Note however that the contribution of Eddy-currents increases with the signal frequency [27], [32], [34], [54], [55] and augment the apparent value of the resistances. This issue is analyzed experimentally hereafter. As a conclusion, the optimum frequency is the maximum satisfying (26) and defining a rotating space vector: \( N_t = 3 \).

IV. ANALYSIS WITH EXPERIMENTAL CASES

A. Experimental Machine And Test Setup

The experimental machine is a three-phase 3 kW in-wheel brushless-DC (BLDC) motor with 14 pairs of surface-mounted permanent-magnets in an outer rotor. It is developed by Technicrёa, France, for the propulsion of small vehicles. Details on the design of similar machines can be found in [66]. The rated stator current in the machine is 1.34 A and the rated rotation speed is 500 rpm. The machine is fed with an IGBT voltage-source inverter (VSI) supplied by a rated \( v_{dc} = 50 \) V DC-voltage. The PWM generator works at \( f_s = 10 \) kHz. The resolution of the current measurements is 0.244 A. Its apparent parameters have been estimated at different frequencies using small pulsating signals along the \( x \) and \( y \)-axes on a standstill unlocked machine. Results are shown in Fig. 5. As expected, the frequency influence the apparent value of the resistances and, a lesser extent, of the inductances.

B. Errors Due To The Resistance

Fig. 6 shows the theoretical error (30) due to the resistance for different signal injection frequencies with respect to the sampling frequency \( \omega_s = 2\pi f_s \). The circles, joined by plain lines, correspond to the error with our experimental BLDC
Fig. 6. Theoretical estimation errors $|\hat{\phi}_x - \phi_x|$ in experimental cases.

## Parameters of Some Machines Found in the Literature.

| $\nu_i$ (Hz) | $|\nu_i - \nu_y|/(\mu \cdot 10^2)$ | $I_{dc}$ [mA] |
|-------------|----------------------------------|---------------|
| [32]-1 1.5 k | 2.691 - 4.431 | 2.34 - 3.61 |
| [32]-1 0.5 k | 0.259 - 0.436 | 2.17 - 2.84 |
| [32]-1 1 k | 0.765 - 1.385 | 2.17 - 2.84 |
| [32]-2 1.5 k | 1.48 - 2.191 | 2.17 - 2.84 |
| [56] 500 | 0.0103 | 101 - 306 |
| [60] 500 | 1.4 | 10 - 70 |
| [67] 500 | 2.2 | 6.5 - 19.6 |
| [67] 500 | 2.275 | 6.5 - 12.75 |
| [67] 500 | 8.25 | 100 - 300 |
| [67] 500 | 1.87 - 1.96 | 7.5 - 9.4 |
| [67] 500 | 0.76 - 0.88 | 420 - 440 |

All machines are permanent-magnets, except [26] that is a switched reluctance machine. These parameters must be taken with care and as information only. (*) If the high-frequency resistance is not mentioned, the DC resistance is taken instead. (**) We take the inductances corresponding to the lowest load. The digital sampling frequency is $\nu_y = 10$ kHz for all drives, except in [26], [53] where this frequency is assumed because not specified. Note that $\omega = 2\pi \nu$.

machine, using the parameters of Fig. 5. It is observed that, even if the apparent resistance tends to increase with the frequency, its relative impact is divided by $\omega_i$ and tends to decrease. This tends to confirm the benefits of using the highest frequency.

Results using the parameters of some machines found in the literature are shown by diamonds and triangles. The parameters can be found in TABLE I. If there is more than one machine in one reference, the reference is followed by a numbering for each machine. In [32], parameters of two machines are given at three different frequencies. The corresponding errors are mentioned in Fig. 6 by diamonds joined by dashed lines. A decrease of the resistance impact is also observed. The other machines are mentioned by triangles. We can see that many papers chose a frequency at $\nu_i = \nu_y/20 = 500$ Hz. Even if the error is typically not much larger than 5\%, using higher frequencies could possibly further reduce the resistance impact.

V. Experiments

A. Measurements And Figures

Experiments are performed on the experimental BLDC machine described above. Having 14 pole-pairs, its rated speed 500 rpm corresponds to a rotation frequency of $14 \cdot 500/60 = 116$ Hz. This indicates the lower limit for the signal injection operations. The current controller bandwidth is around 400 Hz. A speed control is performed for the experiments, with a rather low bandwidth around 10 Hz. Note that the BLDC motors are generally not controlled in speed, but in torque only. The estimated position is filtered through a third order observer, with a 62.6 Hz bandwidth, before it is used in the vector control of the rotation-drive. This observer also provides the speed estimation.

From the top to the bottom, Fig. 7, Fig. 8 and Fig. 9 are organized as follows: 1) a graph of the current samples of the first phase containing both high and low-frequency content (gray dots) and the filtered currents for the normal-drive operations (black dots); 2) the rotation speed (electrical frequency) measured by an external encoder (dashed lines) and the anisotropy speed estimated by the observer (black dots); 3) the rotor-PM angle (electrical degree) measured by an external encoder (dashed lines) and the estimated anisotropy angle using (29) (black dots); 4) the error (electrical degree) between the estimated angle and the rotor-PM angle; 5) the frequency spectrum $|I_e^{(ω)}|/I_e^{(ω)}$ of the current samples. As discussed in Section III, the position information is contained in the negative frequency $\nu = -\nu_i$.

B. Disturbance Sources

The zero-clamping inverter nonlinearity is very annoying in this type of machine: when a phase current crosses zero, the position is lost. In order to prevent this drawback, a current offset is added to the instruction to maintain a margin with respect to the zero-crossing phase lines. This offset is chosen in order to minimize its impact on the torque. As a consequence of the offset, the low-frequency current signal behaviour is far from a sinusoidal signal and the current instruction regularly jumps across the zero-crossing phase lines. This is clearly visible on the current signals. Since the position is lost, the self-sensing operations (but not the signal injection) are interrupted during the jumps. The lower the injected signal frequency, the longer the duration of the interruption. More details can be found in [67].

Oscillations in the estimated anisotropy angle, inherited by the speed estimation, are partly due to significant harmonics in the machine (harmonics in the air-gap magnetic field and in the winding distributions). No compensation is performed here. More details can be found in [24].

C. Results

The two first experiments compare the case $N_i = 20$, i.e. $\nu_i = 500$ Hz, in Fig. 7 with the case $N_i = 3$, i.e. $\nu_i = 333.3$ Hz, in Fig. 8, for low-speed drives at 5 Hz that is 4.3\% the rated speed. The voltage is chosen such that the peak values of the high-frequency current are equal in both cases, around 2 A that is 1.5\% the rated current. For each case respectively it is 0.36 V, i.e. 1.4\% of $V_{dc}/2$, and 2 V, i.e. 8\% of $V_{dc}/2$. The drive operations are based on the encoder measurements and not on the estimated position in order to strictly assess the quality of the estimation and prevent feedback effects. The
errors are around $-10^\circ$ for $N_i = 20$ and close to zero for $N_i = 3$, as theoretically predicted. An important problem with $N_i = 20$ is the interruption due to the zero-crossing, that becomes relatively long compared to the rotation period at higher speeds. The spectra illustrate the better frequency separation between signal injection and rotation-drive signals in the case of $N_i = 3$.

The third experiment Fig. 9 shows the result of self-sensing operations, where the estimated position is used by the vector control. The experiment starts at standstill with the speed instruction step of 60 Hz at $t = 0$, that is 51.7% of the rated speed. Note that 0.2 s is quite short for such an acceleration in vehicle applications. At $t = 0.02$ s, larger errors on the position and the speed (negative) are observed, due to a phase current zero-crossing (inverter nonlinearity) that is not perfectly avoided. Such errors are repeated, especially at higher speeds. Above 60 Hz, the position estimation is strongly degraded because of the zero-crossings and the lower quantity of information, but this speed limitation is not inherent to the self-sensing. A clear spectrum dispersion is observed at $-\nu_i$. Despite the zero-crossing effects, the harmonic oscillations and the spectrum dispersion, these results are very satisfying for that type of machine. Note that the obtained resolution is much better than the one provided by the hall-effect sensors traditionally used with BLDC machines.

VI. CONCLUSIONS

We discussed several benefits using a discrete-time model, the current-difference response instead of the current samples, a moving average for the filtering operations and the highest possible frequency for the signal injection, in order to estimate the position in an optimal way. Based on these considerations, we conclude that the optimal frequency for a rotating voltage injection is one third of the sampling frequency, valid from standstill up to the large range of rotation speeds.

ANNEXES

A. Inversion Of The Anisotropic Relation

Since positive and negative parameters are reals defined by (8), the only imaginary values in the integral-impedances (12) are $z$. Thus: $Z^*_z(z^*) = Z^*_x(z)$. The complex conjugate $I^*_c(z^*)$ computed from the left member of (11) yields then:

$$\delta I^*_c(z^*) = \left(\delta I^*_z(z^*) - Z^*_c(z)e^{-j2\phi}z\delta I_c(z)\right)/Z^*_c(z)$$  \hspace{1cm} (31)
B. Computation Of $D(z)$ Using (8):

Replacing $v_{m}(z) \rightarrow v_{m}(z)$ back in (11) yields:

$$Z_{v}(z) \overline{v}(z) = (Z_{v}^{+}(z) - Z_{v}^{-}(z)) \delta I(z) + Z_{v}(z)e^{j2\varphi_{2}} \overline{v}(z)$$

Using (8): $Z_{v}^{+}(z) - Z_{v}^{-}(z) = Z_{v}(z)Z_{v}(z)$. Then (13) is found if we define the positive and negative derivative-admittance as:

$$
\left\{ \begin{array}{l}
Y_{v}(z) \triangleq \frac{Z_{v}(z)}{Z_{v}(z)Z_{v}(z)}
Y_{v}(z) \triangleq -\frac{Z_{v}(z)}{Z_{v}(z)}Z_{v}(z)
\end{array} \right.
$$

B. Computation Of $D(e^{j\omega T_s})$

Replacing $z$ by $e^{j\xi}$ where $\xi = \omega T_s$, (10) yields (14):

$$D(e^{j\xi}) = \frac{1 - e^{-j\xi}}{1 + e^{-j\xi}} = \frac{e^{-j\xi/2} - e^{-j\xi/2}}{e^{-j\xi/2} + e^{j\xi/2}} = \frac{\sin(\xi/2)}{\cos(\xi/2)}$$

REFERENCES


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