Thermal impedance calculations for micro-electronic structures

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Overview

► Introduction
► Thermal characterization
► Numerical calculations
► Influence of substrate thickness
► Present and future work
Introduction

study of conductive heat transfer

application to electronics is interesting & even indispensable:

- more & more thermal issues in chip design: small dimensions + dissipation ↑ (huge power densities)
- semiconductor behaviour highly T-dependent
  → variable device characteristics, electrothermal coupling

PhD: fundamental research (characterization and physical modelling of relevant phenomena)
Thermal impedance calculations for micro-electronic structures

**Introduction**

**Fundamentals**

\[
k \nabla^2 T - C_v \frac{\partial T}{\partial t} = -p
\]

heat flow

heat storage

heat source
Overview

- Introduction
- **Thermal characterization**
- Numerical calculations
- Influence of substrate thickness
- Present and future work
Thermal characterization
General overview

THERMAL CHARACTERIZATION OF ELECTRONIC PACKAGES

STEADY STATE
THERMAL RESISTANCE Rth

TIME DEPENDENT

THERMAL IMPEDANCE Zth(jω)

EQUIVALENT RC LADDER NETWORK (Székely)

STRUCTURE FUNCTION (Székely)

Bode plot
Nyquist plot

mathematically equivalent

Thermal impedance calculations for micro-electronic structures
Thermal characterization

Thermal impedance

- Thermal resistance by definition:
  \[ R_{th} = \frac{T}{P} \]

- Extended towards frequency domain: thermal impedance (in phasor notation):
  \[ Z_{th}(j\omega) = \frac{T(j\omega)}{P(j\omega)} \]
Thermal characterization
Thermal AC sources and phasors

- oscillating power & temperatures (angular frequency $\omega$)
- described by complex phasors
- in reality: AC components superimposed on DC bias
- thermal frequency response

$$P(t) = P \cos(\omega t)$$

$$T(t) = T \cos(\omega t - \phi)$$
Thermal characterization

What makes $Z_{th}$ so useful?

- captures the entire dynamic thermal behaviour (thermal blueprint of the structure)

\[ T(t) = \int_{-\infty}^{+\infty} P(f)Z_{th}(f)\exp(j2\pi ft)df \]

- familiar: similarities with electrical impedance
- can be measured experimentally (see next slide)
- compact description if using Nyquist representation (see later)
Thermal characterization

Measurement technique

Thermal impedance calculations for micro-electronic structures

Electronic package

Apply power step

Record junction temperature transient

T3ster equipment from Micred

\[ Z_{th}(j\omega) = \frac{j\omega}{P} \int_{0}^{+\infty} T(t) \exp(-j\omega t) dt \]
Thermal characterization

Nyquist curves

- plot of $\text{Im}[Z_{th}]$ versus $\text{Re}[Z_{th}]$

- interesting properties (circular arcs) $\Rightarrow$ Nyquist = V.I.P.
Thermal characterization
Experimental measurement – Diode

BZX-55-C7V5 in JEDEC DO-35 glass package
Free convection cooling conditions
Thermal characterization

Analogy with Debye relaxation

Cole-Cole plot

\[ \varepsilon = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 + (j\omega \tau)^{1-\alpha}} \]

\[ Z_{th}(j\omega) = \sum_{i=1}^{n} \frac{A_i}{1 + (j\omega \tau_i)^{1-\alpha_i}} \]
Thermal characterization

Diode – Curve fitting

<table>
<thead>
<tr>
<th>$i$</th>
<th>$A_i$ [K/W]</th>
<th>$\tau_i$ [s]</th>
<th>$\alpha_i$ [-]</th>
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<td>0.09</td>
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<td>7.08</td>
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<td>3</td>
<td>10.5</td>
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</tbody>
</table>
Overview

Introduction
Thermal characterization
**Numerical calculations**
Influence of substrate thickness
Present and future work
**Numerical calculations**

**Boundary Element Method (1)**

**Conduction equation**

\[
 k \nabla^2 T(\vec{r}) - j\omega C_v T(\vec{r}) = 0
\]

**Fundamental solution**

\[
 G(\vec{r}, \vec{r}') = \frac{1}{4\pi k |\vec{r} - \vec{r}'|} \exp \left( -\frac{j\omega C_v |\vec{r} - \vec{r}'|}{k} \right)
\]

**Green's theorem**

**Boundary integral**

\[
 T(\vec{r}') = k \int_S \left( G(\vec{r}, \vec{r}') \frac{\partial T(\vec{r})}{\partial n} - T(\vec{r}) \frac{\partial G(\vec{r}, \vec{r}')}{\partial n} \right) dS
\]
Numerical calculations
Boundary Element Method (2)

- Boundary integral
- Discretisation
- Boundary conditions

Algebraic set

Unknown functions:

\[ T, \quad \frac{\partial T}{\partial n} \]
Numerical calculations

Boundary conditions – Overview

- **HEAT SOURCE**
  - ingoing flux = power density

- **INTERFACES**
  - flux: continuous (conservation of energy)
  - temp: ??

- **BOTTOM SURFACE**
  - $T = 0$ (ambient)

- **ALL OTHER SURFACES**
  - adiabatic

Thermal impedance calculations for micro-electronic structures
**Numerical calculations**

**Boundary conditions – Interfaces**

- real material interface not perfectly smooth

- temperature drop over air filled cavities

- modelled by thermal contact resistance $r_c$

\[ T_1 - T_2 = r_c \cdot q \]
Overview

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Infl. of substrate thickness

Purpose & setup

- illustrative introduction to the subject
- provides physical insight in thermal AC problems

**UNIFORM HEAT SOURCE**
50μm x 50μm - centered

**SILICON SUBSTRATE**
- $k = 160 \text{ W/mK}$
- $C_v = 1.8 \times 10^6 \text{ J/m}^3\text{K}$

**HEAT SINK (T = 0)**
Infl. of substrate thickness
Thermal impedance for various H values

- 56 kHz (all curves)
- 4.2 kHz
- 1.8 kHz
- 1 kHz
- 562 Hz

- H = 50μm
- H = 100μm
- H = 150μm
- H = 200μm
- H = 250μm

Thermal impedance calculations for micro-electronic structures
Infl. of substrate thickness
High frequency behaviour

- high frequency arc not influenced by substrate thickness
- physical explanation: rule of thumb for AC sources

characteristic dimension of heated zone $\propto \frac{1}{\sqrt{\omega}}$
Infl. of substrate thickness
Thermal resistance (DC analysis)

\[ R_{th} = R_0 + \alpha \cdot H \]

extrapolation: \( R_0 = 38 \, \text{K/W} \)

slope \( \alpha = 2.72 \times 10^5 \, \text{KW}^{-1}\text{m}^{-1} \)
Infl. of substrate thickness

Heat spreading model

\[ R_{th} = R_{geo} + R_{bulk} \]

\[ = R_{geo} + \frac{H - L}{k \cdot W^2} \]

\[ = \left( R_{geo} - \frac{L}{kW^2} \right) + \frac{1}{kW^2} \cdot H \]

\( R_0 \)
Infl. of substrate thickness
Model – Geometrical resistance

\[ dR_{th} = \frac{dz}{k \cdot A(z)} = \frac{dz}{k(w + 2z \tan \phi)^2} \]

\[ R_{GEO} = \int_{z=0}^{z=L} dR_{th} = \frac{1 - w/W}{2w k \tan \phi} \]

\[ \phi = 32.5^\circ \]

\[ L = 78 \, \mu m \]

\[ R_{GEO} = 65.4 \, K/W \]
Infl. of substrate thickness
Comparison: model vs. simulations

**MODEL**

\[ R_0 = R_{geo} - \frac{L}{\kappa \omega^2} \]

\[ = 65.4 - \frac{78 \cdot 10^{-6}}{160 \cdot (150 \cdot 10^{-6})^2} \]

\[ = 43.7 \text{K/W} \]

\[ \alpha = \frac{1}{\kappa \omega^2} = \frac{1}{160 \cdot (150 \cdot 10^{-6})^2} \]

\[ = 2.78 \cdot 10^5 \text{KW}^{-1}\text{m}^{-1} \]

**SIMULATIONS**

(curve fitting data)

\[ R_0 = 38 \text{K/W} \]

\[ \alpha = 2.72 \cdot 10^5 \text{KW}^{-1}\text{m}^{-1} \]
Infl. of substrate thickness

Extension: asymmetric cases

Thermal impedance calculations for micro-electronic structures
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Infl. of substrate thickness

Impedance for off-centre case
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Present and future work
Effects of miniaturisation

► continuous tendency towards smaller & faster electronics (IMEC: CMOS 45nm)

► thermal phenomena occurring in extremely small scale of space & time → 2 main issues:
  – non-Fourier conduction
  – physical modelling of heat transfer in general

► “nano heat transfer”
Present and future work

Non-Fourier conduction

- heat conduction = mechanical process
- limited to speed of sound
  (Silicon: $v = 5 \times 10^3$ m/s = 5 nm/ps)
- Fourier $\Rightarrow$ Cattaneo/Vernotte

$$\tilde{q} = -k \nabla T$$

(cf. $\tilde{J} = \sigma \tilde{E} = -\sigma \nabla V$)

$$\tilde{q} + \tau \frac{\partial \tilde{q}}{\partial t} = -k \nabla T$$

(Silicon: $\tau = 3.5$ ps)

- gives rise to thermal wave effects
Present and future work

Limits of classical theory

- macroscopic parameters \((k, C_v)\) might lose their meaning for very small structures
- definition of “temperature” itself becomes obscure: measure for “average energy”
- nano-scale structure: limited number of atoms \(\rightarrow\) large temp. deviations to be expected
- may have large influence on electric characteristics; substantial increase of thermally induced noise not unlikely
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