Figure 10 Measured radiation pattern of a 4×4 array designed for low side-lobe performance as plotted for frequencies from 1.9 to 2.5 GHz when the desired main-beam direction is 45°

The radiation pattern of the 4×4 array when the $H(u_1, u_2)$ is given by Chebyshev polynomial of Eq. (5) with side-lobe level assumed to be -20 dB (not reached here because of too small array size) are plotted in Figures 6 and 7. The required weighting coefficients, as obtained from Eq. (3), are 1 (dB), 0.56 (-2.5 dB), 0.42 (-3.7 dB), 0.16 (-8 dB), 0.06 (-12.2 dB), and 0.008 (-21 dB). This corresponds to a 12.2-dB voltage dynamic range when the weight of 0.008 is neglected (in practice, the antenna element requiring this small weight is match-terminated). Two situations when the mutual-coupling effect is neglected or included are considered, respectively. When comparing these results with the ones in Figures 4 and 5, it is apparent that the new radiation patterns feature lower side-lobe levels. The lower side-lobe level is obtained at the expense of a wider main beam. Also, the used attenuators (or amplifiers) need to feature a larger dynamic range.

4. EXPERIMENTAL RESULTS

In order to complete the validity of the presented concept of a wideband array antenna with beam-steering capability using real-valued weights, its prototype was developed, as shown in Figure 8. This prototype is capable of realizing the radiation pattern for both maximum gain and low side lobes. The two designs are achieved by employing different weighting coefficients, as explained in the previous section.

In the chosen approach, the weights are realized using attenuators in microstrip technology with the values shown in the previous section. These attenuators were developed by using impedance steps in microstrip. As some of the weighting coefficients are negative, the combination of attenuators and a 180° hybrid (in the form of a 3-dB rat-race coupler) was used to realize them practically.

Figures 9 and 10 show the measured results for the radiation patterns when the main beam is aimed in the 45° off-broadside direction similarly to the simulated cases described in section 3. Comparison between the simulated (Figs. 5 and 7) and measured (Figs. 9 and 10) results show good agreement. As expected, due to the mutual-coupling effects, the measured results (similarly to the simulated ones) feature increased side lobes. The locations of the main beam and nulls were well preserved, as compared with the simulation results. Small discrepancies concerning the side-lobe levels could be due to the nonideal behavior of the developed

attenuators and the 180° hybrid. The advantage of employing the low-side-lobe design is apparent. The measured side lobes for this design stay below -8 dB. This level was unachievable with the array aimed for maximum-gain operation.

V. CONCLUSION

In this paper, the design of a compact array antenna capable to steer a beam in azimuth over a wide frequency band without the use of phase shifters, frequency filters, or delay networks has been presented. By applying the Chebyshev polynomial to synthesizing the radiation pattern, the array offers low side-lobe-level performance while preserving the use of real-valued weighting coefficients. This wideband array antenna can be realized at low cost and therefore should be found to be attractive by designers of wideband smart-antenna applications aiming to mitigate multipath and interference in high-data-rate mobile communication systems.

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PHASE-MODE-BASED CONSTRUCTION OF A COUPLING MATRIX FOR UNIFORM CIRCULAR ARRAYS WITH A CENTER ELEMENT

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ABSTRACT: A rigorous circuit model with a limited number of parameters is presented to describe mutual coupling in uniform circular arrays (UCAs) with a center element. The model, which is based on a phase-mode expansion, is applied to construct a coupling matrix in order to compensate for mutual coupling in this type of arrays. A sample dual-band uniform circular array with center element illustrates the use of the coupling matrix for direction-of-arrival (DOA) estimation. It is explained in which cases mutual coupling is fully taken into account by the coupling matrix and in which cases some residual errors due to mu-

Key words: antenna array mutual coupling; array signal processing; circular arrays; mobile communication

1. INTRODUCTION

Multiantenna systems in principle allow us to drastically improve the theoretical performance in state-of-the-art mobile communications systems. However, in order to increase the performance in practical systems, the communication algorithm needs to take into account the actual electromagnetic (EM) characteristics of the antenna system. Indeed, mutual coupling between elements in the antenna array drastically changes the system's behavior and its communication characteristics. In an array configuration, the EM characteristics of the antenna elements differ from the properties of the standalone elements because of two important effects. First, terminal currents induced in the loads of neighboring elements introduce extra voltages over these loads. This effect is described by the impedance matrix $\bar{Z}$. In [1], this impedance matrix is used to study the effect of mutual coupling on the performance of an adaptive linear array. A second important effect is the deformation of the radiation pattern of the standalone antenna elements due the current distributions induced in the open-circuited neighboring antenna elements (shadowing effects) or neighboring scatterers (platform effects). As this effect occurs for a zero terminal current in the neighboring antenna elements, it only influences the diagonal elements of the impedance matrix $\bar{Z}$, but not the off-diagonal elements. An important consequence of the shadowing or platform effects is that the open-circuit voltage at a particular antenna terminal depends on the direction of the incoming signal, even in the case of antenna elements that radiate omnidirectionally in the standalone configuration. To accommodate for this second phenomenon as well, in [2] the concept of impedance matrix $\bar{Z}$ was generalized to a coupling matrix $\bar{C}$. Based on this approach, the multiple signal classification (MUSIC) algorithm was modified in [3] to take mutual coupling into account. As described in [4, 5], in many cases the coupling matrix is able to compensate for mutual coupling. In [4, 5] it was also demonstrated, however, that in some cases mutual coupling cannot be accounted for by means of a coupling matrix. This is especially so when platform effects play an important role.

Figure 1 Geometry of the UCA with center element

Figure 2 Circuit model of the UCA in receive mode for phase modes of the orders $l(N-1)$ ($l = 0, 1, \dots$)

Uniform circular arrays (UCAs) have some interesting advantages over classical linear arrays, such as a 360° scan angle and a beam-width which is nearly independent from the scan angle. On the other hand, mutual-coupling effects are generally more severe in UCA configurations, as shadowing and platform effects play a larger role than in the linear configuration. In [6], an analytical circuit model based on a limited number of phase modes/spherical modes was presented to describe mutual coupling in UCAs. Dedicated eigenstructure techniques for direction-of-arrival (DOA) estimation with uniform circular arrays were derived that fully take into account the mutual-coupling effects. Expanding the open-circuit voltage of the receiving antenna elements into spherical modes provided physical insight into this problem and allowed us to distinguish between cases for which compensation guarantees accurate results and cases where mutual-coupling, shadowing, and platform effects cannot be compensated.

In this paper, we extend the model developed in [6] to UCAs with an additional antenna element, placed in the center of the array. Nowadays, such UCA configurations have become more and more important for beam steering [7] and direction-of arrival estimation [8]. A phase-mode description for the UCA with center element is derived in section 2. The phase-mode model is then applied in section 3 in order to construct a coupling matrix for the UCA with center element. In section IV, the use of the coupling matrix in DOA estimation is illustrated by considering a relatively complex type of UCA with a center element, where all the elements are dual-band dipole antennas. The way mutual coupling is compensated for by the coupling matrix is explained by observing the phase-mode expansion at 900 and 1800 MHz. Furthermore, we examine in which cases mutual coupling is fully taken into account by the coupling matrix and in which cases some residual errors due to mutual coupling remain in the DOA estimates.

2. PHASE-MODE DESCRIPTION FOR THE UCA WITH CENTER ELEMENT

Consider a UCA (Fig. 1) consisting of N elements, with $N-1$ identical antennas uniformly distributed on a ring with radius r and with one antenna element in the center of the ring. The antenna element in the center does not need to be identical to the antennas on the ring. However, in order to maintain the UCA character of the array, the center element must exhibit rotational symmetry, such that its open-circuit voltage satisfies

$$V_{0,1}(\theta, \phi) = V_{0,1}\left(\theta, \phi - \frac{2\pi}{N-1}\right).$$
 (1)

We number the elements in such a way that the center element is the first element. For this configuration, the input impedance matrix $\bar{Z}$ of the array takes the following form:

$$\bar{Z} = \left(\begin{array}{c|c} Z_{1,1} & Z_{1,2} \mathbf{I}_{1 \times (N-1)} \\ \hline Z_{1,2} \mathbf{I}_{(N-1) \times 1} & \bar{Z}_{\text{outer}} \end{array} \right).$$
 (2)

Figure 3 Circuit model of the UCA in receive mode for phase modes of the orders m different from $l(N-1)$ ($l = 0, 1, \dots$).

The matrix $\bar{Z}_{\text{outer}}$, which describes the interactions between the $N-1$ antenna elements on the ring with radius r , has a Toeplitz structure. A similar symmetry can be seen in the open-circuit voltages which depend on azimuth ϕ and elevation θ . $Z_{1,2}$ characterizes the coupling between the center element and each of the outer elements. $\mathbf{I}_{1 \times (N-1)}$ is an $N-1$ dimensional vector containing ones. The open-circuit voltage $V_{0,1}(\theta, \phi)$ of the central receiving antenna element differs from the open-circuit voltages $V_{0,i}(\theta, \phi)$ ($i = 2, \dots, N$) of the outer antenna elements. The circular nature of the array implies that $V_{0,i}(\theta, \phi) = V_{0,2}(\theta, \phi - (i-2)(2\pi/(N-1)))$, for $i = 2, \dots, N$.

Let us now decompose both the open-circuit voltages and the impedances into phase modes. In [6], it was shown that the number of phase-modes required for an accurate description is bounded by the overall dimensions of the array. Therefore, the decomposition of the open-circuit voltage of antenna element i , given by

$$V_{0,i}(\theta, \phi) = \sum_{m=-M}^{+M} V_{0,i,m}^\phi(\theta) e^{jm\phi}, \quad (3)$$

can be restricted to m running from $-M$ to M , provided that $M \gg (2\pi d/\lambda)$, with d the largest dimension of the array and λ the wavelength. Hence, a limited number of terms in the series for the open-circuit voltage is sufficient to reconstruct the dependency on azimuth angle ϕ and elevation angle θ of the incoming plane wave. Based on the theory outlined in [6], following rule of thumb can be derived in order to obtain an accuracy better than 0.1%:

$$M = \left\lceil 4 \frac{d}{\lambda} + 4 \right\rceil, \quad (4)$$

with $\lceil \dots \rceil$ the ceiling operation. Consequently, besides $Z_{1,1}$ and $Z_{1,2}$, in total $2M+1 = \lceil 8(d/\lambda) + 9 \rceil$ phase-mode coefficients $V_{0,i,m}^\phi(\theta)$ and phase-sequence impedances Z_m^ϕ (defined in the sequel) are sufficient to describe the receiving properties at all ports of the UCA with center element, for all possible incoming plane waves and for a fixed elevation angle θ . Moreover, for the antenna elements on the ring, once we know the series expansion for port 2, we can construct the open-circuit voltage at an arbitrary port $i = 2, \dots, N$ by relying on symmetry:

$$V_{0,i}(\theta, \phi) = V_{0,2}(\theta, \phi - \phi_i) = \sum_{m=-M}^{+M} V_{0,2,m}^\phi(\theta) e^{jm(\phi - \phi_i)}. \quad (5)$$

The phase-mode decomposition of the impedance matrix (2) is somewhat more complicated than in [6]. For each phase mode m ,

Figure 4 Nine-element UCA with center element consisting of nine outer dual-band dipole antennas and center dual-band dipole antenna

the impedance matrix $\bar{Z}_{\text{outer}}$ reduces to the phase-sequence impedance Z_m^ϕ , which, as in [6], is given by

$$Z_{\text{outer},m}^\phi = \sum_{n=1}^{N-1} Z_{\text{outer},i-1,n} e^{-j(2\pi m(i-1-n)/(N-1))}. \quad (6)$$

Because of the symmetry, this impedance is independent of the port number $i = 2, \dots, N$. Because of its location and its rotational symmetry, the inner antenna element 1 only contributes to phase modes of the orders $l(N-1)$ ($l = 0, 1, \dots$). For these modes, its circuit model is described by the impedance $Z_{1,1}$, the phase-mode open-circuit voltage $V_{0,1,l(N-1)}^\phi(\theta)$ and a voltage source describing the coupling to the outer antenna elements, given by $(N-1)Z_{1,2}I_{2,l(N-1)}^\phi$, with $I_{2,l(N-1)}^\phi$ the phase-mode current for the outer antenna elements. The phase-mode current on the center antenna element $I_{1,l(N-1)}^\phi$ contributes to the phase-mode

Figure 5 Open-circuit voltage as a function of the azimuth angle ϕ of the incoming plane wave (incidence in azimuth plane $\theta = \pi/2$), at 900 MHz. $V_{0,1}(\pi/2, \phi)$: open-circuit voltage of center element 1; $V_{0,2}(\pi/2, \phi)$: open-circuit voltage of outer element 2

Figure 6 Open-circuit voltage as a function of the azimuth angle ϕ of the incoming plane wave (incidence in azimuth plane $\theta = \pi/2$), at 1800 MHz. $V_{0,1}(\pi/2, \phi)$: open-circuit voltage of center element 1; $V_{0,2}(\pi/2, \phi)$: open-circuit voltage of outer element 2

voltage of the outer antenna elements under the form of a voltage source $Z_{1,2} I_{1,l(N-1)}^\phi$. For all other orders of phase modes, the outer elements do not interact with the center element and the circuit model derived in [6] still holds. The mutual coupling in a UCA with center antenna element is thus fully described by the phase-mode circuit models shown in Figures 2 and 3. In this model, the phase-mode currents and voltages at the different ports are uncoupled for different mode orders m .

3. CONSTRUCTION OF A COUPLING MATRIX

The use of a coupling matrix is a very popular way to represent mutual coupling in arrays. Often, this matrix is estimated by fitting the actual array response to the ideal array response at a number of discrete angles [4]. In [6], a procedure was outlined to derive the elements of a coupling matrix based on the phase-mode parameters of the UCA, together with a criterion demonstrating for which UCA geometries a coupling matrix is able to correctly compensate for mutual coupling and for which configurations the coupling matrix approach does not guarantee accurate results. We now extend this approach to a UCA with a center antenna element. Let the number of outer antenna elements be at least equal to the number of phase modes required for an accurate description of the mutual coupling effects in the array, that is, $N - 1 = 2M + 1 = \lceil 8(d/\lambda) + 9 \rceil$. Assuming that all signals arrive under the same elevation angle θ and that all antenna ports of the UCA are loaded by an impedance Z_0 , we know that the voltage over the loads is given by

$$\mathbf{V}_{Z_0}(\theta, \phi) = Z_0(\bar{\mathbf{Z}} + Z_0\bar{\mathbf{I}})^{-1} \cdot \mathbf{V}_0(\theta, \phi), \quad (7)$$

with $\bar{\mathbf{I}}$ the $N \times N$ unit matrix and with $\mathbf{V}_0(\theta, \phi) = [V_{0,1}(\theta, \phi), V_{0,2}(\theta, \phi), \dots, V_{0,N}(\theta, \phi)]^T$. By decomposing the voltages and the impedances into the $2M + 1$ lowest-order phase-modes and by relying on the circuit models in Figures 2 and 3, we obtain an equivalent and equally rigorous and accurate description as fol-

lows. From Figure 3, that is, for phase-modes of order $m = \pm 1, \dots, \pm M$, we derive following relation for the phase-mode voltages of order m of the outer antenna elements:

$$V_{Z_0,2,m}^\phi(\theta) = \frac{Z_0}{Z_0 + Z_{0,2,m}} V_{0,2,m}^\phi(\theta). \quad (8)$$

As for the phase-mode contribution of order 0, a more complicated relationship is found as mutual coupling is observed between the outer elements and the center element. From Figure 2, we derive

$$\begin{pmatrix} V_{Z_0,1,0}^\phi(\theta) \\ V_{Z_0,2,0}^\phi(\theta) \end{pmatrix} = Z_0 \begin{pmatrix} Z_0 + Z_{1,1} & (N-1)Z_{1,2} \\ Z_{1,2} & Z_0 + Z_{0,2,0}^\phi \end{pmatrix}^{-1} \cdot \begin{pmatrix} V_{0,1,0}^\phi(\theta) \\ V_{0,2,0}^\phi(\theta) \end{pmatrix}. \quad (9)$$

Note that, for a fixed elevation angle θ , the open-circuit phase-mode voltages are complex numbers and that they are independent of the azimuth angle ϕ . This was not the case for the voltages in (7). Assume now that the response of an ideal uniform circular array with center element but without mutual coupling is given by

$$\begin{aligned} \mathbf{V}_0^{(NC)}(\theta, \phi) &= \mathbf{V}_0^{(1)}(\theta, \phi) \\ &\times (1, e^{j\theta \cos \phi}, e^{j\theta \cos(\phi - (2\pi/(N-1)))}, \dots, e^{j\theta \cos(\phi - (2(N-2)\pi/(N-1)))})^T, \end{aligned} \quad (10)$$

with $V_0^{(1)}(\theta, \phi)$ the open-circuit voltage of a standalone antenna element of the UCA or of a standalone reference antenna element such as a $\lambda/2$ dipole. The first element of the manifold vector (10) corresponds to the ideal center element, whereas the $N - 1$ following entries describe the outer standalone antenna elements. The ideal array manifold (10) can also be decomposed into phase-mode voltages $V_{0,1,m}^{\phi,(NC)}(\theta)$ for the center element and $V_{0,2,m}^{\phi,(NC)}(\theta)$ for the outer elements. As both the realistic and the ideal arrays are about the same size, the number of relevant phase modes is identical; only their relative contribution differs. The correct distribution of phase modes describing the realistic array can be reconstructed from the phase modes modelling the ideal array manifold by loading the ideal array with an appropriate phase-

Figure 7 Black bars: coefficients $|V_{Z_0,2,m}^\phi(\pi/2)|$ at 900 MHz of the phase-mode expansion of the voltage over the load Z_0 of an outer antenna element; white bars: phase-mode components $|V_{0,2,m}^{\phi,(NC)}(\pi/2)|$ of the reference element; grey bars: phase-mode expansion of (14), phase-mode components of the reference element compensated by the coupling matrix

TABLE 1 Phase-Mode Expansion of the Voltage Over the Center Element at 900 MHz

m	$ V_{Z_0,1,m}^\phi(\pi/2) $	$ V_{0,1,m}^{\phi,(NC)}(\pi/2) $, Before Compensation	Phase-Mode Expansion of (14) After Compensation
0	$-0.014325 + 0.024938j$	$0.043037 - 0.0019966j$	$-0.014325 + 0.024938j$
± 9	$6.69 \cdot 10^{-9} + 2.27 \cdot 10^{-8}j$	0	0

sequence impedance for each phase mode. For the outer antenna elements, $2M + 1$ phase modes are to be compensated. In order to compensate for each phase-mode of order $m = \pm 1, \dots, \pm M$ in the ideal array, we obtain a phase-sequence impedance $Z_{\text{outer},m}^{\phi,(MC)}$ for the outer antenna elements by solving

$$V_{Z_0,2,m}^\phi(\theta) = \frac{Z_0}{Z_{\text{outer},m}^\phi + Z_0} V_{0,2,m}^\phi(\theta) = \frac{Z_0}{Z_{\text{outer},m}^{\phi,(MC)} + Z_0} V_{0,2,m}^{\phi,(NC)}(\theta). \quad (11)$$

As for the phase-mode contributions of order zero, we determine a phase-sequence impedance $Z_{1,1}^{(MC)}$ for the center antenna element and a phase-sequence impedance $Z_{\text{outer},0}^{\phi,(MC)}$ for the outer antenna elements by solving

$$V_{Z_0,1,0}^\phi(\theta) = Z_0 \frac{(Z_0 + Z_{\text{outer},0}^\phi) V_{0,1,0}^\phi(\theta) - (N-1) Z_{1,2} V_{0,2,0}^\phi(\theta)}{(Z_0 + Z_{1,1})(Z_0 + Z_{\text{outer},0}^\phi) - (N-1) Z_{1,2}^2} \\ = \frac{Z_0}{Z_{1,1}^{(MC)} + Z_0} V_{0,1,0}^{\phi,(NC)}(\theta), \quad (12)$$

$$V_{Z_0,2,0}^\phi(\theta) = Z_0 \frac{-Z_{1,2} V_{0,1,0}^\phi(\theta) + (Z_0 + Z_{1,1}) V_{0,2,0}^\phi(\theta)}{(Z_0 + Z_{1,1})(Z_0 + Z_{\text{outer},0}^\phi) - (N-1) Z_{1,2}^2} \\ = \frac{Z_0}{Z_{\text{outer},0}^{\phi,(MC)} + Z_0} V_{0,2,0}^{\phi,(NC)}(\theta). \quad (13)$$

Equations (11)–(13) map the ideal phase-mode voltages $V_{0,1,0}^{\phi,(NC)}(\theta)$ and $V_{0,2,0}^{\phi,(NC)}(\theta)$, for a UCA with center element without mutual coupling between the antennas, onto the actual phase-mode voltages $V_{Z_0,1,0}^\phi(\theta)$ and $V_{Z_0,2,0}^\phi(\theta)$ of the realistic array with mutual coupling. For a UCA with one center element and $N - 1$ outer elements, we are able to correct N mode components, that is, order 0 for the inner element (1 mode component) and for the outer elements $N - 1$ components of orders $M = -(N - 2)/2, \dots, (N - 2)/2$ when $N - 1$ is odd, or $M = -(N - 3)/2, \dots, (N - 1)/2$ when $N - 1$ is even. Thus, we can correct mutual coupling with high accuracy by means of a coupling matrix, provided that $N - 1 \gg (2\pi r/\lambda)$. Once all relevant phase-sequence impedances for compensating the ideal array manifold are known, the compensating impedance matrix for the outer elements $\bar{Z}_{\text{outer}}^{MC}$ is found by using an inverse discrete Fourier transform (the inverse of relation (6)). Finally, one obtains the following for the coupling matrix $\bar{C}$:

$$\mathbf{V}_{Z_0}(\theta, \phi) = \bar{C} \cdot \mathbf{V}_{Z_0}^{(NC)}(\theta, \phi) = Z_0(\bar{Z}_{MC} + Z_0 \bar{I})^{-1} \cdot \mathbf{V}_{Z_0}^{(NC)}(\theta, \phi), \quad (14)$$

where the compensating impedance matrix for the UCA with center element $\bar{Z}_{MC}$ is given by

$$\bar{Z}_{MC} = \begin{pmatrix} Z_{1,1}^{MC} & \mathbf{0}_{1 \times (N-1)} \\ \mathbf{0}_{(N-1) \times 1} & \bar{Z}_{\text{outer}}^{MC} \end{pmatrix}. \quad (15)$$

The coupling matrix $\bar{C}$ shall now be used to compensate for mutual coupling in DOA-estimating algorithms such as MUSIC. Suppose

that vertically polarized plane waves coming from L sources emitting bit-sequences $s_l(t)$ impinge on the UCA from azimuth angles ϕ_l and from the elevation angle θ_0 . Over the N antenna loads, we obtain data samples $\mathbf{x}(t)$:

$$\mathbf{x}(t) = \sum_{l=1}^L \mathbf{V}_{Z_0}(\theta_0, \phi_l) s_l(t) + \mathbf{n}(t), \quad (16)$$

where $\mathbf{n}(t)$ is an additive white Gaussian noise component. In this paper, the SNR is defined by $\text{SNR} = (E[|V_{0,1}(\theta_0, \phi)|^2] / (E[|n(t)|^2]))$. Thus, as the reference value for the signal contribution we take the mean square of the open-circuit voltage induced by an incoming plane wave with $|E_z| = 1$. We chose not to use the measured voltage over a load Z_0 , represented by $E[|V_{Z_0,i}(\theta_0, \phi)|^2]$, in the definition of SNR. With our definition of the SNR, severe mismatch with respect to the load Z_0 will result in a reduction of the SNR. This mismatch effect is seldomly taken into account in the literature. The MUSIC algorithm proceeds by calculating the correlation matrix $\bar{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)]$ and by decomposing $\bar{R}$ into signal and noise subspaces by means of the an eigenvalue decomposition or an SVD. Once a coupling matrix $\bar{C}$ has been constructed, the MUSIC spectrum compensated for mutual coupling is found by projecting the compensated manifold $\bar{C} \cdot \mathbf{V}_{Z_0}^{(NC)}(\theta, \phi)$ onto the noise subspace (recall that in the manifold $\mathbf{V}_{Z_0}^{(NC)}(\theta, \phi)$ all mutual-coupling effects were neglected). The DOA estimates are then found as peaks in the MUSIC spectrum. In order to validate the coupling-matrix approach, we shall compare the compensated MUSIC spectrum with the exact MUSIC

Figure 8 Black bars: coefficients $|V_{Z_0,2,m}^\phi(\pi/2)|$ at 1800 MHz of the phase-mode expansion of the voltage over the load Z_0 of an outer antenna element; white bars: phase-mode components $|V_{0,2,m}^{\phi,(NC)}(\pi/2)|$ of the reference element; grey bars: phase-mode expansion of (14), phase-mode components of the reference element compensated by the coupling matrix

TABLE 2 Phase-Mode Expansion of the Voltage Over the Center Element at 1800 MHz

m	$ V_{Z_{0,1,m}}^{\phi}(\pi/2) $	$ V_{0,1,m}^{\phi,(NC)}(\pi/2) $, Before Compensation	Phase-Mode Expansion of (14) After Compensation
0	0.008298 - 0.009236j	0.001257 - 0.006055j	0.0082971 - 0.0092355j
± 9	$-2.40 \cdot 10^{-5} + 1.32 \cdot 10^{-5}j$	0	0

TABLE 3 Mean and Standard Deviation for DOA Estimation at 900 MHz for $N - 1 = 9$

SNR	MUSIC Based on Exact Array Manifold		MUSIC Based on Coupling Matrix	
	3 dB	10 dB	3 dB	10 dB
DOA 1	55.07° (0.49°)	55.02° (0.17°)	55.07° (0.49°)	55.02° (0.17°)
DOA 2	95.02° (0.65°)	95.00° (0.24°)	95.02° (0.64°)	95.00° (0.24°)
DOA 3	142.99° (0.40°)	142.99° (0.15°)	142.99° (0.40°)	142.99° (0.15°)

spectrum, obtained by projecting the exact manifold $V_{Z_0}(\theta_0, \phi_l)$ onto the noise subspace.

4. EXAMPLE

In order to illustrate the use of the coupling matrix in DOA estimation, we consider a UCA with center element, consisting of relatively complex antenna elements. All antenna elements are assumed to be perfectly conducting metallic wire structures, so that the array can be modelled by means of the publicly available NEC-2 code. It has been demonstrated that one can easily compensate for mutual coupling in arrays consisting of simple $\lambda/2$ dipole elements. This is due to the fact that a single mode consisting of a sinusoidal current profile of a half-wavelength on each dipole element mainly determines the radiation characteristics. However, dipole antenna elements are relatively narrowband, which can result in a severe impedance mismatch when using these elements to perform DOA estimation at different frequencies [9]. Therefore, more complex wire antennas should be used to cover multiple frequencies. In Figure 4, a dual-band dipole antenna, consisting of a dipole with length $l = 15.23$ cm surrounded by two parasitic radiators at a distance $d = 1.30$ cm and of length $l_p = 6.65$ cm, was tuned to 50Ω at 900 and 1800 MHz ($S_{11} < -10$ dB), in order to serve as the outer antenna element. For this type of antenna element, compensation of mutual coupling is not straightforward, as currents also flow on the parasitic wires that ensure dual-band behavior. For this UCA geometry, the rule of thumb (4) predicts that 13 outer elements are required to model the EM behavior of the UCA with very high accuracy (better than 0.1%). To illustrate all the effects clearly, we examine the array at lower accuracy by taking into account fewer phase modes and we firstly consider a UCA consisting of a nine outer antenna elements, together with a center antenna element (Fig. 4). The outer antenna feeds are distributed uniformly on a circle of radius $r = 8.06$ cm. As for the center element, we consider a dipole with length $l = 15.35$ cm, surrounded by nine parasitic radiators at a distance $d = 1.32$ cm and of length $l_p = 5.86$ cm. This type of center element ensures dual-band behavior, meanwhile satisfy-

ing the symmetry criterion (1) for the center antenna element. In this type of array configuration, the mutual coupling shall be severe, as the different antenna wires shadow the radiation from the other antennas. This can be seen in the azimuth dependency of the open-circuit voltages, shown in Figure 5 at 900 MHz and Figure 6 at 1800 MHz. The open-circuit voltage of the center element is nearly omnidirectional at both frequencies, given the symmetry of the configuration. However, the open-circuit voltage of the outer elements is clearly dependent on the azimuth angle of the incoming plane waves, because of the proximity of the other antenna elements that act as secondary scatterers.

In a realistic application, all antenna terminals are loaded by an impedance Z_0 , thereby increasing the impact of mutual coupling on the voltages $V_{Z_0}(\theta, \phi)$ over the loads, due to the effects of the impedance matrix $\bar{Z}$, as indicated by (7). Even though the antenna elements in the UCA are relatively complex, for the outer elements and at 900 MHz the most significant expansion coefficients $|V_{Z_{0,2,m}}^{\phi}(\pi/2)|$ are found for $|m| \leq 4$ and thus all relevant components are covered by choosing $M = 4$ and $N - 1 = 2M + 1 = 9$, as shown by the black bars in Figure 7. As for the center antenna element, Table 1 shows that one single phase mode component $V_{Z_{0,1,0}}^{\phi}(\pi/2)$ dominates, since only phase-modes of order $m = 0, \pm 9, \pm 18$ contribute, given Eq. (1), and the pattern is nearly omnidirectional since the contribution at $m = \pm 9$ can be neglected. We shall show that in this case the coupling matrix is able to fully compensate for mutual coupling in a UCA with nine outer elements and one center element. At 1800 MHz, not all the relevant phase-mode voltage components are located in the region $|m| \leq 4$, but extend to $|m| \leq 6$, as indicated by the black bars in Figure 8. Moreover, as shown in Table 2, for the center antenna element, the phase-mode component $V_{Z_{0,1,0}}^{\phi}(\pi/2)$ dominates, although the contribution of the phase mode at order $m = \pm 9$ has clearly increased as 1800 MHz. Given these results, the coupling matrix cannot fully compensate for the mutual-coupling effects in a UCA with nine outer elements and one center element, and some residual error will remain in the estimation of the DOAs.

TABLE 4 Mean and Standard Deviation for DOA Estimation at 1800 MHz for $N - 1 = 9$

SNR	MUSIC Based on Exact Array Manifold		MUSIC Based on Coupling Matrix	
	3 dB	10 dB	3 dB	10 dB
DOA 1	55.00° (0.10°)	55.00° (0.05°)	54.94° (0.10°)	54.95° (0.05°)
DOA 2	95.00° (0.10°)	95.00° (0.04°)	94.74° (0.11°)	94.74° (0.05°)
DOA 3	143.00° (0.10°)	143.00° (0.04°)	143.15° (0.10°)	143.16° (0.05°)

Figure 9 Black bars: coefficients $|V_{Z_{0,2,m}}^{\phi}(\pi/2)|$ at 1800 MHz of the phase-mode expansion of the voltage over the load Z_0 of an outer antenna element; white bars: phase-mode components $|V_{0,2,m}^{\phi,(NC)}(\pi/2)|$ of the reference element; grey bars: phase-mode expansion of (14), phase-mode components of the reference element compensated by the coupling matrix

During the construction of the coupling matrix, we use a standalone $\lambda/2$ dipole antenna with open-circuit voltage $V_0^{(1)}(\theta, \phi)$ as reference element, as both center and outer antenna elements in the array. The white bars in Figures 7 and 8 show the phase-mode expansion $V_{0,2,m}^{\phi,(NC)}(\theta)$ of this reference element at 900 and 1800 MHz, respectively, when located at the position of an outer element of the UCA. One observes that about the same number of relevant phase-mode components (that is, $|m| \leq 4$ at 900 MHz and $|m| \leq 6$ at 1800 MHz) describe the electromagnetic characteristics of both the standalone reference antenna and the more complex realistic antenna element placed in the array. For the reference element located in the center of the UCA, only the lowest-order phase mode contributes at 900 and 1800 MHz, as was approximately the case for the realistic antenna element in the presence of mutual coupling (see Tables 1 and 2). From all these observations, we can conclude that the EM characteristics of the realistic antenna elements in the array and of the standalone reference elements only differ in the way the relevant phase-mode components are distributed. For a sufficiently large number of elements, the use of a coupling matrix will correct each relevant phase mode of the standalone reference element without mutual coupling in order to include all the mutual-coupling effects of the realistic elements in the UCA configuration. The result is given by the grey bars in Figures 7 and 8 and in Tables 1 and 2. One indeed observes that for $N - 1 = 9$, the compensation allows us to match the phase modes of order $|m| \leq 4$. In order to demonstrate the performance of the MUSIC algorithm, we consider three uncorrelated sources emitting 10,000-bit pseudo-random bit sequences. All

signals are incident along the x - y plane (elevation $\theta = 90^\circ$), with directions of arrival $\phi_1 = 55^\circ$, $\phi_2 = 95^\circ$, and $\phi_3 = 143^\circ$. The three signals are received in the presence of additive white Gaussian noise. In Table 3, the mean DOA estimates and the standard deviation are presented at 900 MHz, for an ensemble consisting of 500 implementations, for equally strong signals and for two different SNR levels: 3 and 10 dB. One observes that both the MUSIC algorithm based on the exact array manifold and the MUSIC approach based on the coupling matrix yield equally accurate mean DOA estimates and they lead to about the same standard deviation on the estimates. We can conclude that, at 900 MHz, all mutual coupling effects are fully taken into account by this coupling matrix. In Table 4, the mean DOA estimates and the standard deviation are presented at 1800 MHz. Although both MUSIC implementations result in about the same standard deviation on the DOA estimates, a systematic error is found in the mean DOA estimates when applying the coupling matrix, compared to relying on the exact knowledge of the array manifold. Indeed, Figure 8 shows that the coupling matrix does not compensate the phase modes of orders $m = \pm 5$ and $m = \pm 6$, yet these components still yield relevant contributions to the EM behavior of the outer antenna elements in the UCA.

Let us now increase the number of outer antenna elements from $N - 1 = 9$ to $N - 1 = 13$, leading to a UCA with 14 antenna elements in total. Now, as demonstrated in Figure 9, at 1800 MHz the coupling matrix is able to compensate the phase modes of orders $|m| \leq 6$ for mutual coupling. As all relevant phase-mode components are included in the coupling-matrix formalism, the MUSIC algorithm based on the coupling matrix yields mean DOA estimates that are as accurate as the results obtained from the MUSIC spectrum based on the exact knowledge of the array manifold, as shown in Table 5 for the ensemble consisting of 500 implementations at 1800 MHz.

5. CONCLUSION

A rigorous circuit model with a limited number of parameters has been presented in order to describe mutual coupling in uniform circular arrays (UCAs) with a center element. The model, which is based on a phase-mode expansion, allows us to construct a coupling matrix in order to compensate for mutual coupling in this type of array. When the number of antenna elements is sufficiently large compared to the dimensions of the array, the coupling matrix allows to fully account for mutual coupling. When the number of relevant phase modes is larger than the number of antenna elements, some residual errors due to mutual coupling remain when performing DOA estimation.

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TABLE 5 Mean and Standard Deviation for DOA Estimation at 1800 MHz for $N - 1 = 13$

SNR	MUSIC Based on Exact Array Manifold		MUSIC Based on Coupling Matrix	
	3 dB	10 dB	3 dB	10 dB
DOA 1	55.00° (0.06°)	55.00° (0.02°)	55.00° (0.06°)	55.00° (0.02°)
DOA 2	95.00° (0.06°)	95.00° (0.02°)	95.00° (0.06°)	95.00° (0.02°)
DOA 3	143.00° (0.06°)	143.00° (0.02°)	143.00° (0.06°)	143.00° (0.02°)

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FULL-WAVE ANALYSIS OF SINGLE AND COUPLED STRIPLINES IN MULTILAYERED CYLINDRICAL DIELECTRICS USING THE 3D TLM METHOD

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ABSTRACT: In this paper, an inhomogeneous, multilayered cylindrical stripline is analyzed using the 3D-TLM method. Dispersion characteristics of dominant as well as higher-order modes are obtained for single and coupled cylindrical striplines. Impedance characteristics for the dominant mode are also obtained for the same. © 2005 Wiley Periodicals, Inc. *Microwave Opt Technol Lett* 48: 298–302, 2006; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.21331

Key words: cylindrical stripline; SCN; TLM

1. INTRODUCTION

Using flexible dielectric materials, it is possible to construct non-planar transmission lines that can be warped around with a cylindrical surface and used to excite conformal arrays mounted on a cylindrical object. A cylindrical transmission line is also useful for wideband power-combining applications with considerable size reduction. They can also be used as coaxial-to-planar transition adapters and baluns.

Cylindrical striplines have been analyzed in the microwave literature [1–5]. However, these analyses are based on the assumption of quasi-TEM mode of propagation. In [6, 7], the dispersion

Figure 1 (a) Cylindrical single stripline; (b) cylindrical coupled stripline

characteristics of microstrip line on a cylindrical substrate were reported for an open structure.

In this paper, the dispersion and impedance characteristics of shielded cylindrical stripline are presented using the 3D TLM method. A 3D symmetrical condensed node (SCN) [8] in a 2D array is used to analyze the structure in the time domain using Gaussian pulse excitation. Staircase approximation of the circular boundary is used for the structure.

A numerical advantage of TLM compared to the finite-difference method is that all the six field components are available at each node (rather than one), thus making the boundary condition finer and giving more information at each node. Also, TLM is a one-step method, whereas the finite-difference routine is a two-step method. Conceptually, TLM has the advantage that it is a physical model with exact computer simulation.

The organization of this paper is as follows. Section 2 explains the theoretical approach in implementing 2D array of 3D SCN node for analyzing the inhomogeneous cylindrical stripline structure. Section 3 provides a detailed description of the dispersion and impedance characteristics of single and coupled inhomogeneous cylindrical striplines. The conclusion is presented in Section 4.

2. THEORY

The structures to be analyzed are shown in Figure 7. The TLM method models the electromagnetic-field problem by simulating