SURFACE CURRENT MODELLING OF THE SKIN EFFECT

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Abstract: In this contribution the skin effect for 2D on-chip interconnections is predicted using a new equivalent surface current concept. First, the features of the new approach are briefly outlined and the special case of conductors with rectangular cross-section is treated in more detail. Next, the behaviour of the equivalent surface current as a function of frequency is illustrated. Finally, the new method is used to determine inductance and resistance matrices of 2D on-chip interconnect examples with specifications taken from the International Technology Roadmap for Semiconductors (ITRS).

1 Introduction

The evolution towards smaller chip features and increasing clock rates continues as the International Technology Roadmap for Semiconductors (ITRS) predicts that the smallest on-chip features will shrink from 150 nm in 2003 to 50 nm by 2012 while the clock rate will increase from 1.5GHz to 10GHz. An important issue in the representation of signal conductors and their coupling, is the correct modelling of the so-called skin-effect, also known as current crowding, see e.g. [1], [2], [3]. As in [3], the purpose of this paper is to provide a surface admittance description of the conductor. At each frequency, this description associates a fictitious electric surface current density \( J_s(r, \omega) \) at each point \( r \) on the surface of the conductor to the tangential electric fields \( E_{tan}(r', \omega) \) at every other point on the surface. The surface admittance description allows to replace each conductor by equivalent surface currents and to replace the conductor medium by the medium of the material layer it is embedded in. The remaining field problem can then be solved by solely considering the interactions between the equivalent surface currents. Section 2 very briefly presents the general idea behind the differential admittance concept, restricting ourselves to two-dimensional configurations and to the TM-case. In [3] the TM-case is treated using a finite difference solution of the Helmholtz equation in the conductor’s cross-section. Here, a general solution is obtained in terms of the Dirichlet eigenfunctions of the cross-section. Special attention is devoted to a conductor with rectangular cross-section. The numerical data of Section 3 are intended to provide the reader with some insight into the behaviour as a function of frequency of the equivalent surface admittance. Section 4 briefly states how the equivalent surface current can be used to determine the resistance and inductance matrices of a set of parallel conductors by means of an electric field integral equation (EFIE). This is followed by a set of numerical examples. These examples are derived from the CODESTAR-IST [4] project and take into account the most recent specifications of the ITRS.

2 Equivalent surface current and surface admittance

We restrict ourselves to the time-harmonic \( e^{j\omega t} \) dependence transverse magnetic polarization. \( E_z \) is the electric field inside the homogeneous conductor with constitutive parameters \( \epsilon, \mu_0 \) and \( \sigma \) and cross-section \( S \). The conductor is embedded in a planar stratified medium and the non-conducting layer the conductor is embedded in, is characterized by the constitutive parameters \( \epsilon_{out} \) and \( \mu_0 \). On the boundary \( c \) of \( S \) we have that

\[
H_t = \frac{1}{j\omega\mu_0} \partial_n E_z = \frac{1}{j\omega\mu_0} D_k \cdot E_z,
\]

with \( D_k \) the Dirichlet to Neumann operator and with the index \( t \) referring to the tangential component of the magnetic field and with \( \partial_n E_z \) representing the limit of the normal derivative of the electric field tending from the inside of the conductor to \( c \). We replace the conducting medium by the medium outside

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the conductor, in particular the medium of the material layer the conductor is embedded in. On the boundary \( c \) of \( S \) we now find

\[
H_{z0} = \frac{1}{j\omega\mu_0} \partial_n E_{z0} = \frac{1}{j\omega\mu_0} \mathcal{D}_{k_{out}} \cdot E_{z0}.
\]

Consequently, the conductor can be replaced by the material of its surrounding layer, undoing the discontinuity in conductivity and permittivity due to the conductor’s presence, by introducing an equivalent surface current density \( J_{sz} \), related to the value of the field \( E_{z0} \) on the boundary, through the differential surface admittance operator \( \mathcal{Y} \)

\[
J_{sz} = \mathcal{Y} E_{z0} = \frac{1}{j\omega\mu_0} [\mathcal{D}_k - \mathcal{D}_{k_{out}}] \cdot E_{z0}
\]

To obtain (3) we used the fact that on \( c \) and only on \( c \), \( E_{z0} = E_z \). When solving the field problem external to the conductor, the effect of the conductor is exactly accounted for by the presence of the surface current \( J_{sz} \), provided \( E_{z0} \) and the surface current \( J_{sz} \) are forced to satisfy (3) on \( c \). Inside the conductor a fictitious field is obtained.

A general way to obtain the operator \( \mathcal{Y} \) is to use the Dirichlet eigenfunctions of the cross-section \( S \). Calculations, the details of which are given in [5], show that

\[
J_{sz} = \mathcal{Y} E_z = \tau \sum_{m=1}^{\infty} \partial_n \xi_m \int_c E_z \partial_n \xi_m \, dc
\]

\[
= \frac{\sigma \int_c \int_S |E_z|^2 \, dS}{\int_c \int_S \left( \frac{k_{out}^2 \lambda_m}{\epsilon_m} - \frac{k_{out}^2}{\epsilon_m}\right) \, dc},
\]

with \( \tau = [\sigma + j\omega(\epsilon - \epsilon_{out})] \) and where \( k \) is the wavenumber of the conductor and \( k_{out} \) the wavenumber of the medium replacing the conductor. The \( \xi_m \) are the Dirichlet eigenfunctions of the cross-section \( S \) with corresponding eigenvalues \( \lambda_m \).

The Joule losses associated with the surface current \( J_{sz} \) are

\[
\mathcal{P}_s = \Re \left\{ \int_c J_{sz} E_z^* \, dc \right\} = \sigma \int_S |E_z|^2 \, dS,
\]

which are equal to the Joule volume losses. Moreover, the total surface current is

\[
\int_c J_{sz} \, dc = (\sigma + j\omega) \int_S E_z \, dS - j\omega \epsilon_{out} \int_S E_{z0} \, dS.
\]

For a good conductor, the contribution of the displacement current in the r.h.s. of (6) can be neglected and the total surface current is quasi identical to the total conduction current in the conductor.

For a rectangular conductor \( 0 \leq x \leq a \) and \( 0 \leq y \leq b \), the Dirichlet eigenfunctions and eigenvalues are

\[
\xi_{mn} = \frac{2}{\sqrt{ab}} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right),
\]

with \( \lambda_{mn} = [(m\pi)/a]^2 + [(n\pi)/b]^2 \). In [5] it is shown that an analytical expression for \( J_{sz} \) can be obtained by expanding \( E_z \) on each side of the rectangle in an appropriate Fourier sine series. However, in Section 3 we will apply a MoM approach to obtain R- and L-matrices. To be able to do so, we need a discretized form of the operator \( \mathcal{Y} \). To this end both \( E_z \) and \( J_{sz} \) are expanded in pulse basis functions along the four sides of the rectangle. All the pulse basis amplitudes \( E_j \) for the four sides can be collected into a vector \( \mathbf{E} \) and all the pulse basis amplitudes \( J_j \) can be collected into a vector \( \mathbf{J} \). Long, but completely analytical calculations lead to the discretized analytical form of \( \mathcal{Y} \), viz.

\[
\mathbf{J} = \mathbf{Y}_s \cdot \mathbf{E}.
\]

\( \mathbf{Y}_s \) is the surface admittance matrix. We refer the reader to [5] for detailed expressions. 

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3 Surface admittance matrix as a function of frequency

To illustrate the behaviour of the surface admittance as a function of frequency, consider the 15 mil (381μm) by 1.4 mil (35.56μm) copper conductor (σ = 5.8 × 10^7 (Ωm)^{-1}) example also discussed in [2] (see Fig. 1a). The circumference is subdivided into 2 × 51 horizontal and 2 × 5 vertical intervals. The solid lines in Fig. 2 show the real part of the following elements of the Y_{s}-matrix: Y_{26,26}, Y_{54,54}, Y_{26,82} and Y_{1,1}. Segment 26 is located in the middle of the bottom side, segment 54 in the middle of the right side and segment 82 in the middle of the top side. The (1,1)-element has been selected to illustrate the behaviour near a corner. Results are displayed as a function of the skin depth δ = \sqrt{\frac{2}{ωμσ}} and the frequency ranges between 0.1 MHz (δ = 209.98μm) and 100 GHz (δ = 0.2μm). The plotted values are normalized with respect to the value of the real part of Y_{26,26} at 0.1 MHz, i.e. 1.5 × 10^{-3}. The dashed lines show the corresponding results for the absolute value of the imaginary parts. The numerical results clearly show that the real parts are dominant and almost constant at low frequencies, i.e. when the skin depth is large with respect to the width of the conductor. For increasing frequencies, the imaginary parts gain in importance. When the skin depth becomes very small, all non-diagonal elements become negligible and the absolute value of the real and imaginary parts of the diagonal elements becomes identical. This limiting value is the well-known scalar surface admittance value \sqrt{σ/jωμ}. This result does not only follow from numerical observations but can also be proven analytically [5]. In [2] the behaviour of the internal impedance of a good conductor Z_{i} = r_{i} + jωl_{i} as a function of frequency is examined. For the 15 × 1.4 mil example, the authors observe a difference of 20 percent between the per unit-length internal inductive reactance ωl_{i} and the per unit-length resistance r_{i} when the skin-effect is well developed. The highest frequency considered in [2] is 1GHz. Fig. 2 indicates that at 1GHz (δ = 2.01μm) the real and imaginary part of the diagonal elements of the Y_{s}-matrix also differ considerably. However, for increasing frequencies a near convergence towards the well-known scalar surface admittance value is observed.

4 Resistance and inductance matrices

The relevant EFIE, valid as long as the cross-sectional dimensions of the conductors remain small with respect to the free space wavelength, is [1]

\[ E_{z}(r) = -jωA_{z}(r) - \frac{∂V(r)}{∂z}. \]  

(9)

A_{z} is the vector potential, V is the scalar potential. Using the differential surface admittance, the conductors can be replaced by equivalent surface currents J_{sz}. The vector potential of these currents in free space is given by

\[ A_{z}(r) = -μ_{0} \int_{c} J_{sz}(r') \frac{1}{2π} \ln |r - r'| dc(r'). \]  

(10)

In the low frequency limit the constant voltages on the conductors can be related to the resistance R and inductance matrix L per unit-of-length (p.u.l.). The proper definition of R and L and the way to determine their numerical value by solving (9)-(10) can be found in [5].

To illustrate our method, consider a set of 2D configurations as depicted in Fig. 1b-1e. These four examples are taken from the CODESTAR-JST project [4], with some modifications in the dimensions and are representative of advanced on-chip interconnects as proposed by the ITRS roadmap. All cross-sectional dimensions are in nanometre and the resistivity of the conductors is 2.2μΩcm. In all cases the black shaded centre conductor is taken to be the signal conductor and the other conductors are on zero i.e. reference potential. Fig. 3 displays the resulting resistance p.u.l. in Ω/m between 1 GHz and 1000 GHz. The corresponding inductances p.u.l. in nH/m are shown in Fig. 4. Case 1(2,3,4) corresponds with Fig. 1b(c,d,e). In the MoM solution of (9)-(10), one pulse basis function per 13.5μm was used. To obtain the results for case 4, e.g., we first calculated the (9 × 9) R and L matrix of the complete configuration and then enforced the fact that the 8 conductors surrounding the central one are all kept on reference potential. For the resistance results we clearly observe the transition from the DC case to skin-effect behaviour. For
the inductance results we observe the decrease of the total inductance for increasing frequencies as the magnetic field is forced out of the conductors. In the oral presentation additional data on all elements of the complete R and L matrices will be given. The accuracy on the DC-resistance (which is the value obtained below 1 GHz) is better than 1 promille (case 1: 6.172810^{-5}\,\Omega/m, case 2: 6.166910^{-5}\,\Omega/m, case 3: 7.041610^{-5}\,\Omega/m, case 4: 6.790110^{-5}\,\Omega/m).

References


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