Parameterized Macromodeling and Model Order Reduction for High-Speed Interconnects

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Outline

Introduction

Parameterized Macromodels

New interpolation with scaling-shifting coefficients

Numerical examples
  • Spiral inductor
  • PCB

Conclusions
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Parameterized Macromodels

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Conclusions
Aerodynamics
Automotive
Chemistry

Design variables
width, temperature, angle, frequency, ...

Simulation Model
Fluent®, HSPICE®, CST®, Comsol®, Abaqus®, ...

Response variables
lift, S-parameters, pressure, stress, ...

Costly

Parameterized macromodels
Neural network, Kriging, SVM, rational function, spline,...

Cheap

Prototyping
Optimization
Sensitivity Analysis
CAD/CAM/CAE Environment

Adaptive Modeling
Distributed Computing

Configurable infrastructure

Department of Information Technology – Internet Based Communication Networks and Services (IBCN)
Aerodynamics
Automotive
Chemistry
Aerodynamics
Electronics
Metallurgy

Design variables
width, temperature, angle, frequency, ...

Response variables
lift, S-parameters, pressure, stress, ...

Simulation Model
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Adaptive Modeling

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Design process

- several decisions
  - materials
  - geometrical dimensions
  - shape
  - constraints
    - space
    - cost
    - performance
Simulators
- implementation of models
- describe systems behavior
- help designers

Measurements
- post tuning
- verification
- help designers
A typical design process requires

- design space optimization
- design space exploration
- sensitivity analysis
  - multiple simulations (measurements)
  - different design parameters values (e.g. layout features)
A typical design process requires

- Multiple simulations (measurements)
  - computationally expensive (time and memory)

- Can we do better?

- Yes
  - By parameterized macromodels
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Conclusions
sX = AX + BU
Y = CX + DU

parameterized macromodel

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Model-driven PMOR

discretisation by simulators

U(s,g)

sX = AX + BU
Y = CX + DU

model

real world

model

PDE

Y(s,g)

simulations

Data-driven PMOR

parameterized macromodel

U(s,g)

Y(s,g)

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modeling

discretisation by simulators

data-driven PMOR

Model-driven PMOR

\[ U(s, g) \xrightarrow{\text{model}} X = AX + BU \]
\[ Y(s, g) \xrightarrow{\text{model}} Y = CX + DU \]

Parameterized macromodel

Real world

Data-driven PMOR

\[ U(s, g) \xrightarrow{\text{measurements}} Y(s, g) \xrightarrow{\text{data}} H(s, g) \]

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PMOR concepts

Two design space grids are used in the modeling process

- estimation grid
- validation grid

Design space

\[ g = \left( g^{(n)} \right)_{n=1}^{N} \]
Model-driven PMOR

\[ \begin{align*}
    sX &= AX + BU \\
    Y &= CX + DU
\end{align*} \]

**real world**

**model**

**modeling**

**discretisation by simulators**

**parameterized macromodel**

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**Model Order Reduction**

- PRIMA
- LAGUERRE SVD
- HYBRID TBR
- FAST TBR
- EIGENSPACE PROJECTION

**Interpolation Least Squares**

- SYSTEM MATRICES
- TRANSFER FUNCTIONS
- PROJECTION MATRICES

**model-driven PMOR**

- accuracy
- efficiency
- stability guaranteed
- passivity guaranteed
Model-driven PMOR

\[
\begin{align*}
    &sX = AX + BU \\
    &Y = CX + DU
\end{align*}
\]

Data-driven PMOR

\[
\begin{align*}
    &H(s, g) \\
    &Y(s, g)
\end{align*}
\]

parameterized macromodel
Rational Fitting Methods

- VECTFIT
- MULTIPORT
- STABILITY
- PASSIVITY
- NOISY DATA

Interpolation Least Squares

- SYSTEM MATRICES
- TRANSFER FUNCTIONS
- POLES/RESIDUES

data-driven PMOR

- accuracy
- efficiency
- stability guaranteed
- passivity guaranteed

scattered data

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Rational Fitting Methods + Interpolation Least Squares

\[ H(s,g) = \sum_{p=1}^{P} \frac{Q(g)}{s - \alpha_p(g)} \]

\[ H(s,g) = C(g)(sI - A(g))^{-1}B(g) + D(g) \]

data-driven PMOR

scattered data
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- NOISY DATA

Transfer Functions

Interpolation

- SCALING
- SHIFTING

Data-driven PMOR

\[ H(s,g) \]

Scattered data

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Features

- local approach (cell by cell)
Features

- local approach (cell by cell)
- independent from a specific state-space realization
- stability and passivity guaranteed over the design space
- suitable to robust adaptive sampling
- different flavours
Design space \( g = \left( g^{(n)} \right)^N \)
Compute root macromodels $R(s, g_k^{\Omega_2})$

in the estimation design space grid

Compute scaling and frequency shifting coefficients

$\alpha_{1,k}(g_j^{\Omega_1}), \alpha_{2,k}(g_j^{\Omega_1})$

in the estimation design space grid

Multivariate interpolation of

scaling and frequency shifting coefficients

$\alpha_1(g), \alpha_2(g)$

Multivariate interpolation of

scaled and shifted root macromodels

$\alpha_1(g)R(s\alpha_2(g), g)$
\[ R(s, g_j^{\hat{\Omega}}) = C_0(g_j^{\hat{\Omega}}) + \sum_{n=1}^{N(\hat{\Omega})} \frac{C_n(g_j^{\hat{\Omega}})}{s - p_n(g_j^{\hat{\Omega}})} \]
Compute root macromodels $R(s, g_k^{\Omega_2})$

in the estimation design space grid

Compute scaling and frequency shifting coefficients

$\alpha_{1,k}(g_j^{\Omega_1}), \alpha_{2,k}(g_j^{\Omega_1})$

in the estimation design space grid

Multivariate interpolation of scaling and frequency shifting coefficients

$\alpha_1(g), \alpha_2(g)$

Multivariate interpolation of scaled and shifted root macromodels

$\alpha_1(g)R(s\alpha_2(g), g)$
\[
\min_{\alpha_{1,k}(g_j^\Omega), \alpha_{2,k}(g_j^\Omega)} Err(\tilde{R}(s, g_k^\Omega), R(s, g_j^\Omega))
\]

\[
\tilde{R}(s, g_k^\Omega) = \alpha_{1,k}(g_j^\Omega)R(s\alpha_{2,k}(g_j^\Omega), g_k^\Omega)
\]

\[
\alpha_{1,k}(g_j^\Omega) = \alpha_{2,k}(g_j^\Omega) = 1, \ j = k
\]
\[ \min_{\alpha_{1,k}(g_j^\Omega), \alpha_{2,k}(g_j^\Omega)} \text{Err}(\tilde{R}(s, g_j^\Omega), R(s, g_j^\Omega)) \]

\[ \tilde{R}(s, g_k^\Omega) = \alpha_{1,k}(g_j^\Omega) R(s, \alpha_{2,k}(g_j^\Omega), g_j^\Omega) \]

\[ \alpha_{1,k}(g_j^\Omega) = \alpha_{2,k}(g_j^\Omega) = 1, \ j = k \]

\[ \alpha_{2,\pi}(g_j^\Omega), j = 1, \ldots, 4 \]

\[ \alpha_{1,\pi}(g_j^\Omega), j = 1, \ldots, 4 \]

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Compute root macromodels $R(s, g_k^{\Omega_i})$
in the estimation design space grid

Compute scaling and frequency shifting coefficients
$\alpha_{1,k}(g_j^{\Omega_i}), \alpha_{2,k}(g_j^{\Omega_i})$
in the estimation design space grid

Multivariate interpolation of
scaling and frequency shifting coefficients
$\alpha_1(g), \alpha_2(g)$

Multivariate interpolation of
scaled and shifted root macromodels
$\alpha_1(g)R(s\alpha_2(g), g)$
\[ \alpha_{1,3}(\hat{g}^{\Omega})R(\alpha_{2,3}(\hat{g}^{\Omega})s, g_3^{\Omega}) \]

\[ \alpha_{2,3}(g_3^{\Omega}) = \text{interp}(g_3^{\Omega}, \alpha_{2,3}(g_3^{\Omega}), \hat{g}_3^{\Omega}) \]

\[ \alpha_{1,3}(g_3^{\Omega}) = \text{interp}(g_3^{\Omega}, \alpha_{1,3}(g_3^{\Omega}), \hat{g}_3^{\Omega}) \]

\[ R(s, g_3^{\Omega}) \]

\[ \alpha_{1,4}(\hat{g}^{\Omega})R(\alpha_{2,4}(\hat{g}^{\Omega})s, g_4^{\Omega}) \]

\[ \alpha_{2,4}(g_4^{\Omega}) = \text{interp}(g_4^{\Omega}, \alpha_{2,4}(g_4^{\Omega}), g_4^{\Omega}) \]

\[ \alpha_{1,4}(g_4^{\Omega}) = \text{interp}(g_4^{\Omega}, \alpha_{1,4}(g_4^{\Omega}), g_4^{\Omega}) \]

\[ R(s, g_4^{\Omega}) \]

\[ \alpha_{1,k}(\hat{g}^{\Omega}) = \sum_{k_1=1}^{2} \sum_{k_2=1}^{2} \alpha_{1,k}((g_{k_1}^{(1)}, g_{k_2}^{(2)})^{\Omega}) \ell_{k_1} (g^{(1)}) \ell_{k_2} (g^{(2)}) \]

\[ \alpha_{2,k}(\hat{g}^{\Omega}) = \sum_{k_1=1}^{2} \sum_{k_2=1}^{2} \alpha_{2,k}((g_{k_1}^{(1)}, g_{k_2}^{(2)})^{\Omega}) \ell_{k_1} (g^{(1)}) \ell_{k_2} (g^{(2)}) \]
Compute root macromodels $R(s, g_k^{\Omega_i})$

in the estimation design space grid

Compute scaling and frequency shifting coefficients
$
\alpha_{1,k}(g_j^{\Omega_i}), \alpha_{2,k}(g_j^{\Omega_i})$

in the estimation design space grid

Multivariate interpolation of
scaling and frequency shifting coefficients
$\alpha_1(g), \alpha_2(g)$

Multivariate interpolation of
scaled and shifted root macromodels
$\alpha_1(g)R(s\alpha_2(g), g)$
\( \alpha_{1,3}(\hat{g}^{\hat{\Omega}}) R(\alpha_{2,3}(\hat{g}^{\hat{\Omega}}) s, g_{3}) \) \quad \alpha_{1,4}(\hat{g}^{\hat{\Omega}}) R(\alpha_{2,4}(\hat{g}^{\hat{\Omega}}) s, g_{4}) \)

\[ \hat{\Omega} \]

\( x \hat{g}^{\hat{\Omega}} \)

\( R(s, \hat{g}^{\hat{\Omega}}) = \text{interp}(g_{j}^{\hat{\Omega}}, \alpha_{1,j}(\hat{g}^{\hat{\Omega}})) R(\alpha_{2,j}(\hat{g}^{\hat{\Omega}}) s, g_{j}^{\hat{\Omega}}), \hat{g}^{\hat{\Omega}}) \)

\( \alpha_{1,1}(\hat{g}^{\hat{\Omega}}) R(\alpha_{2,1}(\hat{g}^{\hat{\Omega}}) s, g_{1}^{\hat{\Omega}}) \) \quad \alpha_{1,2}(\hat{g}^{\hat{\Omega}}) R(\alpha_{2,2}(\hat{g}^{\hat{\Omega}}) s, g_{2}^{\hat{\Omega}}) \)

\[ R(s, \hat{g}^{\hat{\Omega}}) = \sum_{k_{1}=1}^{2} \sum_{k_{2}=1}^{2} \tilde{R}(s, (g_{k_{1}}^{(1)}, g_{k_{2}}^{(2)})^{\hat{\Omega}}) \ell_{k_{1}}(g^{(1)}) \ell_{k_{2}}(g^{(2)}) \]
Standard Interpolation

Compute root macromodels $R(s, g_k^{\Omega})$
in the estimation design space grid

Multivariate interpolation of
root macromodels
$R(s, g)$
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Rational Fitting Methods

- VECTFIT
- MULTIPORT
- STABILITY
- PASSIVITY
- NOISY DATA

Interpolation

- TRANSFER FUNCTIONS
- SCALING
- SHIFTING

data-driven PMOR

scattered data

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3D example: Spiral inductor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency ((f_{req}))</td>
<td>10 kHz</td>
<td>30 GHz</td>
</tr>
<tr>
<td>Horizontal length ((L_x))</td>
<td>0.57 mm</td>
<td>0.74 mm</td>
</tr>
<tr>
<td>Vertical length ((L_y))</td>
<td>0.57 mm</td>
<td>0.74 mm</td>
</tr>
</tbody>
</table>
3D example: Spiral inductor

Ly = 0.57 mm

Ly = 0.74 mm
3D example: Spiral inductor \textbf{(Without scaling-shifting coefficients)}

\[ L_x = [0.57, 0.60, 0.63] \text{ mm}, \quad Ly = 0.57 \text{ mm} \]
3D example: Spiral inductor (Without scaling-shifting coefficients)

$L_x = [0.57, 0.60, 0.63]$ mm, $L_y = 0.57$ mm
3D example: Spiral inductor *(With scaling-shifting coefficients)*

\[ L_x = [0.57, 0.60, 0.63] \text{ mm}, \; L_y = 0.57 \text{ mm} \]
3D example: Spiral inductor *(With scaling-shifting coefficients)*

\[ L_x = [0.57, 0.60, 0.63] \text{ mm}, \quad L_y = 0.57 \text{ mm} \]
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data-driven PMOR

scattered data

H(s,g)
### 3D example: PCB

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<th>Parameter</th>
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</thead>
<tbody>
<tr>
<td>Frequency (freq)</td>
<td>0 Hz</td>
<td>20 GHz</td>
</tr>
<tr>
<td>Antipads radius (R)</td>
<td>0.4826 mm</td>
<td>0.6026 mm</td>
</tr>
<tr>
<td>Distance (D)</td>
<td>1.2525 mm</td>
<td>2.4525 mm</td>
</tr>
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</table>

- **R** = antipads radius
- **D** = distance between signal vias and ground vias (center to center)

- 7.7 mm
- 6.6 mm
3D example: PCB

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<tr>
<td>Step</td>
<td>CPU time</td>
<td></td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>-------------------</td>
<td></td>
</tr>
<tr>
<td>Estimation grid by solver (4 × 6) (R,D)</td>
<td>3 h 6 min</td>
<td></td>
</tr>
<tr>
<td>Validation grid by solver (3 × 5) (R,D)</td>
<td>1 h 56 min 15 s</td>
<td></td>
</tr>
<tr>
<td>Building model</td>
<td>5 min 49 s</td>
<td></td>
</tr>
<tr>
<td>Validating model</td>
<td>11 s</td>
<td></td>
</tr>
<tr>
<td>Evaluating solver (one frequency response)</td>
<td>7 min 45 s</td>
<td></td>
</tr>
<tr>
<td>Evaluating model (one frequency response)</td>
<td>0.1 s</td>
<td></td>
</tr>
<tr>
<td>Step</td>
<td>CPU time</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------</td>
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Speed-up 4650 x
D = 1.8525 mm

R = 0.543 mm
D = 1.8525 mm

R = 0.543 mm

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D = 1.8525 mm

R = 0.543 mm

\[ \left| S_{21} \right| \]

Frequency [GHz]

\[ R = 0.5826 \text{ mm} \]

\[ R = 0.5026 \text{ mm} \]

\[ D = 1.3725 \text{ mm} \]

\[ D = 2.3325 \text{ mm} \]
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Conclusions
telecom

input → output = f(in) → output

electronics

fluid dynamics

chemistry

biomodeling

geology

automotive

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Department of Information Technology – Internet Based Communication Networks and Services (IBCN)
Aerodynamics  
Automotive  
Chemistry  
Aerodynamics  
Electronics  
Metallurgy  

**Design variables**
- width, temperature, angle, frequency, ...

**Response variables**
- lift, S-parameters, pressure, stress, ...

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**Simulation Model**
- Fluent®, HSPICE®, CST®, Comsol®, Abaqus®, ...  

---

**Parameterized macromodels**
- Neural network, Kriging, SVM, rational function, spline, ...

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**Adaptive Modeling**
- Costly

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**Distributed Computing**
- Cheap

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**Configurable infrastructure**
- Department of Information Technology – Internet Based Communication Networks and Services (IBCN)
Parameterized macromodels

Multiple design variables

Compact models

Efficient design activities (excellent speed-ups)
  - Multiple simulations (measurements)
    - Design space optimization, exploration, sensitivity analysis
Parameterized macromodels

Time-domain simulations
- Non-linear drivers and receivers

Stochastic modeling
- impact of manufacturing tolerances

High number of dimensions

Models from measurements
- noise to handle

Applications in different domains
Questions

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Recent publications


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