Solving exactly solvable pairing models

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Abstract

Since the pioneering work of Richardson more than forty years ago, it has been known that the eigenequations for the constant pairing Hamiltonian can be reduced to a set of non-linear equations for a limited number of complex variables. Therefore the constant pairing model was said to be ‘exactly solvable’. However, for fermionic particles the solution of the set of non-linear equations turned out to be hampered by uncontrollable divergencies. Here we show how one can handle these divergencies and obtain the full solution of the exactly solvable pairing models.

Introduction

The isovector $J = 0$ pairing Hamiltonian is known to constitute a dominant part of the effective interaction between valence nucleons in atomic nuclei [1]. The same Hamiltonian also describes the correlations between electron pairs in superconductors, according to the BCS theory [2]. One can write the pairing Hamiltonian in a general form as

$$H = \sum_{j,m} e_j a_{jm}^\dagger a_{jm} - \frac{1}{4} \sum_{j,m,j',m'} g_{jj'} a_{jm}^\dagger a_{j'm'}^\dagger a_{jm} a_{j'm'},$$

with $e_j$ a set of single-particle energies, $g_{jj'}$ the (level dependent) pairing interaction strength and the indices $j$ $m$ referring to the time reverse of the state with indices $jm$. In 1963, Richardson showed [3] that the eigenvalue equations for the constant pairing Hamiltonian ($g_{jj'} = g$, for all $j$, $j'$) could be reduced to a set of non-linear equations, which were easy to solve in the case of non-degenerate levels. This made the constant pairing interaction quasi exactly solvable! However, there was a problem for fermions in multiply degenerate levels: the solutions of the sets of non-linear equations turned out to exhibit singularities at certain critical values of the pairing interaction strength [4]. These singularities are related to the Pauli principle for fermions: the exact many-body wave functions can be seen as a product of pair states. At the critical interaction strengths, the product wave function seems to put more pairs into a single level than allowed by the Pauli principle. The algebraic properties of the fermion pair operators assure that the multiply occupied part of the wave function is projected out. However, this makes that the wave function behaves non-analytically around the critical interaction strengths, and this translates into singularities in the solutions of the sets of non-linear equations. The
trick to avoid these singularities and to solve the equations for fermions in multiply degenerate levels consists of making a change of variables such that the singularities are canceled out at the level of the non-linear equations [5].

1 Exactly solvable pairing models

1.1 quasi-spin operators

The solvability of the pairing Hamiltonian is based on the algebraic properties of \( SU(2) \) operators. Let us consider the quasi-spin operators

\[
S_0^j = \frac{1}{2} \sum_{m > 0} (a_{j,m}^+ a_{j,m} + a_{j,m}^+ a_{j,m}^*) - 1,
\]

\[
S^+_j = \sum_{m > 0} a_{j,m}^+ a_{j,m}^*,
\]

\[
S^-_j = \sum_{m > 0} a_{j,m} a_{j,m}.
\]

One easily checks that these operators fulfill the \( SU(2) \) commutation relations:

\[
[S^+_j, S^-_j] = 2S_0^j,
\]

\[
[S^0_j, S^\pm_j] = \pm S^\pm_j.
\]

With these operators one can write down a Hamiltonian of the following form:

\[
H = E_0 + \sum_j \varepsilon_j S^0_j + \sum_j \delta_j \left[ S^0_j (S^0_j - 1) + S^+_j S^-_j \right] + \gamma_1 \sum_j S^0_j
\]

\[+ \gamma_2 \left( \sum_j S^0_j \right)^2 + \sum_{ij} (\varepsilon_i - \varepsilon_j) u_{ij} S^0_i S^0_j + \sum_{ij} (\varepsilon_i - \varepsilon_j) v_{ij} S^+_i S^-_j,
\]

where \( E_0 \) is a constant, the \( \varepsilon_j \) are related to the single-particle energies, the \( \delta_j \) are quasi-spin parameters and \( u_{ij} \) and \( v_{ij} \) are skew symmetric matrices of interaction parameters. It will be shown later on that the pairing Hamiltonian of Eq.(1) can be written in the form of Eq.(7). The specific form of the operators \( S^+_j, S^-_j, S^0_j \) guarantees that particles will be moving always in time-conjugated pairs. Particles occupying a site \( jm \) for which the time-conjugated site \( j\bar{m} \) is empty, will be inert under the action of \( H \). Such particles are said to be 'unpaired'. Any state \( |\nu\rangle \) with only unpaired particles or without particles at all will be a vacuum state for the \( S^0_j \) operators,

\[
S^0_j |\nu\rangle = 0, \ \forall j.
\]

The vacuum states can be labeled by a set of quantum numbers \( \nu \) that reflect the number of unpaired particles for each orbital \( j \). These quantum numbers are called seniorities [6]. One can define \( SU(2) \) operators analogous to Eq.(4) for bosons too (in fact, these will be \( SU(1,1) \) operators in disguise). Therefore each fermionic exactly solvable pairing model has a bosonic equivalent [7]. However, the vacuum states of the \( S^-_j \) operators are completely different because of the possible double occupancies of boson states. The resulting sets of non-linear equations for bosons do not suffer from singularities. Therefore we will limit the present discussion to fermions only.
1.2 integrability

Hamiltonians of the form of Eq.(7) with the same coefficients \( u_{ij} \) and \( v_{ij} \) but with different values for the other parameters, have the interesting property that they commute provided that

\[
(u_{ij} - u_{ik}) v_{jk} - v_{ij} v_{ik} = 0, \quad \forall \ i, j, k; \ i \neq j, i \neq k, j \neq k. \tag{9}
\]

This condition is known as the Gaudin relation [8]. It is related to an algebraic structure known as a Gaudin algebra, which is at the heart of the exactly solvable pairing models, but beyond the scope of this article. See Ref. [7] for a review. The interesting aspect of Eq.(9) is that it gives a sufficient condition for a Hamiltonian of the form Eq.(7) to be integrable: by taking all but one \( \epsilon_j \)'s equal to zero one can construct a complete set of operators \( H_i \) that all commute with the Hamiltonian, and hence are constants of motion [9]. This means that there are as much independent constants of motion as degrees of freedom (one for each single-particle level), which is a way to express the integrability of the model [10].

1.3 solvability

At the same time, Eq.(9) is a sufficient condition for the exact (algebraic) solvability of the Hamiltonian \( H \) of Eq.(7). This requires the extension of the matrices \( u_{ij} \) and \( v_{ij} \) to the complete complex plane: one has to define a set of parameters \( \eta_j \) and two functions \( u(x,y) \) and \( v(x,y) \) such that

\[
\begin{align*}
    u(x,y) &= -u(y,x), \quad u(\eta_i, \eta_j) = u_{ij}, \\
    v(x,y) &= -v(y,x), \quad v(\eta_i, \eta_j) = v_{ij},
\end{align*}
\]

\[
[u(x,y) - u(x,z)] v(y,z) - v(x,y) v(x,z) = 0, \quad \forall x, y, z \in \mathbb{C}. \tag{10}
\]

Using these functions, one can write down a Bethe ansatz \( |x\rangle \) for the eigenstates:

\[
|x\rangle = \left( \prod_{i=1}^{N_p} S^+(x_i) \right) |\nu\rangle, \tag{11}
\]

with \( |\nu\rangle \) a vacuum state, \( N \) the number of active pairs, and

\[
S^+(x) = \sum_j v(\eta_j, x) S^+_j. \tag{12}
\]

This defines a wave function for \( N = 2N_p + \sum_j \nu_j \) particles. Acting on this wave function with the Hamiltonian of Eq.(7), one obtains after some algebra (using the \( SU(2) \) commutation rules and the Gaudin relation of Eq.(10)):

\[
\begin{align*}
    H |x\rangle &= \langle \nu | H |\nu\rangle \langle x | + \left[ H, \prod_{i=1}^{N_p} S^+(x_i) \right] |\nu\rangle \\
    &= E(x) |x\rangle + \sum_i \left( 1 - 2 \sum_j s^0_j u(\eta_j, x_i) - 2 \sum_{k,k \neq i} u(x_k, x_i) \right) \\
    & \quad \times \left( \sum_{j'} \epsilon_{j'} v_{\eta_j x_i} S^+_j \right) \left( \prod_{k'=1,k' \neq i}^{N_p} S^+(x_{k'}) \right) |\nu\rangle,
\end{align*} \tag{13}
\]
where $E(x)$ is given by
\[ E(x) = E_0 + \sum_j \left[ \epsilon_j s_j^0 \left( 1 + 2 \sum_i u(\eta_j, x_i) \right) + d_j s_j^0 (s_j^0 - 1) \right] + \gamma_1 s^0 + \gamma_2 (s^0)^2, \tag{14} \]
and $s_j^0$ is the vacuum expectation value of $S_j^0$,
\[ s_j^0 = \langle \nu | s_j^0 | \nu \rangle = \frac{v_j}{2} - \frac{D_j}{4}, \tag{15} \]
with $D_j$ the single-particle degeneracy of orbital $j$, and $s^0$ is the total quasi-spin projection, which relates directly to the total particle number $N = 2N_p + \nu$, and the total number of single-particle states $M = \sum_j D_j$, through
\[ S^0 = \frac{N}{2} - \frac{M}{2}. \tag{16} \]
From Eq.(13) one sees that the state $|x\rangle$ will be an eigenstate of $H$ provided that the $N_p$ complex variables $x_i$ fulfill the following set of nonlinear equations:
\[ \sum_j s_j^0 u(\eta_j, x_i) + \sum_{k,k\neq i} u(x_k, x_i) - \frac{1}{2} = 0, \quad \forall i. \tag{17} \]
These are the Richardson-Gaudin equations in their general form. Solving these equations is equivalent to solving the eigenproblem for the integrable pairing Hamiltonians of the form Eq.(7). The corresponding eigenvalues are given by $E(x)$ from Eq.(14). Expectation values for certain operators can be obtained by evaluating the eigenvalues for the constants of motion, whose eigenstates are given by the same set of variables $x_i$. More observables can be evaluated through the Helmann-Feynman theorem, by varying parameters in the Hamiltonian or in the constants of motion.

### 1.4 exactly solvable models

The condition for solvability and integrability is given by the Gaudin relation, Eq.(9) or Eq.(10). Any set of functions $u(x,y)$ and $v(x,y)$ fulfilling this relation, will lead to an exactly solvable Hamiltonian of the form Eq.(7), for any parameter sets $\epsilon_j$ and $d_j$. A general parameterization of the functions $u(x,y)$ and $v(x,y)$ is given by
\[ u(x,y) = g \frac{1+sxy}{x-y}, \quad v(x,y) = g \sqrt{(1+sx^2)(1+sy^2)} \frac{1}{x-y}, \tag{18} \]
with $s$ either equal to 0, +1 or −1. Any other solution of the Gaudin relation can be transformed into this form through a redefinition of the variables. Note that the functions $u(x,y)$ and $v(x,y)$ must fulfill the relation
\[ v(x,y)^2 - u(x,y)^2 = sg^2, \quad \forall x, y \in \mathbb{C}. \tag{19} \]
Depending on the sign $s$ of the right-hand side, one obtains the rational ($s = 0$), trigonometric ($s = +1$) or hyperbolic solution ($s = -1$). Many different integrable and exactly-solvable models can be derived from this parameterization [11, 5].
The constant pairing Hamiltonian, as in Eq.(1), with $g_{j\bar{j}} = g$, can be obtained from the rational model with the following choice of parameters:

$$\varepsilon_j = 2e_j, \quad E_0 = \frac{1}{2} \sum_j e_j D_j,$$

$$u_{ij} = v_{ij} = -\frac{g}{\varepsilon_i - \varepsilon_j};$$

$$\delta_j = -g, \quad \gamma_1 = -g, \quad \gamma_2 = g.$$ (20)

This results in a constant pairing Hamiltonian $H_{CP}$ of the form Eq.(1),

$$H_{CP} = \sum_{jm} e_j a_{jm}^\dagger a_{jm}, -\frac{g}{4} \sum_{jj'm'n'} a_{jm}^\dagger a_{jm}^\dagger a_{jj'} a_{jj'}.$$ (21)

The non-linear equations of Eq.(17) reduce to

$$\sum_{k,k\neq i} \frac{1}{x_k - x_i} + \sum_j \frac{s_j^0}{2e_j - x_i} + \frac{1}{2g} = 0, \quad \forall i.$$ (22)

At the same time, Eq.(14) for the eigenvalues can be drastically simplified:

$$E_{CP}(x) = \sum_j x_j + \sum_j e_j v_j.$$ (23)

Hence, the variables $x_i$ can be interpreted as complex pair energies. Because the $x_i$ will always come in conjugated pairs, the total energy will be real, as it should be for a Hermitian Hamiltonian.

2 Solving the set of nonlinear equations

2.1 singularities

The eigenproblem for the constant pairing interaction is reduced to solving a set of $N_p$ nonlinear equations for $N_p$ complex unknowns, Eq.(22). To understand the issue of the singularities, it is instructive to take a look at a solution for a specific model: consider neutrons in the $sd + fp$ shell with the parameters listed in table 1, at zero seniority. Figure 1 shows the evolution of the variables $x_i$ for an interaction strength $g$ ranging from 0 to 1.5 MeV. One observes that a real variable can cross a level parameter $2e_j$ only if at the same time other variables come together at that level. This results in diverging terms in Eq.(22). These divergencies must cancel out, which leads to the condition

$$n_j = -2s_j^0 + 1,$$ (24)

with $n_j$ the number of variables that actually converge to $2e_j$. For fermions the value $(-2s_j^0)$ corresponds to the pair degeneracy of the level $e_j$: because of the Pauli principle, no more fermion pairs can occupy the level. A variable near to a level parameter $2e_j$ normally leads to a strong dominance in the term for that level in the Bethe ansatz of Eq.(11), so one can interpret this as a pair occupying that specific level. The fact that one more variable converges to $2e_j$ would suggest that there is an extra pair occupying that level. But this is unphysical because of the Pauli principle. In fact, the part of the Bethe ansatz wave function that would result in an $n_j$-fold occupation of the level is projected out because of the algebraic properties of the fermionic operators in Eq.(12). So at the level of the wave function and the energies, the singularities do not show up. They are however present in the nonlinear equations, which makes them hard to solve.
To solve the nonlinear equations in Eq. (22), one can adopt the following strategy:

- Start from the noninteracting limit: for very weak interactions the pairs will occupy specific levels, which results in variables $x_i$ clustered around specific level parameters $2e_j$. Each distribution of pairs over levels will result in a different eigenvector.

- Construct an approximate solution in the weakly interacting limit. The mixing between different levels is of second order in $g$, so up to first order one can solve for each level separately. The equations for one level can be reduced to a single polynomial equation, whose roots correspond to the variables $x_i$ of the cluster.

- Express the nonlinear equations in terms of new variables for which the singularities in Eq. (22) disappear. The new variables should be completely symmetric in terms of the old variables, in order to become analytic around the singularity.

- Solve the equations in the new variables using a standard method, e.g. multidimensional Newton-Raphson. Because the coupling between levels is weak compared to the couplings in one cluster, one can apply this method iteratively, solving for each cluster separately until the results converge globally.

- Increase the interaction strength $g$ by a small amount.

- Reorder the clusters at each step: convert the new variables back to the $x_i$ variables and check whether they are still near to the original level parameter, or if they have crossed over to another cluster.

- The analytic character of the new variables can be used to extrapolate previous solutions to a new guess at new values of $g$. This guess can then be improved using the Newton-Raphson method.

- Evolve the solution adiabatically up to the desired interaction strength by gradually increasing the value of $g$. It might be useful to take smaller steps around the singularities, because there the solutions are very sensitive to $g$.

### Table 1: Parameters for neutrons in the $sd + fpg$ shell. The single-particle energies are taken from Ref. [12].

<table>
<thead>
<tr>
<th>level</th>
<th>energy $(\text{MeV})$</th>
<th>degeneracy $2j+1$</th>
<th>initial pair occupations $s_j^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1d_{5/2}$</td>
<td>-21.5607</td>
<td>6</td>
<td>-3/2</td>
</tr>
<tr>
<td>$1d_{3/2}$</td>
<td>-19.6359</td>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>$2s_{1/2}$</td>
<td>-19.1840</td>
<td>2</td>
<td>-1/2</td>
</tr>
<tr>
<td>$1f_{7/2}$</td>
<td>-10.4576</td>
<td>8</td>
<td>-2</td>
</tr>
<tr>
<td>$2p_{3/2}$</td>
<td>-8.4804</td>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>$1f_{5/2}$</td>
<td>-7.7003</td>
<td>6</td>
<td>-3/2</td>
</tr>
<tr>
<td>$2p_{1/2}$</td>
<td>-7.6512</td>
<td>2</td>
<td>-1/2</td>
</tr>
<tr>
<td>$1g_{9/2}$</td>
<td>0.5631</td>
<td>10</td>
<td>-5/2</td>
</tr>
</tbody>
</table>
A practical way to organize the clusters is to link each variable to its nearest single-particle level $2e_j$, and to consider a cluster $C_j$ for each level $2e_j$ that has variables around it. The question is then how to solve the equations for each cluster. Consider the equations around level $2e_j$:

$$\frac{s_j^0}{2e_j - x_i} + \sum_{k \in C_j, j \neq i} \frac{1}{x_k - x_i} + F_j(x_i) = 0, \quad \forall \alpha \in C_j,$$

with

$$F_j(x) = \frac{1}{2g} + \sum_{j', j' \neq j} \frac{s_{j'}^0}{2e_{j'} - x} + \sum_{k', k' \in C_j} \frac{1}{x_{k'} - x}.$$  

(25)

(26)

Note that $F_j(x)$ is a smoothly varying function for $x$ near to $2e_j$, while the other terms in Eq.(25) might exhibit divergencies of the form $1/(2e_j - x_i)$. This suggests a way to remove the singularities from
the equations: multiply the equation with \((2e_j - x_i)^p\), for some power \(p\), and sum over all variables \(x_i\) in the cluster \(C_j\). The resulting equations become

\[
s_j^0 \sum_{i \in C_j} (2e_j - x_i)^{p-1} + \sum_{i, k \in C_j, i \neq k} \frac{(2e_j - x_i)^p}{(2e_j - x_i) - (2e_j - x_k)} = \sum_{i \in C_j} (2e_j - x_i)^p F_j(x_i). \tag{27}
\]

For \(p > 1\) one can write

\[
\sum_{x,y, x \neq y} \frac{x^p}{x-y} = \frac{1}{2} \sum_{x,y, x \neq y} \frac{x^p - y^p}{x-y} = \frac{1}{2} \sum_{x,y, x \neq y} \sum_{k=1}^{p} x^{k-1} y^{p-k} = \frac{1}{2} \sum_{x=1}^{p} \left( \sum_{y} x^{k-1} \right) \left( \sum_{y} y^{p-k} \right) - \frac{p}{2} \sum_{x} x^{p-1}. \]

This allows us to remove the singularity also in the second term of Eq.(27), for \(p > 1\):

\[
\left( s_j^0 + n_j - \frac{p}{2} \right) \sigma_{p-1} + \frac{1}{2} \sum_{k=2}^{p-1} \sigma_{k-1} \sigma_{p-k} + R_p = 0, \tag{28}
\]

where

\[
\sigma_p = \sum_{i \in C_j} (2e_j - x_i)^p, \quad R_p = \sum_{i \in C_j} (2e_j - x_i)^p F_j(x_i).
\]

The variables \(\sigma_p\) form a new set of variables, for which the eigenequations Eq.(28) are free from singularities. These equations are easily solved using a gradient method, e.g. multidimensional Newton-Raphson [13]. The gradient of the first two terms in Eq.(28) is easily evaluated, the gradient of \(R_p\) can be evaluated using the corresponding variables \(x_i\):

\[
\frac{\delta R_p}{\delta \sigma_k} = \sum_{i \in C_j} \frac{\delta R_p}{\delta x_i} \frac{\delta x_i}{\delta \sigma_k}. \tag{29}
\]

Given the variables \(\sigma_p\), one can recover the variables \(x_i\) by constructing a polynomial \(P(x)\) through a Faddeev-Leverrier sequence,

\[
P(x) = x^{n_j} + \sum_{k=1}^{n_j} p_k x^{n_j-k}
\]

\[
p_1 = -\sigma_1, \quad p_k = -\frac{1}{k} \left( \sigma_k + \sum_{m < k} \sigma_{k-m} p_m \right), \quad k = 2, \ldots n_j. \tag{30}
\]

The roots of the polynomial will be related to the variables \(x_i\) by

\[
P(2e_j - x_i) = 0. \tag{31}
\]
A polynomial of order \( n_j \) has exactly \( n_j \) different roots, so this yields the \( n_j \) distinct variables \( x_i \) in the cluster \( C_j \). Furthermore it shows that the variables \( x_i \) will be real or come in complex conjugated pairs.

The new set of equations, Eq.(28), has one weak point: in case of a singularity, when Eq.(24) holds, the first term of the equation for \( p = n_j \) vanishes. The resulting equation is linearly dependent on the other equations (for \( p = 1, \ldots, n_j - 1 \)), except for the smoothly varying term \( R_p \). This means that the set of equations becomes ill-conditioned. A way out of this problem, is not to use the equation for \( p = n_j \), and instead to work with the equation for \( p = -1 \). The singularities cancel out also for negative values of \( p \), and one obtains the equation

\[
s_j^0 g \sigma_{-1} + g R_0 = 0,
\]

with \( \sigma_{-1} = \sum_{i \in C_j} 1/x_i \), and \( R_0 = \sum_{i \in C_j} F_j(x_i) \). So it is more practical to work with the variable \( g \sigma_{-1} \) instead of \( \sigma_{n_j} \) (the factor \( g \) is included in order to avoid a singularity at \( g = 0 \)). The coefficients of the polynomial \( P(x) \) of Eq.(31) can still be evaluated using the relation \( p_{n_j} = -p_{n_j-1}/\sigma_{-1} \).

### 2.3 the weakly interacting limit

For weak interaction strengths the function \( F_j(x) \) is dominated by the constant term \( 1/2g \). Hence one can take \( F_j \) to be a constant. The resulting functions \( R_p \) take the simple form \( R_p = \frac{\sigma_p}{2g} \). Now the equations Eq.(28) can be solved straightforwardly, to yield the variables \( \sigma_l \), from which one can construct the polynomial \( P(x) \) that gives a unique set of variables \( x_i \) for each cluster. In fact, one can show that \( P(x) \) is an associated Laguerre polynomial of negative upper index [14]. Note that each specific distribution of pairs over level clusters \( C_j \) results in a unique solution. This establishes a one-to-one correspondence between the eigenstates of the noninteracting system (\( g = 0 \)) and the eigenstates of the weakly interacting system. One can conclude that the Bethe ansatz solutions are complete: they cover all eigenstates, and there are no spurious solutions.

### 2.4 the extrapolation step

One more ingredient is needed to avoid problems with the singularities: when the interaction strength passes through a critical value, the variables \( x_i \) passing through a singularity can change from real to complex or vice versa. At the same time the variables \( \sigma_l \) will become very small, except for \( l = -1 \). Therefore also the Newton-Raphson steps become very small, and the iteration never converges to the solution on the right side of the singularity if it starts out on the other side. This problem can be overcome by an extrapolation step each time the value of \( g \) is increased: assume that converged solutions \( x'_i \) and \( x''_i \) were obtained for the values \( g' \) and \( g'' \). For each cluster, one evaluates the variables \( \sigma'_p \) and \( \sigma''_p \). Then one extrapolates these to new values for \( \sigma_p \) at the new interaction strength \( g \):

\[
\sigma_p = \frac{(g - g')\sigma''_p - (g - g'')\sigma'_p}{g'' - g'}
\]

The extrapolation step avoids the problems with the singularities and greatly improves the convergence of the method.
3 Conclusions

Using the method outlined here, one can obtain any eigenstate by starting from the right seniorities and pair occupancies in the noninteracting limit. Any eigenstate can be evaluated, excited states are not more difficult than the ground state. Figure 2 shows the lowest lying states for the model discussed in subsection 2.1. The method can deal with very large systems, well beyond the limits of large-scale exact diagonalisations [15]: our largest run so far for a system with multiply degenerate levels has been for 100 pairs distributed over 40 levels. The limiting factor is the size of the pair clusters around one level, which is proportional to the degeneracy of the levels. The present implementation (in 64-bit precision) becomes unstable when clusters get as large as 60 to 80 pairs. The reason is that switching from the $\sigma$ to the $x$ variables requires finding all roots of a polynomial equation with high enough accuracy.

This work shows that the exactly solvable pairing models are indeed solvable in practice, even for fermions with multiple degeneracies. It opens up a whole new range of applications for these models. Furthermore, similar techniques might be useful to solve the non-linear equations for other exactly-solvable pairing and spin models [16, 17, 18, 19, 20].

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