1-systems of $Q^+(7, q)$ and trialities

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A 1-system $\mathcal{M}$ of the hyperbolic quadric $Q^+(7, q)$ in $PG(7, q)$ is a set $\{L_0, L_1, \ldots, L_{q^3}\}$ consisting of $q^3 + 1$ lines on $Q^+(7, q)$ with the property that the tangent space of $Q^+(7, q)$ at $L_i$ has no point in common with $(L_0 \cup L_1 \cup \ldots \cup L_{q^3}) \setminus L_i$, for $i = 0, 1, \ldots, q^3$. If $\mathcal{M}$ consists of $q^2$ reguli through a common line $L_i \in \mathcal{M}$, then the 1-system is called locally hermitian at $L_i$.

We will consider the action of a triality $\tau$ of $Q^+(7, q)$ on a 1-system $\mathcal{M}$ of an induced $Q(6, q) \subseteq Q^+(7, q)$. Special attention will be paid to the case of a locally hermitian, semiclassical 1-system of $Q(6, q)$. In particular, it will be explained that its image under a triality is again locally hermitian and semiclassical and that it is contained in a hyperplane of $PG(7, q)$ if and only if $\mathcal{M}$ is a spread of a classical generalized hexagon $H(q)$ on $Q(6, q)$. Since we are able to show that whenever $\mathcal{M}$ is not a spread of some $Q^-(5, q) \subseteq Q(6, q)$, nor a spread of a classical generalized hexagon $H(q)$ on $Q(6, q)$, the 1-system $\mathcal{M}^\tau$ was not previously known, this construction using a triality yields new examples of locally hermitian, semiclassical 1-systems of $Q^+(7, q)$.

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