Maximal partial spreads of $H(4n + 1, q^2)$

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Since $H(4n + 1, q^2)$ does not admit spreads, it is useful to study its maximal partial spreads. A maximal partial spread $S$ of $H(4n + 1, q^2)$ is a set of pairwise disjoint generators (i.e. maximal totally isotropic subspaces) of $H(4n + 1, q^2)$ such that every other generator of $H(4n + 1, q^2)$ has a non-empty intersection with at least one element of $S$.

For $n = 1$, a new upper bound on the size of a maximal partial spread of $H(5, q^2)$ will be given. In fact, there is only one example of a maximal partial spread of $H(5, q^2)$ known, which has size $q^3 + 1$, and for $n > 1$, no examples are known. It will be shown that the example living on $H(5, q^2)$ can be generalized to $H(4n + 1, q^2)$ for all $n$, thus providing a class of maximal partial spreads of $H(4n + 1, q^2)$ of size $q^{2n+1} + 1$. 