The smallest minimal blocking sets of $Q(2n, q)$, for small odd $q$

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In [2] we used results on the size of the smallest minimal blocking sets of $Q(4, q)$, $q$ even (from [1]) and projection arguments to find the following characterization of the smallest minimal blocking sets of $Q(6, q)$, $q$ even, $q \geq 32$:

**Theorem 1** Let $K$ be a minimal blocking set of $Q(6, q)$, different from an ovoid of $Q(6, q)$, $|K| \leq q^3 + q$. Then there is a point $p \in Q(6, q) \setminus K$ with the following property: $T_p(Q(6, q)) \cap Q(6, q) = pQ(4, q)$ and $K$ consists of all the points of the lines $L$ on $p$ meeting $Q(4, q)$ in an ovoid $O$, minus the point $p$ itself, and $|K| = q^3 + q$.

The results of [1] could be proven for $q = 3$ and replaced by computer results for $q = 5, 7$. Using then the same projection arguments we found the above characterization for $q = 3, 5, 7$.

Using inductive arguments we can find results for $Q(2n, q)$, $q = 3, 5, 7$. The situation is now very dependent of $q$, since for example $Q(6, 3)$ has an ovoid, but $Q(6, q)$, $q = 5, 7$, not. For $q = 5, 7$, we found the following characterization.

**Theorem 2** Let $K$ be a minimal blocking set of $Q(2n + 2, q)$, $n \geq 2$, $|K| \leq q^{n+1} + q^{n-1}$. Then there is an $(n-2)$-dimensional space $\pi$, $\pi \subseteq Q(2n + 2, q)$, $\pi \cap K = \emptyset$, with the following property: $T_\pi(Q(2n + 2, q)) \cap Q(2n + 2, q) = \pi Q(4, q)$ and $K$ consists of all the points of the lines $M$ on $p_i$, $p_i \in \pi$, meeting $Q(4, q)$ in an ovoid $O$, minus the points $p_i$ themselves, and $|K| = q^{n+1} + q^{n-1}$.

For $q = 3$ we found a characterization using ovoids of $Q(6, 3)$.

**Theorem 3** Let $K$ be a minimal blocking set of $Q(2n + 2, q)$, $n \geq 3$, $|K| \leq q^{n+1} + q^{n-2}$. Then there is an $(n-3)$-dimensional space $\pi$, $\pi \subseteq Q(2n + 2, 3)$, $\pi \cap K = \emptyset$, with the following property: $T_\pi(Q(2n + 2, 3)) \cap Q(2n + 2, 3) = \pi Q(6, 3)$ and $K$ consists of all the points of the lines $M$ on $p_i$, $p_i \in \pi$, meeting $Q(6, 3)$ in an ovoid $O$, minus the points $p_i$ themselves, and $|K| = q^{n+1} + q^{n-2}$.

**References**
