Accurate Modeling of Antennas Using Variable-Fidelity EM Simulations and Co-Kriging

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Abstract—We present an accurate and low-cost modeling of antenna structures using variable-fidelity electromagnetic (EM) simulations. Our approach exploits sparsely sampled high-fidelity (accurate) EM data as well as densely sampled coarse-discretization (low-fidelity) EM simulations that are accommodated into one model using co-kriging technique. By using coarse-discretization simulations, the computational cost of creating the antenna model is greatly reduced compared to conventional approach, where high-fidelity simulations are directly used to set up the model. To our knowledge, this is the first application of co-kriging to antenna modeling. Numerical verification and comparisons with kriging interpolation are given.

Keywords—Antenna modeling; electromagnetic (EM) simulation; kriging; co-kriging; computer-aided design (CAD).

I. INTRODUCTION

Reliable evaluation of antenna structures can be obtained through electromagnetic (EM) simulation. High-fidelity simulation is CPU intensive, which is a bottleneck for EM-based design tasks such as parametric optimization, statistical analysis, or yield-driven design. Thus, accurate and computationally cheap models of antennas (so-called surrogates) are indispensable.

Cheap antenna models can be obtained using approximation techniques such as polynomial regression [1], radial basis functions [2], kriging [2], [3], support vector regression [4]-[6], artificial neural networks [7]-[10], fuzzy systems [11], or multidimensional Cauchy approximation [12]. However, for good accuracy, these techniques require a large number of training points, particularly if the number of design variables is large.

Here, we consider antenna models constructed using both high- and low-fidelity EM simulations. Simulation of coarsely-discretized antenna structure may not be accurate; however, it is much faster than the high-fidelity one. As we demonstrate, such low-fidelity data can be combined with sparsely sampled high-fidelity simulations using co-kriging [13]. The resulting antenna model is as accurate as the conventional approximation surrogate using much larger number of training data points. The proposed modelling technique is demonstrated using two examples: a ultrawideband planar dipole antenna and a rectangular dielectric resonator antenna. Comparison with conventional kriging interpolation is also given.

II. ANTENNA MODELING USING CO-KRIGING

A. Antenna Models

We consider two types of antenna models. Let \( R_f(x) \) denote an EM-simulated high-fidelity model, which is an accurate representation of the antenna structure. \( R_f \) is expensive to evaluate (typical simulation time measured in hours). Here, \( x \) is a vector of designable (e.g., geometry) parameters. The components of \( R_f \) may represent, e.g., the antenna reflection coefficient \( |S_{11}| \) over the frequency band of interest. We also consider an auxiliary (low-fidelity) model \( R_c \), which may be evaluated using the same EM solver, however, with coarser discretization. The low-fidelity model \( R_c \) is much faster than \( R_f \), but not as accurate. Therefore, it cannot be normally directly used instead of the high-fidelity model to perform tasks such as design optimization. In this paper, we combine the low- and high-fidelity simulations to create the surrogate model that is almost as accurate as \( R_f \) but requires much smaller number of high-fidelity training points than the approximation model created using only \( R_f \) samples.

B. Kriging Interpolation

Kriging is a popular technique to interpolate deterministic noise-free data [2], [14]. These Gaussian Process based surrogate models are compact and cheap to evaluate. Here, we use kriging as a benchmark technique for comparison with the co-kriging of Section II.C. Let \( X_{R_f} = \{x_{R_f}^1, x_{R_f}^2, \ldots, x_{R_f}^{N_{R_f}}\} \subset X_R \) be the base (training) set and \( R_f(X_{R_f}) \) the associated fine model responses. Then, the kriging interpolant, also known as the Best Linear Unbiased Predictor (BLUP), is derived as,

\[
R_{\text{K}}(x) = M \alpha + r(x) \cdot \Psi^{-1} \cdot (R_f(X_{R_f}) - F \alpha)
\]

where \( M \) and \( F \) are Vandermonde matrices of the test point \( x \) and the base set \( X_{R_f} \), respectively. The coefficient vector \( \alpha \) is determined by Generalized Least Squares (GLS). \( r(x) \) is an \( 1 \times N_{R_f} \) vector of correlations between the point \( x \) and the base set \( X_{R_f} \), where the entries are \( r_f(x) = \psi(x, x_{R_f}) \), and \( \Psi \) is a
Co-Kriging Modeling

Co-kriging [13] is a type of kriging where the \( R_f \) and \( R_c \) model data are combined to enhance the prediction accuracy. Co-kriging is a two-steps process: first a kriging model \( R_{s,KRC} \) of the coarse data \((X_{B,SR}, R_c(X_{B,SR}))\) is constructed and on the residuals of the fine data \((X_{B,FR}, R_f)\) a second kriging model \( R_{s,KRF} \) is applied, where \( R_d = R_c(X_{B,SR}) - \rho R_c(X_{B,FR}) \). The parameter \( \rho \) is included in the MLE. Note that if the response values \( R_c(X_{B,SR}) \) are not available, they can be approximated by using the first kriging model \( R_{s,KRC} \), namely, \( R_c(X_{B,SR}) \approx R_{s,KRF}(X_{B,SR}) \). The resulting co-kriging interpolant is defined as

\[
R_{s,CO}(x) = M\alpha + r(x) \cdot \Psi^{-1} \cdot (R_f - FA)
\]

where the block matrices \( M, F, r(x) \) and \( \Psi \) can be written in function of the two separate kriging models \( R_{s,KRC} \) and \( R_{s,KRF} \):

\[
r(x) = [\rho \cdot \sigma_f^2 \cdot r_f(x), \rho \cdot \sigma_c^2 \cdot r_c(x, X_{B,SR}) + \sigma_c^2 \cdot r_c(x)]
\]

\[
\Psi = \begin{bmatrix}
\sigma_c^2 \Psi_c & \rho \cdot \sigma_c^2 \cdot \Psi_{c,c}(X_{B,SR}, X_{B,FR}) \\
0 & \rho \cdot \sigma_c^2 \cdot \Psi_{c,c}(X_{B,SR}, X_{B,FR}) + \sigma_c^2 \cdot \Psi_d
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
F_c & 0 \\
\rho \cdot F_f & F_d
\end{bmatrix}, \quad M = [\rho \cdot M_c \quad M_d]
\]

where \((F_c, \sigma_c^2 \Psi_{c,c}, M_c)\) and \((F_d, \sigma_d^2 \Psi_{d,d}, M_d)\) are matrices obtained from the kriging models \( R_{s,KRC} \) and \( R_{s,KRF} \) respectively (see Section II.B). In particular, \( \sigma_c^2 \) and \( \sigma_d^2 \) are process variances, while \( \Psi_{c,c} \) and \( \Psi_{d,d} \) denote correlation matrices of two datasets with the optimized \( \theta_i, \ldots, \theta_i \) parameters and correlation function of the kriging models \( R_{s,KRC} \) and \( R_{s,KRF} \) respectively.

III. VERIFICATION EXAMPLES

A. UWB Planar Dipole Antenna

Consider the planar dipole antenna [15] (Fig. 1). The design variables are \( x = [l_0 w_0 a_0 l_p w_p s_0]^T \). The high-fidelity model \( R_f \) (~10 mln mesh cells, evaluation time 44 minutes) is simulated using the CST MWS transient solver [16]. The low-fidelity model \( R_c \) is also evaluated in CST (~100,000 mesh cells, evaluation time 43 seconds). The antenna models are set up in the region with the center at \( x^0 = [19 13 0.5 13 6 1]^T \) and size \( \delta = [1 1 0.2 1 0.2 1]^T \). The kriging and co-kriging models \( (R_{s,KRF}, R_{s,CO}) \) are constructed using various numbers of training points (from \( N_{SR} = 20 \) to \( N_{SR} = 400 \)). Co-kriging models are configured using 400 \( R_f \) samples (the CPU cost of which corresponds to around 6 evaluations of \( R_f \)). The quality of the surrogate is assessed using a relative error measure \( \|R(x) - R(\hat{x})\|/\|R(x)\| \) expressed in percent. The error is averaged over 50 test designs.

The modeling errors are given in Table I (see also Fig. 2). Note that the co-kriging model accuracy obtained with 20 (50) \( R_f \) samples is as good as that of the kriging model obtained for 100 (200) samples, which proves that co-kriging and the use of coarse-discretization EM data allows us to greatly reduce the CPU cost of creating accurate antenna model compared to conventional method using solely \( R_f \) information.

B. Rectangular Dielectric Resonator Antenna

Consider the rectangular suspended DRA [17] (Fig. 3). The design variables are \( x = [e_1 h_1 h_2 s_1 s_2]^T \). Other parameters are fixed. The high- and low-fidelity models are evaluated in CST [16] with the following evaluation times: \( R_f \) 11 minutes, and \( R_c \) 20 sec. The antenna models are set up in the region with the center at \( x^0 = [10 8.5 0.5 3 10]^T \) and size \( \delta = [1 1 0.5 1 2]^T \) mm. Similarly as for the previous example, co-kriging allows to substantially reduce the computational cost of creating the accurate antenna model compared to approximation of the high-fidelity model data only (cf. Table II and Fig. 4).
IV. CONCLUSION

We presented an antenna modeling methodology using co-kriging. We demonstrate that by combining the low- and high-fidelity EM simulations, it is possible to create an accurate model of an antenna structure while using limited number of high-fidelity data points.

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REFERENCES


TABLE I.  UWB DIPOLE ANTENNA: MODELING RESULTS

<table>
<thead>
<tr>
<th>Model</th>
<th>Average Modeling Error [%]</th>
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<tr>
<td>N_{ex} = 20</td>
<td>17.5 5.6 4.3 2.8 2.0</td>
</tr>
<tr>
<td>N_{ex} = 50</td>
<td>4.2 2.6 2.4 2.0 1.9</td>
</tr>
</tbody>
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Fig. 2. Rectangular DRA: responses of $R$, $|S_11|$ and co-kriging surrogate (o) at selected test points. Co-kriging model created using 50 evaluations of $R$, and 400 evaluations of $R_{ex}$.

TABLE II.  RECTANGULAR DRA: MODELING RESULTS

<table>
<thead>
<tr>
<th>Model</th>
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</tr>
</thead>
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<tr>
<td>N_{ex} = 20</td>
<td>12.1 8.8 6.9 5.2 3.6</td>
</tr>
<tr>
<td>N_{ex} = 50</td>
<td>6.7 5.4 5.0 4.1 3.5</td>
</tr>
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