Implementing a Vendor Managed Inventory Policy in a Two-Stage Supply Chain with Stochastic Demands

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Abstract: In a two-stage supply chain, implementing Vendor Managed Inventory policies (VMI), the supplier assumes, in addition to his own inbound inventory, the responsibility of maintaining inventory at its retailers and ensuring that they will not run out of stock at any moment. This paper discusses an optimization approach, considering the impact of demand uncertainty on the inbound as well as the outbound inventory, for a two-stage supply chain implementing VMI. In the proposed approach, retailers are first clustered to minimize the within-cluster travel costs and/or distances and are then replenished using an optimal direct shipping strategy satisfying some additional restrictions.

Keywords: Inventory Control, Distribution, Supply Chain Optimization, Stochastic Demand.

1. INTRODUCTION

Vendor Managed Inventory (VMI) is an inventory management policy, in which the supplier assumes, in addition to its inbound inventory, the responsibility of maintaining inventory at the retailers and ensures that they will not run out of stock at any moment. The delivery times and quantities to deliver to a retailer is no longer done after the retailer's orders, the supplier determine the quantity and when the delivery takes place. The replenishment is thus proactive as it is based on the available inventory information instead of being reactive in response to retailers' orders. This policy has many advantages for both the supplier and the retailers. The supplier has the possibility of combining multiple deliveries to optimize the truck loading and the routing cost. Moreover as the deliveries become more uniform, the amount of inventory that must be held at the supplier can be drastically reduced. On the other hand, the retailers need no longer to dedicate resources to the management of their inventories. Also, the service level (i.e. product availability) increases, as the supplier can track inventory levels at the retailers to determine the precise replenishment urgency.

One reason VMI gains more popularity is the current enabling technologies to monitor retailer inventories in an online and cost effective manner. Inventory data can be made available much easier. However, implementing VMI does not in all case lead to improved results. Failure can happen due, for example, to the unavailability of the necessary information or the inability of the supplier to make the right decisions. The large amount of data makes it extremely hard to optimize this problem. It involves managing inventory in a supply chains and optimizing distribution which are two particularly challenging problems.

The focus of this paper is to analyze the impact of demand and lead time stochasticity on a two-stage supply system implementing VMI. An approach is proposed to minimize the

overall inventory and distribution costs while taking into account both retailers demands and lead-times variability at the supplier. The problem is tackled by repeating the steps described below. In the first step, retailers are clustered to minimize the traveled distance or equivalently distribution costs. Then, a direct-shipping procedure is used to determine the optimal replenishment schedule for the fixed retailer groups. In a third step, retailers can be switched from group to group to again optimize the total costs by local search combined with a simulation. The inventory model of this study is shown as Fig. 1.

Fig.1.A two-echelon inventory system

2. A BRIEF LITERATURE REVIEW

One stream of research related to this problem is the single warehouse and multiple-retailers inventory models taking transportation cost into account. Example of such studies are carried by Golden, Assad, and Dahl 1984; Dror and Levy 1986; Dror and Ball 1987; Chandra and Fisher 1994; Gallego and Simchi-Levi 1990; Herer and Roundy 1997; Viswanathan and Mathur 1997; Chan et al. 2002; Chan and Simchi-Levi 1998; Aghezzaf 2008.

An extension of this research line is concerned with models, involving location-inventory network design, that integrate the location and inventory decisions. Barahona and Jensen (1998) studied a practical distribution network design problem for computer spare parts. Their model takes into account the inventory cost at the various warehouses. Erlebacher and Meller (2000), developed an analytical model to minimize the total fixed operating costs and inventory holding costs incurred by warehouses, together with the transportation costs. The model is solved heuristically. Max Shen, Coullard, and Daskin (2003) and Daskin, Coullard, and Max Shen (2002) considered the case where retailers are facing uncertain demands following a Poisson distribution. Shu, Teo, and Max Shen (2005) solved the problem for general demand distribution.

In all models, the inventory holding costs at the retailers are ignored. The model considered here, does not consider the design issue, however it takes all inventories at the warehouse as well as at the retailers into account. The VMI policy addresses the issue of coordinating the warehouse and retailers inventory replenishment activities to minimize the system-wide multi-echelon ordering and holding costs.

3. THE INTEGRATED DETERMINISTIC MODEL

For the model development, let R be the set of retailers, indexed by i . Let $R^+ = R \cup \{0\}$, where 0 indicates the warehouse and V the set of available vehicle. We also define the following notations:

- ψ^v : is the fixed operating and maintenance costs of vehicle $v \in V$;
- t_{ij} : is the duration of a trip from retailer $i \in R^+ = R \cup \{0\}$ to retailer $j \in R^+$;
- φ_0 : is a fixed ordering cost incurred by the warehouse each time it places an order; the ordering cost is independent of the order quantity;
- φ_i : is a fixed ordering cost incurred by each retailer $i \in R$ each time it places an order from the warehouse; the fixed ordering cost is independent of the order quantity;
- τ_{ij} : is a per unit transportation cost from the warehouse or retailer i to retailer j ;
- h_0 : is the per unit per year inventory holding cost rate in warehouse 0;
- h_j : is the per unit per year inventory holding cost rate in retailer j ;
- d_j : is a constant demand rate per year faced by retailer j ;
- T_0 : the replenishment interval at warehouse 0;
- T_j : the replenishment interval at retailer j .

Assume that customers are clustered and served by the set of vehicles v in V^* , and let R^v be the set customers served by vehicle v . If customer j is served by vehicle v , then $T^v = T_j$. The objective function to be optimized is:

SVRP:

$$Z_D = \sum_{v \in V^*} \psi^v + \frac{\varphi_0}{T_0} + \sum_{v \in V^*} \left(\frac{1}{2} h_0 \left(\sum_{j \in R^v} d_j \right) \left[\max(T_0, T^v) - T^v \right] \right) + \sum_{v \in V^*} \left(\frac{1}{T^v} \left(\tau^v + \sum_{j \in R^v} \varphi_j \right) + \frac{1}{2} \left(\sum_{j \in R^v} h_j d_j \right) T^v \right) \quad (1)$$

where $\tau^v = \sum_{(i,j) \in \text{Trip}(v)} \tau_{ij}$ is the total travel cost of the complete trip made by vehicle v , satisfying the restrictions that $\sum_{(i,j) \in \text{Trip}(v)} t_{ij} \leq T^v$ and that the total amount delivered to the customers in each tour made by the vehicle during its trip, $\text{Trip}(v)$, should not exceed the vehicle's capacity.

Assuming the power-of-two inventory stationary policy, in which each retailer is replenished at equally distant time intervals which are power-of-two multiples of a common base planning period. In absence of the travel and vehicle capacity restrictions on T^v , Roundy (1985) showed that the convex programming relaxation of (1) approximates the optimal solution value to 98% accuracy.

If we assume also that $(h_j - h_0) > 0$ for every retailer j , we can summarize the main results of Roundy (1985) as is done in Shu (2010) for the basic model Single Warehouse Multiple Retailers (SWMR): The solution of (1) is a lower bound on the average cost of any feasible inventory control policy, and the solution can be rounded off to obtain a feasible integer-ratio policy with a cost within 98% of the minimum of (1). Such a policy can be computed in $O(n \times \log(n))$ time (see the algorithm below). Furthermore, in the solution to (1), the retailers can be divided into three groups: G , L , and E .

For retailers in G , the replenishment interval is given by:

$$\hat{T}^v = \sqrt{\frac{2 \left[\tau^v + \sum_{j \in R^v} \varphi_j \right]}{\sum_{j \in R^v} h_j d_j}} > \hat{T}_0$$

For retailers in L , the replenishment interval is given by:

$$\hat{T}^v = \sqrt{\frac{2 \left[\tau^v + \sum_{j \in R^v} \varphi_j \right]}{\sum_{j \in R^v} (h_j - h_0) d_j}} < \hat{T}_0$$

Finally, for retailers in E , the replenishment interval is the same as that at the warehouse and given by:

$$\hat{T}^v = \hat{T}_0 = \sqrt{\frac{2 \left(\varphi_0 + \sum_{v \in E} \left[\tau^v + \sum_{j \in R^v} \varphi_j \right] \right)}{\sum_{v \in E} \sum_{j \in R^v} h_j d_j + \sum_{v \in L} \sum_{j \in R^v} h_0 d_j}}$$

If we start from a feasible partition $(R^v)_{v \in V^*}$ of retailers, that satisfies $T_{\min}^v = \sum_{(i,j) \in \text{Trip}(v)} t_{ij} \leq T_{\max}^v$ defining the smallest cycle obtained from the total amount delivered to the retailers served during each sub-tour made by the vehicle. We can determine the optimal values for each vehicle v , as follows:

$$T_{opt}^v = \begin{cases} \hat{T}^v, & \text{if } T_{\min}^v \leq \hat{T}^v \leq T_{\max}^v \\ T_{\min}^v, & \text{if } T_{\min}^v > \hat{T}^v \\ T_{\max}^v, & \text{if } \hat{T}^v > T_{\max}^v \end{cases}$$

To complete the procedure we need to develop an algorithm that determines the optimal feasible partition of retailers. This can be achieved by means of a combined solution method for SWMR problem combined with any effective heuristic for the vehicle routing problem. In this paper, the constrained vehicle routing problem is solved using a constructive local search procedure that maintains for each vehicle the condition $T_{\min}^v = \sum_{(i,j) \in \text{Trip}(v)} t_{ij} \leq T_{\max}^v$ on its cycle time.

To solve problem the complete SVRP, firstly, we adapted the algorithm proposed by Roundy (1985) to minimize inventory cost and determine the possible retailers set partitions G , E , L and then we solve the constrained VRP problem for elements in E to cluster retailers as much as possible with the objective of minimizing transportation cost. These two steps are repeated again, but now with the clustered sets of retailers until no further clustering is possible and then the procedure stops. Detailed steps of the algorithm are given below:

Algorithm for G , E , L , deterministic case (Roundy 1985)

Step 1. Calculate and Sort the Breakpoints.

Calculate the breakpoints $\tau_j = \sqrt{\frac{2k_j}{h_j \cdot D_j}}$ and $\tau_j = \sqrt{\frac{2k_j}{(h_j - h_0^j) D_j}}$.

Sort them to form a non-decreasing sequence of $2N$ numbers. Label each breakpoints with the value of n and with an indicator showing whether it is left breakpoint τ_j' or the right breakpoint τ_j .

Step 2. Initialize E , G , L , K and H .

Set $E = G = \emptyset$, $L = \{1, \dots, N\}$, $K = K_0$, and $H = \sum_{j=1}^n \frac{1}{2} h_0^j D_j$.

Step 3. Cross the Largest Uncrossed Breakpoint.

Let τ be the largest previously uncrossed breakpoint. If $\tau^2 \geq K/H$ and $\tau = \tau_j$ is a right breakpoint, cross τ and update E , L , K and H by $E \leftarrow E \cup \{n\}$, $L \leftarrow L \setminus \{n\}$, $K \leftarrow K + K_j$ and $H \leftarrow H + h_j$. Then go step 3. If $\tau^2 > K/H$ and $\tau = \tau_j'$ is a left breakpoint, cross τ and update E , G , K and H by $E \leftarrow E \cup \{n\}$, $G \leftarrow G \cup \{n\}$, $K \leftarrow K - K_j$ and $H \leftarrow H - h_j - h_0^j$ go to step 3. Otherwise T_j^* is in the current piece. Go to step 4.

Step 4. Calculate T^* .

Set $T^* = \sqrt{\frac{K}{H}}$. Then T_j^* for all retailers $j \in R$.

It remains to be shown that step 3 will be executed before the last breakpoint is crossed. Otherwise we would have $H = 0$ and $E \cup L = \emptyset$. If $\tau = \sqrt{\frac{K_j}{h_j}}$ is the only uncrossed breakpoint, it is a left breakpoint. Then in step 3 $K_0 + K_j$ and $H = h_j$, so $K/H > \tau^2$. Therefore step 4 will be executed and the algorithm will terminate.

Step 5. Post Processing.

Grouping the retailers in E into one group because as they all have the same replenishment cycle time T_j^* and trying to group the retailers in G and L with closely the same cycle time T_j^* .

4. INCORPORATION OF DEMAND VARIABILITY

To consider the effect of demand variability and analyze its impact on the distribution strategy, we introduce safety stocks in the model. One possible way is to express the objective function as is done in Chu and Shen (2010) as follows:

$$\begin{aligned} & \sum_{v \in V^*} \psi^v + \frac{\varphi_0}{T_0} + \sum_{v \in V^*} \left(\frac{1}{2} h_0 \left(\sum_{j \in R^v} d_j \right) \left[\max(T_0, T^v) - T^v \right] \right) + h_0 s_0 \\ & + \sum_{v \in V^*} \left(\frac{1}{T^v} \left(\tau^v + \sum_{j \in R^v} \varphi_j \right) + \frac{1}{2} \left(\sum_{j \in R^v} h_j d_j \right) T^v + \sum_{j \in R^v} \frac{h_j d_j}{\sum_{j \in R^v} d_j} s_v \right) \end{aligned} \quad (2)$$

If d_j is the average demand at retailer j and σ_j^2 the demand variance per unit time, then:

$$s^v = \sqrt{T^v \sum_{j \in R^v} (\sigma_j^2)}$$

and

$$s_0 = \theta_0 \sqrt{L_0 \sum_{v \in V^*} \sum_{j \in R^v} (\sigma_j^2) + \sum_{v \in V^*} \sum_{j \in R^v} (\sigma_j^2) \max\{0, T_0 - T^v\}}$$

where s^v is the safety stock level at the retailers, s_0 is the safety stock level at the warehouse, L_0 is the order lead time at the warehouse, θ_0 depends on the specified service level at the warehouse and θ_v depends on the specified Type I service level at retailer group served by the vehicle v .

In the remainder of this section we consider the well-known class of the so-called power-of-two policies. In such a policy the replenishments take place at constant reorder intervals that are power-of-two multiples of some fixed base planning period. Chu and Shen (2010) have shown that an optimal power-of-two policy can be found easily in which the cost is guaranteed to be within 2% of optimality. Their approach can be extended to the case we are considering in this paper.

Based on T^{v*} , the optimal solution of the relaxed problem (2), one can construct an ordering policy satisfying $T^v = 2^{kv} T_L$ where kv is integer with good performance using the procedure below. For simplicity we assume that T_L to be one day.

Clustering algorithm, stochastic case (Chu and Shen 2010)

- (i) Let T^{v*} denote the optimal solution of the relaxation problem. Define $q_v = \lfloor \log_2(T^{v*}) \rfloor$ and define $r_v = \lfloor \log_2(T^{v*}) \rfloor - q_v$.
- (ii) For $v \in \{0\} \cup E$, if $r_v < \log_2((\sqrt{5} + 1)/2)$, set $T^v = 2^{q_v}$, and set $T^v = 2^{q_v+1}$ otherwise.
- (iii) For $v \in G$, if $r_v \leq 0.5$, set $T^v = 2^{q_v}$, and set $T^v = 2^{q_v+1}$ otherwise;

(iv) For $v \in L$, we consider two cases: $q_v < q_0$ and $q_v = q_0$.

- $q_v < q_0$: set $T^v = 2^{q_v}$ if $r_v < 0.5$, and set $T^v = 2^{q_v+1}$ otherwise;
- $q_v = q_0$:
 - if $r_0 < \log_2((\sqrt{5} + 1)/2)$, set $T^v = 2^{q_v} = T_0$,
 - if $r_0 \geq \log_2((\sqrt{5} + 1)/2)$, set $T^v = 2^{q_v}$ if $r_v \leq 1 - \log_2((\sqrt{5} + 1)/2)$, and set $T^v = 2^{q_v+1}$ otherwise.

Once the cycle times are determined, the transportation cost can be improved using the well-known heuristic for VRP.

5. AN ILLUSTRATIVE EXAMPLE

We will illustrate the approach using the following example. Consider seven retailers as shown in fig. 2. These retailers are scattered around the warehouse having demand rates assumed to be stable between 1 or 2 ton/hour, totaling 10 ton/hour. We assume that a fleet of vehicle is available for product replenishment from the warehouse.

Fig. 2. A simple example with seven retailers

Table 1 shows the distances (in kilometers) between the different retailers. Travel times can be obtained from the table 1 by considering an average speed of 50 km/hour for each vehicle. We assume that all vehicles in the fleet have a capacity of 60 ton, a fixed operating and maintenance costs of €50 per hour and a transportation cost of €0.10 per kilometer. We also assume that fixed ordering cost at the warehouse is €75/hour and all retailers have also the same ordering cost of €50/hour. Finally, we consider the inventory holding cost rates are different for each retailer.

For the illustrative 7-retailers example (see Fig. 2), we consider the distribution pattern in which all retailers are served in a direct shipping tour, in which the vehicle leaves the warehouse, serves a retailer and then returns back to the warehouse after visiting each retailer. If only one vehicle is used, the minimal cycle time of the vehicle is its total travel time, i.e. 29 hours, while the maximal cycle time is 30 hours. This solution is feasible, but it is far from being optimal, in terms of inventory and transportation costs as well as

capacity usage, because the vehicle load is only 30 ton (or 50% of the vehicle capacity) for the tour to retailer 2, 4, 5 and 7. To use the vehicles capacity more efficiently and thus obtain a better solution, some retailers must be clustered en served in one sub-tour of the vehicle.

Table 1. Distances in kilometers between the different retailers

	RW	1	2	3	4	5	6	7
RW	0	100	175	175	100	50	75	50
1		0	50	150	175	150	175	150
2			0	50	150	225	250	225
3				0	100	225	250	225
4					0	125	175	150
5						0	50	75
6							0	50
7								0

Now, if a vehicle makes a “TSP” single-tour through the seven retailers, as is shown on the illustrative network in Fig. 3. The best single-tour starts from the warehouse and goes to the each retailers and then back to the warehouse at the end of the tour. Here we obtain an infeasible solution, since minimal cycle time of this tour is 11.5 hours, while the maximal cycle time is $60/(2+1+2+1+1+2+1) = 6$ hours. Because the minimal cycle time exceeds the maximal cycle time, the simple tour solution is infeasible. For the distribution tour solution to be feasible, the vehicle capacity has to be at least $10 \text{ ton/hour} \times 11.5 \text{ hours} = 115 \text{ ton}$. Therefore, when using the single tour solution for routing vehicles, a second vehicle would be necessary for replenishing the seven retailers or a vehicle with a larger capacity.

Fig. 3. An infeasible vehicle tour

To get a feasible and a better solution (Fig. 4), we adapted method developed in Roundy (1985) to determine the retailers set partitions G , E , L of retailers and then carry out the steps of the algorithm presented in section 3.

We now consider retailer clusters resulting from the set partitions G , E , L and we allow a vehicle to make multiple tours, the resulting solution is infeasible (see Fig. 4). The minimal cycle time is 20.5 hours (6.5 hours for the first tour

+ 10.5 hours for the second tour + 3.5 hours for the third tour) and the maximal cycle time is 12 hours. The maximal cycle time of the multiple tours is the minimum of $(60/(2+1), 60/(2+1+2), 60/(1+1)) = 12$ hours. The single vehicle multiple tours solution is still infeasible because the minimal cycle time exceeds the maximal cycle time. Therefore, for the VRP solution to be feasible, we have to add more vehicles to replenishing the retailers.

Thus, for the direct shipping solution in Fig. 4, the clustered retailers are initially served by three vehicles. Vehicle 1 makes the tour consisting of the tour $V_1(1, 2)$. The tour has $T_{\min} = 6.5$ hours, $T_{\max} = 20$ hours and $T_{\text{opt}}^v = 18.81$ hours. The theoretical optimal cycle time is between the minimum and maximal cycle time, so the actual cycle time is 18.81 hours. This gives a travelling cost rate for vehicle 1 is €66.93. Next, vehicle 2 makes the tour consisting of the tour $V_2(3, 4, 6)$. The tour has $T_{\min} = 10.5$ hours, $T_{\max} = 12$ hours and $T_{\text{opt}}^v = 18.05$ hours. Here, the theoretical optimal cycle time is bigger than the maximal cycle time, so the actual cycle time that will minimize cost rate will be equal to the maximal cycle time, i.e. 12 hours. Then, cost rate for vehicle 2 is €81.92.

Fig. 4. A feasible solution with three vehicles

Lastly, vehicle 3 makes the tour consisting of the tour V_3 (5,7). The tour has $T_{\min} = 3.5$ hours, $T_{\max} = 30$ hours and $T'_{\text{opt}} = 26.57$ hours. The theoretical optimal cycle time is also between the minimum and maximal cycle time, so the actual cycle time is 26.57 hours. Cost rate for travelling by vehicle 3 is €61.15.

So, the total cost rate for traveling tours with three vehicles is €210. Of course the total cost rate will decrease if we can now combine some of the tours.

Table 2 gives the quantities that are delivered to each of the retailers, which are given by the cycle time of the tour covering the retailers, multiplied by the retailers demand rates. We also can calculate the vehicle load when starting the different tours. In Table 3, we can see that vehicle load for this tour is utilized efficiently but the total cost rate for these travelling tours with three vehicles is expensive. To decrease the total cost rate for traveling tours, we attempt to combine some tours into a multiple tours trip using for some of the vehicles used in the initial solution.

Fig. 5, shows that only two vehicles are used to make multiple tours and the tours are combined using the same vehicle. The dotted lines show the tours that the first vehicle has to make, while the full line shows the tour of the second vehicle.

Fig. 5. A feasible solution with two vehicles

In this new solution, vehicle 1 makes the multi-tour consisting of the tour V_4 (1, 2, 5, 7). The multi-tour contains two simple tours. The simple tour (1, 2) has $T_{\min} = 6.5$ hours, $T_{\max} = 20$ hours and $T_{\text{opt}}^* = 18.81$ hours, while the simple tour (5, 7) has $T_{\min} = 3.5$ hours, $T_{\max} = 30$ hours and $T_{\text{opt}}^* = 26.57$ hours. Thus, the minimal cycle replenishment cycle time for this multi-tour is given by $T_{\min} = 10$ hours and the maximal replenishment cycle time is given by $T_{\max} = 20$ hours. The theoretical optimal cycle time is 18.81 hours. The theoretical optimal cycle time is between the minimum and maximal cycle time. Therefore, the actual replenishment cycle time is 18.81 hours. This results in a cost rate for vehicle 1 that is equal to €74.91.

Table 2. Delivery quantities at the retailers

Retailers	Delivery (ton)
1	37.62
2	18.81
3	24.00
4	12.00
5	26.57
6	24.00
7	26.57

Table 3. Vehicle load for the different tours

Tours	Vehicle load (ton)
V_1	56.43
V_2	60.00
V_3	53.14
Total	169.57

Then, vehicle 2 makes the same tour, consisting of the tour V_5 (3, 4, 6), exactly as in the previous solution (see Fig. 4). This tour has $T_{\min} = 10.5$ hours, $T_{\max} = 12$ hours and $T^v_{\text{opt}} = 18.05$

hours. The theoretical optimal cycle time is higher than the maximal cycle time. So, the actual replenishment cycle time is 12 hours. Then, cost rate for vehicle 2 is €81.92.

So, the total transportation cost rate for these multi-tours with the two vehicles is only €156.83 which is lower than the initial solution with three vehicles.

In Table 4, the replenishment cycle time of the tour is multiplied by the retailers demand rates. We thus obtain the quantities that are delivered to each of the retailers. To get a better solution, we tried to combine the tours using the same vehicle. Vehicle 1 delivers 94.05 ton in 10 hours of travel and vehicle 2 distributes 60 ton in 10.5 hours of travel as shown in Table 5. We realized that a saving of about 25.32% since the total cost rate decreases from €210 for the solution with tours to €156.83 for the solution with multiple tours.

This detailed analysis of this small case shows the potential savings resulting from the proposed approach. It explains in details the steps of the implementation of the approach.

Table 4. Quantities delivered to each of the retailers

Retailers	Delivery (ton)
1	37.62
2	18.81
3	24.00
4	12.00
5	18.81
6	24.00
7	18.81

Table 5. Vehicle capacity for the multiple tours

Tours	Vehicle load (ton)
V_4	94.05
V_5	60.00
Total	154.05

6. CONCLUSIONS

Managing inventory and routing in a supply network is a very challenging optimization problem. In this paper, we propose a global solution approach for a two-stage supply chain implementing VMI. The approach is based on some effective algorithms for inventory and routing sub-problems. In particular, the algorithm to solve the single-warehouse multi-retailer problem, proposed by Roundy (1985) and then extended by Shu (2010) and Chu and Shen (2010) for the stochastic case, are taken advantage of in our approach. The complex component in the proposed approach is still the VRP sub-problem which is heuristically solved in this paper.

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