Abstract The impact of the design parameters of electric power distribution systems on the propagation of harmonic distortion is investigated. This conceptual study is based on simulations on a generalized distribution system model, and leads to an increased insight in the mechanisms of the generation and propagation of voltage distortion. Moreover, analytical expressions are presented that predict the impact of changing design parameters on voltage distortion.

Keywords Distribution systems · Voltage distortion · Harmonics · Power quality · Power system impedance

1 Introduction

Because of the increasing penetration of non-linear loads in electric power distribution systems, utilities and equipment manufacturers are increasingly concerned with harmonic pollution of the voltage waveform. Voltage distortion is known to exhibit many adverse effects [1], and especially in areas where the electricity trade is being liberalized, it is feared that voltage distortion will increase in the near future [2]. In many parts of the world, the actual voltage distortion levels are maintained within planning levels by imposing appropriate emission limits to the harmonic line currents of equipment [3].

To determine appropriate equipment emission limits, both measurement campaigns [4] and simulations [5, 6] are required to study the harmonic propagation in an actual network. At first sight, it seems easier and cheaper to conduct simulations instead of measurements. However, the parameters and design strategies may be very different from one network to another [7], making a large number of simulations necessary. Moreover, many parameters are unknown and are therefore estimated or even neglected, rendering results that may differ largely from measured data [8]. Finally, as harmonic propagation studies are mostly limited to rather specific cases (e.g., [5]), fundamental insight in the mechanisms of harmonic propagation and the influence of the distribution system design is not obtained.

In this paper, the impact of different distribution system design parameters on harmonic propagation is investigated. The study is based on the analysis of a generalized distribution feeder model of which the parameters are varied. Some preliminary and qualitative results, based on simulations of a specific case study, are reported in [9]. In the present paper, however, a more fundamental approach is adopted. Analytical expressions and quantitative results are presented that approximately predict the impact of changing design parameters on the distortion levels. The validity of the analytical predictions is confirmed by simulations. Subsequently, the analysis is extended to include the effects of shunt capacitance, which are strongly related with power system resonances.

2 Basic power system setup

2.1 Network topology

In order to investigate the influence of different distribution system parameters on harmonic propagation, a simplified model of a typical medium voltage (MV) distribution system is adopted (Fig. 1). The high-to-medium voltage (HV/MV) transformer, which is feeding the point of common coupling (PCC), is represented by its short-circuit impedance $Z_m$. Several parallel and identical radial feeders, having five nodes each, connect to the loads. The conductor segments interconnecting...
the feeder nodes are represented by their series impedances $Z_{s,k}$.

The MV/LV transformers may be located between the MV bus and the feeders ($Z_l$, Fig. 1a) or between the feeder nodes and the loads ($Z_c$, Fig. 1b). The former practice calls for extended low voltage (LV) feeders, which is the practice in many North American regions. The latter practice calls for extended MV feeders, which is the practice in large parts of Europe. The latter arrangement is also found in many parts of the Western world.

The loads connected to the nodes are modeled as ideal current sources; this approximation is allowed as for moderate voltage distortion levels (total harmonic distortion (THD) < 10%) the accuracy is sufficient to draw general conclusions on the propagation of voltage distortion [10].

2.2 Transformer parameters

The following typical transformer impedance values are adopted:

$$Z_m = |Z_m|e^{j\theta_m}$$

= 0.1 pu < 80° (large HV/MV transformer)

$$Z_l = |Z_l|e^{j\theta_l}$$

= 0.1 pu < 45° (small HV/MV transformer)

$$Z_c = |Z_c|e^{j\theta_c}$$

= 0.04 pu < 80° (large MV/LV transformer)

The (fundamental) transformer impedances $Z_m$, $Z_l$, and $Z_c$ are considered to be resistive-inductive. The effects of skin effect and proximity losses are neglected. Considering $\theta_{Z_m} > 45^\circ$, the harmonic impedance for harmonics $h \geq 3$ is approximately inductive and equal to:

$$Z_{m,l,c}(h) \approx jh|Z_{m,l,c}| \sin \theta_{Z_m,l,c}$$

Consequently, the harmonic transformer impedance depends on both the magnitude and phase angle of the fundamental transformer impedance. For instance, it follows from (1) that the small HV/MV and MV/LV transformers exhibit an harmonic impedance that is 1.39 and 3.48 times smaller, respectively, than the harmonic impedance of the large HV/MV transformer.

2.3 Conductor parameters

Because of the considerable length of the conductors in many distribution systems, voltage drop is generally more important than power loss. The maximal allowable magnitude $|Z_s|$ of the (fundamental) conductor impedance is determined to limit the (fundamental) voltage drop $\Delta V = |V_{F1}| - |V_{F5}|$ to 0.12 pu along the feeder, while the voltage at the beginning of the feeder equals $|V_{F1}| = 1.06$ pu. The total feeder load is supposed to equal 1 pu at a power factor of $\cos \phi = 0.8$, and is equally divided among the nodes. These values are typical for distribution system operation.

The conductor type (overhead or cable) of all conductor segments is considered equal, but the conductor section may be tapered. For simplicity, the impedance of the feeder segments is supposed to increase linearly towards the end of the feeder. $M_t$ denotes the tapering factor determining the impedance of the last feeder segment:

$$Z_{s,t} = M_t Z_{s,1}$$

A non-tapered line is obtained for $M_t = 1$. The following maximal conductor impedance values are then found and serve in the following sections to obtain different feeder arrangements with equal (fundamental) voltage drops $\Delta V$:

$$Z_s = |Z_s|e^{j\theta_s} = 0.04815$ pu < 55° (regular overhead line, no tapering)

$$Z_s = |Z_s|e^{j\theta_s} = 0.06089$ pu < 80° (widely spaced overhead line, no tapering)

$$Z_s = |Z_s|e^{j\theta_s} = 0.04626$ pu < 30° (cable, no tapering)

$$Z_{s,1} = |Z_{s,1}|e^{j\theta_{s,1}} = 0.02898$ pu < 55° (regular overhead line, $M_t = 3$)
Z_{s,1} = |Z_{s,1}| e^{j \theta_{Z_{s,1}}} = 0.02070 \text{ pu } < 55^\circ 
(\text{regular overhead line}, M_t = 5)

Z_s = |Z_s| e^{j \theta_{Z_s}} = 0.09300 \text{ pu } < 55^\circ 
(\text{regular overhead line}, 
\text{no tapering, expected load power factor } \cos \phi = 1)

The pu values are referred to the rating of a single feeder. As with the transformer impedance, the conductor impedance is considered to be resistive-inductive. Because \( \theta_{Z_s} \geq 30^\circ \), the harmonic impedance of the conductors is approximately inductive:

\[ Z_s(h) \approx j h |Z_s| \sin \theta_{Z_s} \]  \hspace{1cm} (3)

Consequently, when the regular overhead line is replaced by the cable or the widely spaced overhead line, the magnitude of the harmonic impedance (3) decreases with a factor 1.71 or increases with a factor 1.52, respectively. When expected power factor of the feeder load is increased from 0.8 to 1, the harmonic impedance of the regular overhead line increases by 1.93.

2.4 Modeling peak rectifier loads

In this paper, peak rectifiers are supposed to be the most common non-linear load. Because of the limited number of nodes in a single feeder, a large number of loads is aggregated in every single node. A realistic load current spectrum should therefore include the effects of attenuation and diversity [11]. A typical current spectrum complying with these requirements is listed in Table 1 [12].

It should be remembered that the numerical results of voltage distortion calculations depend on the actual load current spectrum. Some examples, illustrating in more detail the specific influence of the zero-sequence \((h = 3, 9, 15, ...)\) and higher-order \((h \geq 17)\) harmonic load currents, are given in [9].

### Table 1 Line current spectrum of the applied non-linear load

| \( h \) | \( |I(h)|/|I(1)| \) |
|---|---|
| 1 | 1.000 |
| 3 | 0.820 |
| 5 | 0.534 |
| 7 | 0.316 |
| 9 | 0.166 |
| 11 | 0.082 |
| 13 | 0.015 |
| 15 | 0.010 |
| 17 | 0.0060 |
| 19 | 0.0050 |
| 21 | 0.0030 |
| 23 | 0.0010 |
| 25 | 0.0010 |
| 27 | 0, \( h > 25 \) |

### 3 Influence of neutral conductor practice

When the load current contains a zero-sequence component (e.g., caused by load unbalance or triplen harmonics), the impact of the neutral conductor practice on the voltage distortion throughout the distribution system becomes important. Both the arrangements of the feeder conductor and transformers influence the neutral conductor practice.

#### 3.1 Feeder conductor arrangement

In this paper, two typical feeder conductor arrangements are studied. For a symmetrical three-phase, four-wire arrangement with equal conductor sections, the impedance for zero-sequence currents equals four times the impedance for the positive- and negative-sequence components. In such systems, triplen harmonics (behaving as zero-sequence currents in balanced systems) may cause considerable voltage distortion [13].

When the three-phase conductors are split into three single-phase, two-wire, conductor sets, a three-phase, six-wire arrangement is formed, for which the zero-sequence impedance of the conductors equals the positive- and negative-sequence impedances. Therefore, zero-sequence currents cause considerably less voltage distortion than in a three-phase, four-wire arrangement.

To assess the influence of the conductor arrangement, the total harmonic voltage drop \( \Delta V_h \) of a single conductor segment is defined as the RMS value of the harmonic voltage drop:

\[ \Delta V_h = \sqrt{\sum_{h=2}^{\infty} |Z_s(h)|^2 |I(h)|^2} \]  \hspace{1cm} (4)

with \( I(h) \) denoting the harmonic current components flowing in the conductor. If balanced loads are assumed, the reduction of the total harmonic voltage drop \( \Delta V_h \) when changing a four-wire for a six-wire arrangement depends on the importance of the triplen harmonic components of the load current spectrum and can be approximated using (4) and (3):

\[ \Delta V_{h,6w} \approx \Delta V_{h,4w} \approx \left( \sum_{h \neq 3, h > 1} h^2 |I(h)|^2 + k_s^2 \sum_{h=3, h > 1} h^2 |I(h)|^2 \right)^{1/2} \]  \hspace{1cm} (5)

where \( k_s \) denotes the ratio of the zero-sequence to positive-sequence impedances of the conductors in the four-wire arrangement. For the load current spectrum of Table 1 and \( k_s = 4 \) (symmetrical four-wire arrangement), the reduction factor (5) equals 2.62. In practice, the neutral conductor section is often chosen smaller than the phase conductor sections, causing \( k_s > 4 \) and hence resulting in an even greater reduction factor.

In the same manner, the reduction factor of the total harmonic voltage drop of a conductor segment can be
approximated when the zero-sequence current components are eliminated before entering the feeder conductors (e.g., by means of an intermediate MV/LV transformer connected in ∆Y):

$$\frac{\Delta V_{h,\text{neutral}}}{\Delta V_{h,\text{no neutral}}} \approx \sqrt{\sum_{h \neq 3/h > 1} h^2|I(h)|^2 + k_m^2 \sum_{h = 3/h > 1} h^2|I(h)|^2} / \sum_{h \neq 3/h > 1} h^2|I(h)|^2 \quad (6)$$

For the line current spectrum of Table 1, the reduction factor (6) equals 1.28 for $k_s = 1$ (six-wire arrangement) and becomes 3.36 for $k_s = 4$ (symmetrical four-wire arrangement). In practice, the removal of the zero-sequence currents from six-wire arrangements is not practical and is therefore never encountered; the associated reduction factor is therefore of purely academic value.

3.2 Transformer arrangements

The neutral conductor may be interrupted by the MV/LV transformer, e.g., by connecting it in ∆Y arrangement, thereby reducing the harmonic voltage drop of the HV/MV transformer located upstream (in the PCC). Using (1), and assuming balanced loads, the reduction factor of the voltage distortion in the PCC can be calculated:

$$\frac{\text{THD}(V_{\text{PCC}})_{\text{neutral}}}{\text{THD}(V_{\text{PCC}})_{\text{no neutral}}} \approx \sqrt{\sum_{h \neq 3/h > 1} h^2|I(h)|^2 + k_m^2 \sum_{h = 3/h > 1} h^2|I(h)|^2} / \sum_{h \neq 3/h > 1} h^2|I(h)|^2 \quad (7)$$

where $k_m$ denotes the ratio of the zero-sequence to the positive-sequence impedances of the HV/MV transformer. For the load current spectrum of Table 1 and $k_m = 1$, the reduction of the voltage distortion in the PCC equals 1.28.

4 Basic factors governing power system harmonic propagation

To assess the influence of the power system parameters presented in the previous sections, simulations have been performed on the network of Fig. 1. In this section, the impedance of the MV/LV transformers is neglected (i.e., $|Z_c| = |Z| = 0$). Non-zero MV/LV transformer impedances will be assumed in Sect. 5, where a case study is presented. In this section, also the effects of shunt capacitance are neglected; for moderate feeder conductor lengths (up to several kilometers for cable conductors, and up to a few tens of kilometers for overhead conductors) the shunt capacitance becomes important for higher harmonics only, for which the injected current is usually small. The effects of capacitance are discussed in Sect. 6.

The total (fundamental) load of a single feeder equals 1 pu at $\cos \phi = 1$, and is balanced and equally divided among the feeder nodes. The voltage in the first feeder node (closest to the PCC) is controlled to $|V_{f1}| = 1.06$ pu for all simulations. The source voltage from the HV bus is considered to be purely sinusoidal. The total amount of distorting loads (with a line current spectrum as in Table 1) represents 10% of the total fundamental load, while the remaining (linear) load only draws fundamental current.

In accordance with IEC regulations [14], only the first 40 harmonics are considered when calculating THD values. By analogy with (4), the total harmonic voltage drop $\Delta V_{h}$ of the feeder is defined as:

$$\Delta V_{h} = \sqrt{\sum_{h = 2}^{40} |V_{f1}(h) - V_{f5}(h)|^2} \quad (8)$$

This measure allows to explain the difference between the voltage distortion in the PCC and at the end of the feeder. Indeed, except when the neutral conductor is interrupted, the operating conditions encountered in this paper allow for the following approximation if $\Delta V_{Fh}$ is expressed in “per units”:

$$\text{THD}(V_5) \approx \text{THD}(V_{\text{PCC}}) + \Delta V_{Fh} \quad (9)$$

This is confirmed by the simulation results, which are summarized in Table 2 and are discussed in the following subsections. Simulation no. 1 is considered to be the base case, to which the effects of all parameter variations are compared.

4.1 Transformer choice

The impact of the transformer parameters on voltage distortion is explained by comparison of simulation nos. 1–3 from Table 2. The results are graphically represented in Fig. 2a. From Sect. 2.2, it follows that for simulation nos. 2 and 3 the magnitude of the harmonic transformer impedance is 3.48 and 1.39 times smaller, respectively, than the harmonic impedance for simulation no. 1 (base case). As expected, this causes the voltage THD in the PCC to decrease with about the same factor.

At the end of the feeder, the voltage THD is several times greater than in the PCC because of the harmonic voltage drop of the feeder conductors. This result is in accordance with recent measurements in the French LV system [13]. Because the total harmonic voltage drop of the feeder conductor $\Delta V_{Fh}$ remains constant between simulation nos. 1, 2, and 3, the reduction of the voltage THD at the end of the feeder is rather small and is approximately equal to the reduction of the voltage THD in the PCC.
Concluding, the impact of the transformer impedance on voltage distortion is quite important in the PCC, but less important at the end of the distribution feeder, where the total harmonic voltage drop of the feeder conductors becomes dominant. It turns out that the voltage distortion in the PCC increases when the (fundamental) transformer impedance increases or becomes more inductive.

4.2 Feeder conductor type

The impact of the feeder conductor type on voltage distortion is explained by comparing simulation nos. 4–6 from Table 2. The results are graphically represented in Fig. 2b. For simulation no. 4, the phase angle of the conductor impedance is decreased from 55° to 30°, as compared with simulation no. 1. According to Sect. 2.3, this causes the harmonic impedance of the feeder to reduce with a factor 1.71. As expected, the total harmonic voltage drop of the feeder $\Delta V_{fh}$ is reduced by about the same factor as well. In turn, this causes a significant reduction of the voltage THD at the end of the feeder. Similar results are obtained when comparing simulation nos. 1 and 5, where the conductor impedance angle is increased from 55° to 80° (causing the harmonic impedance of the feeder to increase by about 1.52), and for simulation no. 6, where the expected displacement factor is increased from 0.8 (inductive) to 1.0 (causing the harmonic impedance of the feeder to increase by about 1.93). It is to be noted that the increase of the expected fundamental displacement factor (simulation nos. 1 and 6) has more impact than the increase of the phase angle of the conductor impedance (simulation nos. 1 and 5).

Table 2 Voltage THD for different network parameters

<table>
<thead>
<tr>
<th>Sim. no.</th>
<th>$Z_m$ (% pu)</th>
<th>$\theta_{Z_m}$ (deg)</th>
<th>$Z_{s,l}$ (% pu)</th>
<th>$\theta_{Z_s}$ (deg)</th>
<th>$M_l$</th>
<th>Voltage THD</th>
<th>$\Delta V_{fh}$ (% pu)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$V_{PCC}$ (%)</td>
<td>$V_5$ (%)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10.0</td>
<td>80</td>
<td>4.815</td>
<td>55</td>
<td>1</td>
<td>4.12</td>
<td>13.31</td>
<td>9.57</td>
</tr>
<tr>
<td>2</td>
<td>4.0</td>
<td>45</td>
<td>4.815</td>
<td>55</td>
<td>1</td>
<td>1.21</td>
<td>10.63</td>
<td>9.57</td>
</tr>
<tr>
<td>3</td>
<td>10.0</td>
<td>45</td>
<td>4.815</td>
<td>55</td>
<td>1</td>
<td>3.03</td>
<td>12.34</td>
<td>9.57</td>
</tr>
<tr>
<td>4</td>
<td>10.0</td>
<td>80</td>
<td>4.626</td>
<td>30</td>
<td>1</td>
<td>4.22</td>
<td>10.28</td>
<td>6.27</td>
</tr>
<tr>
<td>5</td>
<td>10.0</td>
<td>80</td>
<td>6.089</td>
<td>80</td>
<td>1</td>
<td>3.98</td>
<td>16.92</td>
<td>13.88</td>
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<tr>
<td>6</td>
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<td>80</td>
<td>9.300</td>
<td>55</td>
<td>1</td>
<td>4.12</td>
<td>24.43</td>
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</tr>
<tr>
<td>7</td>
<td>10.0</td>
<td>80</td>
<td>2.898</td>
<td>55</td>
<td>3</td>
<td>4.10</td>
<td>13.28</td>
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<td>8</td>
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<td>9</td>
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<td>80</td>
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<td>2.81</td>
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<td>80</td>
<td>4.815</td>
<td>55</td>
<td>1</td>
<td>3.21</td>
<td>11.00</td>
<td>9.57</td>
</tr>
<tr>
<td>11</td>
<td>10.0</td>
<td>80</td>
<td>4.815</td>
<td>55</td>
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<td>4.12</td>
<td>7.92</td>
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<td>10.0</td>
<td>80</td>
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<td>1</td>
<td>3.21</td>
<td>6.57</td>
<td>3.62</td>
</tr>
</tbody>
</table>

Fig. 2 Influence of different parameters: a transformer parameters, b feeder conductor parameters, c tapering

![Fig. 2](image_url)
The impact on the voltage THD in the PCC is rather small. However, it is noticed that the voltage THD in the PCC is slightly reduced, when the conductor impedance angle is increased. This is caused by phase angle diversity between the nodes [11], and becomes more important for increasing harmonic orders and increasingly inductive conductors. The stronger the higher harmonics of the load current, the more this effect becomes noticeable; an example is given in [9].

Concluding, the impact of the feeder conductor impedance on voltage distortion is quite important at the end of the feeder, but very small in the PCC. Similar as with the transformer impedance, it turns out that the harmonic voltage drop of the feeder increases when the (fundamental) conductor impedance increases or becomes more inductive.

4.3 Feeder conductor tapering

The impact of feeder conductor tapering on the voltage distortion is explained by comparing simulation nos. 1, 7, and 8 from Table 2. The results are graphically represented in Fig. 2c. For simulation nos. 7 and 8, the tapering factor $M_t$ is increased from 1 to 3 and 5, respectively, as compared with simulation no. 1. The impact on the voltage THD in the PCC and at the end of the feeder is negligible. This is mainly due to the equal fundamental voltage drop criterion for calculating the conductor segment impedances $Z_{s,k}$ (Sect. 2.3). Therefore, feeder conductor tapering will not be explored any further in this paper.

4.4 Neutral conductor practice

4.4.1 Four-wire conductor arrangement

The impact of interrupting the neutral conductor on voltage distortion is explained by comparing simulation nos. 1, 9, and 10 from Table 2. The results are graphically represented in Fig. 3a. In simulation no. 9, the neutral conductor is interrupted between the feeder and load nodes by $\Delta Y$ transformers with negligible impedance $Z_c=0$. It follows that the expected decrease of the voltage THD in the PCC by about 1.28 (Sect. 3.2) matches the simulations quite well. Also, the reduction of the harmonic voltage drop of the feeder (about the factor 3.36 as predicted in Sect. 3.1) causes a considerable reduction of the voltage THD at the end of the feeder.

The same harmonic voltage reduction in the PCC as above is found when comparing simulation nos. 1 and 10, where, in the latter, the neutral conductor is interrupted in the PCC by a $\Delta Y$ transformer with negligible impedance $Z_l=0$. However, the harmonic voltage drop of the feeder conductor is not influenced leading to only a small reduction of voltage THD at the end of the feeder.

4.4.2 Six-wire conductor arrangement

In simulation nos. 11 and 12, a six-wire conductor arrangement is applied instead of the four-wire conductor arrangement of simulation no. 1. The results are graphically represented in Fig. 3b. The simulation results match well the predicted reduction of the total harmonic voltage drop of the conductor (with a factor 2.62, Sect. 3.1), again causing a considerable reduction of the voltage THD at the end of the feeder.

Finally, in simulation no. 12, the neutral conductor is interrupted in the PCC by a $\Delta Y$ transformer with negligible impedance $Z_l=0$. As with the four-wire conductor arrangement, this reduces the voltage THD in the PCC (and equals the value of simulation nos. 9 and 10), but not the harmonic voltage drop of the conductor $\Delta V_{Fh}$ (which equals the value of simulation no. 11), yielding only a small reduction of the voltage THD at the end of the feeder.

4.4.3 Conclusion

Concluding, the neutral conductor practice has significant impact on the voltage distortion throughout the distribution system, because it affects the harmonic
voltage drop caused by the zero-sequence currents. Interrupting the neutral conductor always reduces the voltage distortion in the PCC, and also the harmonic voltage drop of the feeder conductors upstream of the interruption. Also switching between four- and six-wire conductor arrangements reduces the harmonic voltage drop of the feeder conductors significantly.

### 5 Case study: differences between power system design practice in Europe and North America

#### 5.1 Power system setup and typical parameters

In this section, a comparison is made between the harmonic propagation properties of the two distribution system structures of Fig. 1, which differ in the placement and arrangement of the MV/LV transformers. The transformers are either placed at the beginning of the feeders (which is common to most European distribution systems, Fig. 1a), or between the feeder and load nodes (which is common to many North American distribution systems, Fig. 1b).

When the MV/LV transformers are located at the beginning of the feeders ($Z_l$), their size is usually quite large. In this case, the neutral conductor is interrupted in the MV/LV transformer, which is common to European practice. When the MV/LV transformers are located between the feeder and load nodes ($Z_c$), their size is usually rather small. In this case, the neutral conductor is not interrupted in the MV/LV transformer, which is common to North American practice.

The resulting distribution systems, including the considered transformer arrangements, are depicted in Fig. 4. Note that in both design styles under consideration the distribution feeders always contain a neutral conductor. The conductor type may be regular overhead or cable, while tapering is not considered. The conductor arrangement may be four wire or six wire. The parameters of the conductors and of the transformers are taken from Sects. 2.2 and 2.3. The loading of the feeders is the same as in Sect. 4, the results are summarized in Table 3.

#### 5.2 Propagation of voltage distortion

The simulation results of Table 3 show that the influence of the different parameters is in accordance with the findings of Sect. 4. In any case, when comparing equal conductor types and arrangements (comparing simulation nos. C1 with C5, C2 with C6, and so on), the resulting voltage THD in the PCC and in the load node at the end of the feeder is always worse for the North American distribution style. This is no surprise, as the neutral conductor is interrupted in the MV/LV transformers in the European style, contrary to the North American style.

Moreover, the North American design style typically results in single-phase (or six-wire) feeders operated in MV (simulation nos. C2 and C4, Table 3), whereas the European style is mainly resulting in three-phase (or four-wire) feeders operated in LV (simulation nos. C5 and C7, Table 3) [15]. The comparison between the North American and European design styles is graphically represented in Fig. 3c.

Both in Europe and North America, overhead lines are still widely present in rural areas. Comparing simulation nos. C2 and C5, it is easily seen that the resulting voltage THD in the PCC would be larger for the North American distribution style because the neutral conductor is not interrupted before reaching the HV/MV transformer. On the other hand, the voltage THD at the end of the feeder is quite smaller for the North American style. This is mainly caused by the reduced zero-sequence feeder impedance of the associated six-wire arrangement. In practice, this effect will be even more pronounced than predicted from Table 3 because the section of the neutral conductor is often chosen smaller than the section of the phase conductors.

In domestic and commercial areas, cable conductors are preferred. The comparison of simulation nos. C4 and C7 shows the same trend as for overhead lines.

### Table 3 Case study - summary of the simulation results: voltage distortion in the PCC and in the load node at the end of the feeder

<table>
<thead>
<tr>
<th>Sim. no.</th>
<th>MV/LV transformer</th>
<th>Conductor arrangement</th>
<th>$V_{\text{PCC}}$</th>
<th>$V_{\text{s}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Small, located in nodes</td>
<td>Overhead 4-w</td>
<td>4.25</td>
<td>15.51</td>
</tr>
<tr>
<td>C2</td>
<td>Small, located in nodes</td>
<td>Overhead 6-w</td>
<td>4.25</td>
<td>9.84</td>
</tr>
<tr>
<td>C3</td>
<td>Small, located in nodes</td>
<td>Cable 4-w</td>
<td>4.35</td>
<td>12.43</td>
</tr>
<tr>
<td>C4</td>
<td>Small, located in nodes</td>
<td>Cable 6-w</td>
<td>4.35</td>
<td>8.778</td>
</tr>
<tr>
<td>C5</td>
<td>Large, located at beginning of feeder</td>
<td>Overhead 4-w</td>
<td>3.24</td>
<td>12.90</td>
</tr>
<tr>
<td>C6</td>
<td>Large, located at beginning of feeder</td>
<td>Overhead 6-w</td>
<td>3.24</td>
<td>8.38</td>
</tr>
<tr>
<td>C7</td>
<td>Large, located at beginning of feeder</td>
<td>Cable 4-w</td>
<td>3.32</td>
<td>10.03</td>
</tr>
<tr>
<td>C8</td>
<td>Large, located at beginning of feeder</td>
<td>Cable 6-w</td>
<td>3.32</td>
<td>7.33</td>
</tr>
</tbody>
</table>
The possible presence of shunt capacitance in the feeders causes resonances. Resonance conditions are introduced in the power system of Fig. 1 by adding capacitors to the feeder nodes. The capacitance is represented by its total admittance $jY_c$, which is equally divided among the feeder nodes, and represents both conductor capacitance and other capacitances (e.g., power factor correction capacitors). Moreover, the feeder nodes are shunted with equally divided parasitic resistances, the total admittance of which equals 0.01 pu. The remaining power system parameters are chosen as in Sect. 4. To assess the impact of resonance on voltage distortion, a harmonic voltage amplification factor $M$ is introduced by comparing the actual voltage distortion (under resonance) to the distortion present if the capacitance were removed:

$$M = \frac{\text{THD}(V)_{Y_c 
eq 0}}{\text{THD}(V)_{Y_c = 0}}$$

(10)

The $M$ values calculated in this section are obtained using the load current spectrum of Table 1. The quality factor $Q$ of a certain resonance condition is assessed by calculating (10) using a load current spectrum containing only the fundamental and a single harmonic of the same harmonic order as the resonance frequency of the power system. The simulation results are presented in Table 4 and are explained in the following subsections. A more detailed analysis, including a more elaborate discussion on damping effects, is presented in [16].

### 6.1 Required capacitance for resonance

The required total capacitive admittance $jY_c$ to obtain resonance at a given harmonic order is of course function of the inductance present in the system. Simulation no. R1 of Table 4 corresponds to the base case of Sect. 4. In simulation no. R2 (six-wire instead of four-wire feeders), the feeder inductance for zero-sequence currents is reduced by a factor 4, giving rise to a largely increased $Y_c$ requirement to obtain resonance at the 3rd and 9th harmonics. The same conclusion is drawn from simulation no. R3 (neutral conductor interrupted at the PCC by a ΔY transformer), where the inductance of the HV/MV transformer is virtually removed for zero-sequence currents. Similar conclusions are drawn for all harmonics in simulation no. R4 (cable feeder), where the feeder inductance is significantly reduced as compared to the base case. It is also noticed that the large zero-sequence inductance of the base case (simulation no. R1) causes the required capacitance for 9th harmonic resonance to be smaller than for 11th harmonic resonance.

<table>
<thead>
<tr>
<th>Case no.</th>
<th>$h$</th>
<th>$Y_{feeder}$</th>
<th>$Y_{PCC}$</th>
<th>$Y_{RCC}$</th>
<th>$M_{RCC}$</th>
<th>$M_{PCC}$</th>
<th>$M_{OCC}$</th>
<th>$M_{0CC}$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>3</td>
<td>(4.36)</td>
<td>$Y_c = 0.389$</td>
<td>(11.50)</td>
<td>$Y_c = 0.321$</td>
<td>(15.12)</td>
<td>$Y_c = 0.164$</td>
<td>(23.12)</td>
<td>$Y_c = 0.164$</td>
</tr>
<tr>
<td>R2</td>
<td>5</td>
<td>(4.03)</td>
<td>(11.78)</td>
<td>(15.02)</td>
<td>(15.12)</td>
<td>(11.50)</td>
<td>(7.12)</td>
<td>(2.75)</td>
<td>Base case—regular overhead feeder</td>
</tr>
<tr>
<td>R3</td>
<td>6</td>
<td>(4.25)</td>
<td>(8.02)</td>
<td>(11.50)</td>
<td>(15.02)</td>
<td>(15.12)</td>
<td>(11.50)</td>
<td>(15.12)</td>
<td>Neutral conductor interrupted in PCC</td>
</tr>
<tr>
<td>R4</td>
<td>7</td>
<td>(4.25)</td>
<td>(8.02)</td>
<td>(11.50)</td>
<td>(15.02)</td>
<td>(15.12)</td>
<td>(11.50)</td>
<td>(15.12)</td>
<td>Neutral conductor interrupted in PCC</td>
</tr>
<tr>
<td>R5</td>
<td>8</td>
<td>(4.25)</td>
<td>(8.02)</td>
<td>(11.50)</td>
<td>(15.02)</td>
<td>(15.12)</td>
<td>(11.50)</td>
<td>(15.12)</td>
<td>Neutral conductor interrupted in PCC</td>
</tr>
<tr>
<td>R6</td>
<td>9</td>
<td>(4.25)</td>
<td>(8.02)</td>
<td>(11.50)</td>
<td>(15.02)</td>
<td>(15.12)</td>
<td>(11.50)</td>
<td>(15.12)</td>
<td>Neutral conductor interrupted in PCC</td>
</tr>
</tbody>
</table>

Table 4 Quality factors and amplification of voltage distortion at different resonant conditions
6.2 Quality factor of resonance

The quality factor associated with resonance is determined by the amount of damping present in the power system. In the power system under study and with the assumed load condition, damping is primarily provided by the series losses of conductor and transformer impedances. Therefore, the quality factor is increasing with increasing harmonic orders [16]. However, increased parasitic losses in simulation no. R5 (shunt loss, total resistive shunt admittance of 0.05 pu) and simulation no. R6 (series loss in capacitors, fundamental loss factor of 0.01) considerably reduce the quality factor, especially for higher harmonic orders.

6.3 Peak rectifier current spectrum

In practice, the actual load current spectrum contains several strong harmonics. Moreover, resonance also affects the power system impedance at other frequencies around the resonance frequency. Consequently, the analytical prediction of the amplification $M$ of voltage distortion under resonance conditions is not possible from the mere knowledge of the quality factor alone. The actual voltage distortion level is therefore obtained from simulations. The results from Table 4 are in accordance with the conclusions of Sect. 4. Additional remarkable features are explained below.

From Table 4, it follows that the actual voltage distortion amplification factors $M$ tend to decrease with increasing order of the harmonic resonance, contrary to the quality factors $Q$. This results from the rapidly decreasing amplitude of the line current harmonics of the applied non-linear load spectrum. Moreover, the actual amplification factors $M$ are always smaller than the corresponding quality factors $Q$, because the quality factor only takes load current components at the resonance frequency into account.

In the base case (simulation no. R1), it was noticed that 3rd harmonic resonance is not visible. Indeed, it is masked out by the closely neighboring 5th harmonic resonance, as is illustrated in Fig.5. The same effect is encountered in simulation nos. R4–R6. In simulation nos. R3 and R6, the 11th harmonic resonance is also masked out by the 9th harmonic resonance at the end of the feeder.

Finally, it is no surprise that interrupting the neutral conductor before it reaches the PCC (simulation no. R3) eliminates 3rd and 9th harmonic resonances (caused by zero-sequence currents) in the PCC.

7 Conclusion

In this paper, the impact of distribution system parameters on the propagation of harmonic distortion has been considered. Under the condition that the feeder conductor parameters of different designs are chosen to obtain the same fundamental voltage drop of the feeder conductors, the following conclusions can be drawn when the power system capacitance is negligible:

- The more inductive the (fundamental) impedance of the transformers and the feeder conductors become, the higher is the resulting voltage distortion. At the end of the feeder, the harmonic impedance of the feeder conductors (and its associated harmonic voltage drop) becomes dominant.
- Overhead lines cause greater harmonic voltage drop than cables, therefore the voltage THD in the load nodes near the end of the feeders is greater with overhead lines (and, consequently, also throughout the feeders).
- Feeder conductor tapering has very little influence on the voltage THD in the PCC and at the end of the feeder.
- When zero-sequence currents are present, the application of six-wire conductor arrangements results in less voltage THD in the load nodes near the end of the feeder (and, consequently, also throughout the feeders) as compared with four-wire conductor arrangements.
- Interrupting the neutral conductor significantly reduces the harmonic voltage drop of transformers and feeder conductors upstream of the interrupting point.

Under reasonable assumptions, these effects can be analytically predicted and match the simulation results quite well. The results strongly depend on the harmonic content of the load currents. These considerations help to explain the different harmonic propagation behavior of different design approaches, as was shown in a case study, comparing typical European and North American design styles.

When the power system capacitance is taken into account, the following conclusions can be drawn:
– The required capacitance to obtain resonance at a given harmonic order increases when cable feeders are applied instead of overhead feeders. In the case of four-wire feeders, the required capacitance for resonance of zero-sequence harmonic currents is much reduced as compared to six-wire feeders.

– At low power system loading levels, the quality factor of the resonance tends to increase with increasing harmonic order. However, this effect is noticeably reduced if parasitic losses are taken into account.

– The amplification of voltage distortion (as compared to the situation with negligible power system capacitance) is always smaller than the corresponding quality factor.

– Closely neighboring resonances may cause one of them to be masked out by the other when sweeping the capacitance value.

– If the neutral conductor is interrupted, the resonance of zero-sequence current components is not visible upstream of the interruption.

The actual distortion level is not only influenced by the quality factor at the resonance frequency. Also the influence of resonance at neighboring frequencies and the influence of other load current harmonics is to be taken into account. This makes the analytical prediction of the amplification of voltage distortion at resonance rather complicated and was therefore not pursued in this paper.

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References