Optical transmission model for thin two-dimensional layers

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Abstract: A simplified simulation method to calculate the transmission of an oblique incident plane wave through a thin two-dimensional layer is presented. The algorithm, based on a ray-tracing technique, uses the Extended Jones-calculus and neglects diffraction within the layer. The algorithm is applied to an in-plane switching liquid crystal display. The transmission at the display surface is compared with the more accurate reduced-grating method (RGM) [5].

Keywords: diffraction, liquid crystal display, in-plane switching

I. INTRODUCTION

Calculating the optical transmission of a plane wave through a liquid crystal (LC) medium is a difficult task. For one-dimensional layers, simple matrix formalisms known as the Jones-calculus [1] and the Berreman-method [2] were developed years ago. These methods take phenomena such as refraction and birefringence into account and are sufficient for one-dimensional layered structures such as LC devices and anti-reflection coatings.

In twodimensional media, propagation is more complicated due to diffraction effects leading to refraction or scattering. Therefore, more precise calculation algorithms based on the finite-difference time-domain algorithm [3], beam-propagation [4] or the reduced-grating method (RGM) [5] were developed.

In this article, a simplified algorithm for oblique light incidence using the Extended Jones-calculus (EJC) [6] is explained. The algorithm for the propagation through the LC layer is explained in detail and the possibilities and limitations of the simplified method are examined.

II. SIMULATION MODEL

The simulation model starts from a two-dimensional LC medium that is invariable along the y-axis with the director orientation given on a regular rectangular grid in the x,z plane. The optical parameters of the uniaxial LC are given by the ordinary and extra-ordinary refractive indices \( n_o \) and \( n_e \). To assume the LC properties homogeneous inside each grid box, the grid must be sufficiently fine. Our goal is to calculate the propagation of an obliquely incident plane wave through the LC layer. At the bottom surface of the LC layer, refraction occurs. Because of refraction the propagation direction in air of the plane wave \( k_0 \), determined by the inclination angle \( \theta_k \) (angle with the z-axis) and the azimuth angle \( \varphi_k \) (angle between the x-axis and the projection of \( k_0 \) on the x,y plane), is changed to a different direction \( k \) inside the medium. Therefore the inclination angle \( \theta_k \) is changed according to Snell’s refraction law. As refractive index for the LC medium, we choose the ordinary refractive index \( n_o \). In the case of small birefringence, the difference in wave vector for the ordinary and extra-ordinary wave is negligible. The EJC has been developed for one-dimensional media. To be able to use the EJC we assume that in our structure the thickness of the layer measured along the z-axis is small compared to the lateral LC dimensions and not much larger than the wavelength of the light. When the variations of the LC director are relatively slow, the EJC provides a good approximation of the correct result [7]. In this case, we may calculate the propagation through the LC only taking into account birefringence and neglecting diffraction.

To handle a plane wave incident on the two-dimensional medium, a ray tracing technique is used. The plane wave is constructed as a large number of rays parallel to the vector \( k_0 \) in air or \( k \) inside the medium. From each of the grid boxes at the bottom of the layer a ray propagates towards the other side in the direction of \( k \).

For each rectangle in which a ray passes on its way, a propagation matrix based on the EJC is calculated. By doing so, the variation of the LC in the x-direction is neglected. If the variation of the LC director along the x-axis is sufficiently slow, this is a good approximation.

III. TRANSMISSION NEAR THE LAYER SURFACE

The simulation method is tested by applying it to an in-plane switching liquid crystal display (IPS-LCD). An IPS-LCD consists of a thin LC layer between two glass substrates. The transparent electrodes form an interdigitated pattern of positive and negative stripes on the bottom glass substrate parallel to the y-axis. The LC layer has a thickness of 4 µm, the electrode width is 6 µm and the gap in between 18 µm. The rubbing direction of the alignment layers, which fixes the orientation of the molecules at the glass substrates, makes an angle of 10° with the y-axis. The pretilt, the angle of the director with the glass surface is about 1°. The LC-material is Merck ZLI-4792. When a voltage is applied between the electrodes, the calamatic LC molecules tend to orient themselves along the field lines of the two-dimensional electric field that appears. The director simulations for the LC medium were performed using the software package 2dimMOS [8]. The polarizer at the bottom of the LCD is oriented perpendicular to the rubbing direction and the analyzer at the top parallel. The optical
simulations presented in this article, use a monochromatic incident plane wave with a wavelength of 632.8 nm and the incident direction is given by $\theta_i = \phi_i = 25^\circ$.

We verify our simulation results by comparing the transmission of an unpolarized plane wave through the IPS-LCD, with the more accurate RGM. Fig. 1 shows the intensity of the transmission at the surface relative to the intensity of the incident unpolarized plane wave when voltages of 5 to 25 V are applied between the positive and the negative electrodes. The plotted region starts in the middle of one electrode and ends in the middle of the next one. The graph shows that the overall correspondence between the method using the EJC and the RGM is quite good. At 5 V, the applied voltage is just above the threshold voltage and only at the edges of the electrodes the electric field is strong enough to reorient the LC. The transmission reaches a maximum at 10 V and decreases for higher voltages. In the bulk between the two electrodes, the electric field is parallel to the glass surface. In this area the director is almost uniform and the correspondence between both methods is excellent. At higher voltages the refractive index variations above the electrodes increase and diffraction within the LC becomes important. This explains the deviations for higher voltages. From Fig. 1 we can conclude that the method using the Jones-calculus is accurate in predicting the amplitude of the electric field. However the calculation of the phase variation of the electric field before propagating through the analyzer, shown in Fig. 2, is less good. Even for quite low voltages, there is a small deviation in the phase variation and for increasing voltage this deviation becomes larger. For the analysis of this imprecision, the phase of the electric field is divided in an absolute and a relative phase variation.

The origin of the relative phase lies in the birefringence of the LC. When propagating through a birefringent medium, a plane wave is split up in two modes which propagate with a different speed. This retardation is responsible for the change in polarization state of the plane wave and is directly related to the transmission of a LC cell between crossed polarizers. Although the phases of $E_x$ and $E_y$ differ for both methods at high voltages, the relative phase $E_y - E_x$ corresponds very well. This corresponds with the fact that birefringence is correctly taken into account by the Jones-calculus. The absolute phase variation of the electric field is related to the propagation distance of the rays in the medium. If diffraction is absent, the absolute phase variation is the same for all rays. The differences in the absolute phase variation are mainly due to focusing of the wave.

IV. LIMITATIONS OF THE METHOD

To ensure that the method with the EJC gives accurate results diffraction effects have to be negligible. Therefore, the layer should be thin enough so that the focusing has not yet occurred and the lateral LC variations should be low within a distance in the order of one wavelength. Although the right-hand side of the LC inside the IPS-LCD is practically symmetric to the left-hand side, the transmission curves in Fig. 1 show an asymmetry. Also the differences in the absolute phase are mainly present on the right-hand side.

![Fig. 2. Variation of the phase of the electric field components $E_x$, $E_y$ and the relative phase $E_y - E_x$ before propagation through the analyzer.](image)

Thereinafter, a quantitative way to evaluate the lateral variations is explained. In fact the variations of the refractive index of the medium are important. For LCs this is rather complex since the material is birefringent. When a plane wave propagates through a birefringent medium a superposition has to be made of the ordinary and the extra-ordinary wave. The extra-ordinary wave feels an effective refractive index $n_{eff}$, which depends on the angle between the direction of propagation $k$ and the local orientation of the LC director and has a value between $n_o$ and $n_e$. Fig. 3 represents the effective refractive index of the extra-ordinary wave corresponding with the propagation direction $k$ for the molecules in the middle section of the liquid crystal for the same voltages as in Fig. 1. Due to the oblique incidence the variations of the refractive index are much larger at the right-hand side and we expect the largest differences in that region.

V. CONCLUSIONS

A simple and fast simulation algorithm to calculate the transmission through thin two-dimensional layers is presented. The method is based on neglecting the diffraction inside the thin layer and shows good results in predicting the transmission at the top surface of an IPS-LCD. Due to focusing of the light, the calculation of the phase of the electric field is less accurate in the regions with the largest changes in refractive index. The method is useful in most display applications which work with thin layers and low voltages. Finally, the algorithm is fast and easily expandable to the three-dimensional case.

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REFERENCES


![Fig. 3. Variation of the effective refractive index of the extra-ordinary wave for the molecules in the middle of the LC layer.](image)