**Problem Setting**
- In an fMRI data analysis, many brain regions are tested simultaneously for activation.
- Several multiple testing procedures (MTPs) exist that prevent an inflation of type I errors while accounting for the spatial structure of the data.
- Such procedures consider testing peaks or clusters of activation (Chumbley, 2010).
- Not only the peak-wise error rates are of interest, but also an estimate of the true positive rate (power).
- We focus on the analysis of maxima (or peaks) of activation.
- The goal is to estimate the proportion of non-active peaks ($\pi_0$).
- Given $\pi_0$:
  1. The type I error and power following a thresholding technique can be estimated, enabling a direct trade-off between sensitivity and specificity.
  2. Widely used MTPs can be made adaptive and more powerful.

**A Measure of Significance for Peaks**
- Random Field Theory (RFT) allows to obtain $p$-values for peaks to test the null hypothesis of no activation ($H_0$) against the alternative of activation ($H_1$).
- Consider a t-map with a cluster defining threshold of $t_c$ with $P(T > t_c) = 0.05$. Peaks above $t_c$ are considered to be activated.
- This is equivalent with comparing $p$-values of peaks with an $\alpha$-threshold. Under RFT, $p$-values for peaks can be obtained.
- Under $H_0$, RFT $p$-values are uniformly distributed.
- Results of testing $H_0$ for peaks can be summarized as follows:

\[
\begin{array}{c|c|c|c|c|c}
\hline
& \text{true } H_0 & \text{true } H_1 & \# \text{ selected} & \# \text{not selected} \\
\hline
\text{SNR} = \text{LOW} & \text{SNR} = \text{HIGH} & \text{SNR} = \text{LOW} & \text{SNR} = \text{HIGH} \\
\text{blue STOREY} & 0.584 & 0.558 & 0.594 & 0.564 \\
\text{blue Bootstrap} & 0.174 & 0.144 & 0.1 & 0.069 \\
\text{blue Bootstrap (ad)} & 0.093 & 0.057 & 0.09 & 0.067 \\
\text{blue Bootstrap (s)} & 0.067 & 0.068 & 0.068 & 0.114 \\
\text{base LS(L)} & 0.289 & 0.045 & 0.179 & 0.001 \\
\text{base SEP (ad)} & 0.133 & 0.042 & 0.059 & 0.117 \\
\text{base SEP (s)} & 0.093 & 0.095 & 0.095 & 0.097 \\
\hline
\end{array}
\]

**Discussion and Conclusion**
- Estimating the proportion of null voxels to make voxel-based procedures adaptive is not useful since this proportion is close to 1 in fMRI studies (Chen et al., 2009).
- However, when performing inference on other topological features, such as peaks, more-active features can be expected, reducing the proportion of null voxels.
- We present a technique to estimate the proportion of activated peaks.
- This allows to estimate error measures and power in a single fMRI-study.
- Overall, we find that $\pi_0$ and operating characteristics of selection procedures are estimated adequately.
- Given an estimate of specificity and sensitivity, a direct trade-off between both measures can guide thresholding peaks of brain activation in fMRI studies. This allows researchers to reconsider the balance between true positive and false negative rates in function of study goals.

**Error Measures and Power**
- We consider:
  - Type I error rate
  - False Positive Rate (FPR): $\text{FPR} = \frac{FP}{m}$
  - False Discovery Rate (FDR): $\text{FDR} = \frac{FP}{m}$
- If $\alpha$- and 0-0 and other conditions.
- Power
  - True Positive Rate (TPR): $\text{TPR} = \frac{TP}{m}$
- Signal-to-noise ratio (SNR): $\text{SNR} = \frac{t_c}{\sigma}$
  - High ($\sim 0.5$) vs. Low ($\sim 0.1$)
- True and false positives and negatives are defined as in Figure 2, where the grey area equals the activation field after smoothing.

**Application 2: Adaptive FDR Control**
- False discovery rate on peaks: topological FDR (Chumbley, 2010).
- FDR can be made less conservative when taking into account $\pi_0$ (Benjamini et al., 1995).
- In our simulation, we find that the FDR is controlled conservatively using our evaluation criteria as in figure 2.
- However, adaptive FDR control leads to less conservative conclusions.
- Results with SNR=0.75 and smoothness 2.5, FDR control of 0.05.

**References**

**Corresponding Author**
Joke Durnez, Joke.Durnez@UGent.be
Department of Data Analysis, Ghent University, 9000 Ghent