A RATE-INDEPENDENT NON-ASSOCIATED CONSTITUTIVE MODEL FOR FINITE ELEMENT SIMULATION OF SHEET METAL FORMING

Mohsen Safaei*, Wim De Waele*, Shun-lai Zang**

*** Laboratorium Soete, Department of Mechanical Construction and Production, Ghent University, Technologiepark Zwijnaarde 903, B-9052 Zwijnaarde, Belgium**
mohsen.safaei@ugent.be; wim.dewaele@ugent.be
****School of Mechanical Engineering, Xi'an Jiaotong University, No. 28, Xianning Road, Xi'an, Shaanxi, China, shunlai.zang@gmail.com**

ABSTRACT This paper presents a plane stress anisotropic constitutive model based on a non-associated flow rule (non-AFR) and a one-surface non-linear mixed isotropic-kinematic hardening law. A fully implicit stress update algorithm was used to implement the developed continuum formulation as a user material subroutine (UMAT) into the commercial finite element code ABAQUS. This model is capable of predicting the permanent softening of metals in addition to the Bauschinger effect and transient behavior.

INTRODUCTION: Under the assumption of associated flow, the plastic flow and yield stress functions follow the same formulation. Thus, using a plastic potential function to address zero dilatancy (i.e. zero or negligible volume change after plastic deformation), automatically implies using the same pressure-insensitive function as yield stress function (normality condition). This is why an associated flow rule (AFR) is not capable of modeling both zero dilatancy and pressure-sensitive yield stresses. However, uniaxial tension and compression tests on iron based metals and aluminum, reported by Spitzig and Richmond [1984], revealed the (linear) dependency of yield stress on the superimposed hydrostatic pressure. This observation together with the negligible plastic dilatancy assumption in metals, opened a discussion about the validity of associated plasticity. Attention has been focused on non-associative flow rules (non-AFR) describing the plastic flow independently of the yield stress function in constitutive models for metals. The aim of this work is to develop a non-associated flow rule with a mixed hardening formulation that captures the permanent softening effect, as well as the transient behavior and Bauschinger effects.

PROCEDURES, RESULTS AND DISCUSSION: This work is based on a non-AFR originally developed by Stoughton [2002], which is a typical version of a more generalized and pressure sensitive one developed by Stoughton and Yoon [2004]. The quadratic anisotropic plastic potential function is identical to the one proposed by Hill:

$$f_p = \left(\bar{\sigma}_{11}^2 + \lambda_p \bar{\sigma}_{22}^2 - 2\nu_p \bar{\sigma}_{11}\bar{\sigma}_{22} + 2\rho_p \bar{\sigma}_{12}^2\right)^{\frac{1}{2}} \tag{1}$$

where $\bar{\sigma}$ is equal to $\sigma - \alpha$ with α the back-stress tensor. The yield stress function is also a quadratic function of the stress tensor, but is defined independently from the plastic potential function in terms of measured yield stresses. The yield function, F , is given by

$$F = f_y - \sigma^{iso} = \left(\bar{\sigma}_{11}^2 + \lambda_y \bar{\sigma}_{22}^2 - 2\nu_y \bar{\sigma}_{11}\bar{\sigma}_{22} + 2\rho_y \bar{\sigma}_{12}^2\right)^{\frac{1}{2}} - \sigma^{iso}(\bar{\varepsilon}^p) \leq 0 \tag{2}$$

where σ^{iso} is the isotropic hardening function and $\bar{\varepsilon}^p$ is the effective plastic strain. In Eqns. (1) and (2) λ, ν and ρ are material parameters (functions of the Lanckford coefficients) of which the subscripts p and y indicate their relation to plastic potential or yield stress function respectively.

The employed hardening model was developed by Zang et al [2011] and includes a two term Chaboche kinematic hardening and a modified isotropic hardening function. The back-stress tensor evolves according to the following function

$$\alpha = \alpha_1 + \alpha_2 \tag{3}$$

$$d\alpha_1 = c_1 \frac{\bar{\sigma}}{f_p} d\bar{\varepsilon}^p - \gamma \alpha_1 d\bar{\varepsilon}^p \tag{4}$$

$$d\alpha_2 = c_2 \frac{\bar{\sigma}}{f_p} d\bar{\varepsilon}^p \tag{5}$$

where α_1 and α_2 are the first and second terms of the total back-stress function and c_1, c_2 and γ are material parameters. As seen in Eqns. (4) and (5), the kinematic hardening uses the plastic potential function whereas the yield stress function is used when associated flow is considered.

The modified isotropic hardening is described as follows:

$$\sigma^{iso}(\bar{\varepsilon}^p) = \sigma_0 + Q\left(1 - e^{-b\bar{\varepsilon}^p}\right) - {}^{c_1}\!/_{\gamma}\left(1 - e^{-\gamma\bar{\varepsilon}^p}\right) \tag{6}$$

where Q and b are material parameters. The hardening parameters of the AA5754-O aluminum alloy (Table 1) were optimized by SiDoLo software and based on results of Tension/Compression (T/C) tests at different pre-strains performed by Lee et al [2007].

Table 1: Material Constants

E	σ_0	σ_{45}	σ_{90}	σ_b	r_0	r_{45}	r_{90}	c_1	c_2	γ	Q	b
70000	94.8	94.6	96	102	0.76	0.7	0.79	4665.3	204.8	212	126.4	16.1

CONCLUSIONS: The accuracy of the developed UMAT is evaluated by comparing the results of the one-element uniaxial T/C finite element model and explicit code written for uniaxial cyclic loading using very small strain increments (Fig.1). In the same figure, the accuracy of the model is shown by comparing numerical results with experimental data of a T/C test at 0.078 pre-strain. A very good agreement is observed between non-AFR finite element simulation and experimental result; the implemented anisotropic hardening very well captures the permanent softening effect. The strength of the non-AFR is revealed by modeling both direction dependent yield stresses and strain ratios due to using two independent functions for plastic potential and yield stress, Fig 2. Excellent agreement between both yield stress and strain ratios at different orientations is observed. The combination of the mixed hardening and non-AFR is expected to result in a better prediction of phenomena such as springback and earing in sheet metal products.

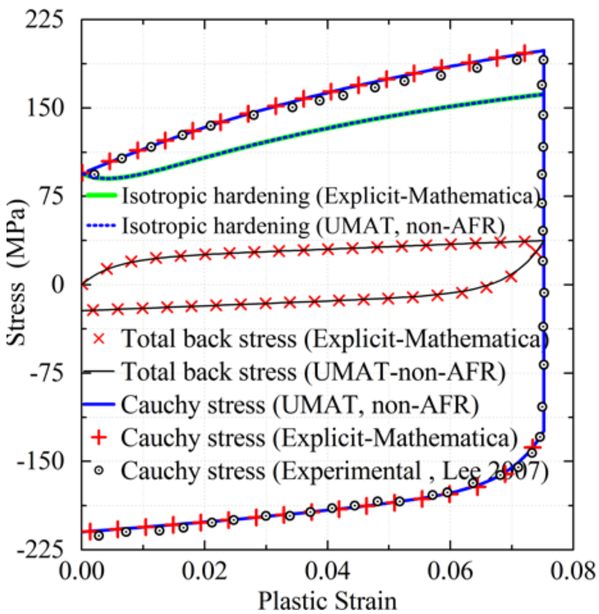

Fig. 1 Tension/Compression results obtained from UMAT are evaluated by explicit code and experimental results.

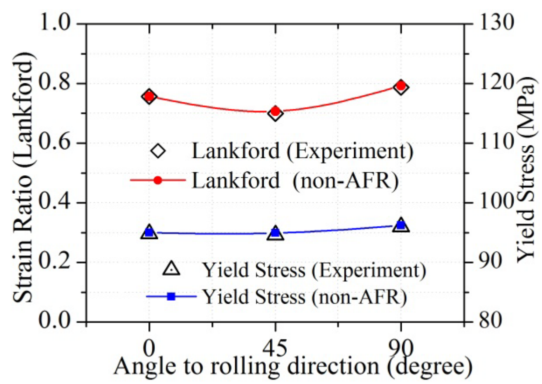

Fig. 2 Both strain ratios and yield stress agree very well with experimental results.

Acknowledgement: The financial support from the Ghent University Research Fund (BOF08/24J/106) is greatly appreciated.

REFERENCES:

Lee, M.G., Kim, D., Kim, C., Wenner, M.L., Wagoner, R.H., Chung, K.S., 2007. A Practical Two-Surface Plasticity Model and Its Application to Spring-Back Prediction.International Journal of Plasticity **23**, 1189-1212.

Spitzig, W.A., Richmond, O., 1984. The Effect of Pressure on the Flow-Stress of Metals.Acta metallurgica **32**, 457-463.

Stoughton, T.B., 2002. A Non-Associated Flow Rule for Sheet Metal Forming. International Journal of Plasticity **18**, 687-714.

Stoughton, T.B., Yoon, J.W., 2004. A Pressure-Sensitive Yield Criterion under a Non-Associated Flow Rule for Sheet Metal Forming. International Journal of Plasticity **20**, 705-731.

Zang, S.L., Guo, C., Thuillier, S., Lee, M.G., 2011. A Model of One-Surface Cyclic Plasticity and Its Application to Springback Prediction. International Journal of Mechanical Sciences **53**, 425-435.