Test of a majority-based reversible (quantum) 4 bits ripple-carry adder in adiabatic calculation

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Abstract—Quantum computing and circuits are of growing interest and so is reversible logic as it plays an important role in the synthesis of circuits dedicated to quantum computation. Moreover, reversible logic provides an alternative to classical computing machines, that may overcome many of the power dissipation problems in the near future. As a proof of concept we designed and tested a reversible 4 bits ripple-carry adder based on a do-spy-undo structure. This paper presents some performances obtained with such a chip processed in standard 0.35 μm CMOS technology and used in real reversible calculation (in this study, computations are performed in both directions such that addition and subtraction are made reversibly with the same chip). We also discuss the superiority of using adiabatic signals over classical rectangular pulses when using dual-line pass-transistor logic gates. Adiabatic signals allow the signal energy stored on the various capacitances of the circuit to be redistributed rather than being dissipated as heat. Finally, we show that adiabatic signals allow to avoid calculation errors introduced by the use of conventional rectangular pulses and allow to drastically reduce the number of pulse resynchronization in large circuits.

Index Terms—reversible computation, design, implementation, pass-transistor logic, ripple-carry adder, Spectre simulation, quantum computation, adiabatic signal, test and measurement.

I. INTRODUCTION

Reversible computing is useful both in lossless classical computing [1] and in quantum computing [2]. Reversible circuits also present less power consumption against classical ones [3]–[5].

The physical implementation of the adder we are presenting is reversible dual-line complementary pass-transistor CMOS logic [6] and does not make use of buffer of any sort nor level restorer. This adder has been extended and embedded in larger components such as a multiplier [7] and, more recently, in a H264 AVCC encoder [8].

We present for the very first time to our knowledge, extensive electrical tests performed on a reversible CMOS chip computing in both directions: forward (adder) and backward (subtractor) and compare the efficiency of both classical rectangular pulses and adiabatic signals.

II. SHORT HISTORY AND DESCRIPTION OF THE REVERSIBLE ADDER CHIP

A. Short history

In 2005, Cuccaro et Al. [9] presented a new linear-depth ripple-carry quantum addition circuit making use only of controlled-NOT (CNOT or Feynman) gates and controlled-controlled-NOT (CCNOT or Toffoli) gates which was an improved version of a V-shaped reversible adder presented by [10]. In 2008, [7] presented the synthesis and design of a reversible Fourier transform making use of such a reversible adder, but using a do-spy-undo (majority-umajority) scheme structure as firstly presented by [11]. This design was making use only of controlled-NOT (Feynman) gates and controlled-SWAP (Fredkin) gates[11]. Unfortunately, this 8 bits adder was embedded in a larger 8 bits multiplier making it impossible specific access to measurements.

The circuit we present in this paper is a 4 bits majority-based reversible ripple-carry adder. As in [7], it makes use only of Feynman and Fredkin gates and presents, as a general structure, a do-spy-undo scheme [11]. Full details about synthesis discussion, structure and theoretical consumption are already presented in [12].

B. Block structure

As a summary, let us recall the block structure of the studied adder. Let $n$ denote the size of both numbers to be added. First, as a do-undo structure is used, each bit addition implemented, except the most significant bit (MSB), necessitates one do-undo circuit. The 3 inputs majority do circuit and the 3 inputs umajority undo circuit are presented in Fig.1 and Fig.2 respectively.

The do-undo block constitutes a one bit adder when used forward (respectively a one bit subtractor if used backward). The numbers $a$ and $b$ (respectively $S$ and $A$) are the main inputs and $C_{in}$ ($C_{out}$) is the carry-in. The number $S$ ($b$) represents the computation result (sum $a + b$ respectively.

1Both Fredkin and Toffoli gates are universal gates, which means that any logical or arithmetic operation can be constructed entirely of one of those gates.
difference $S-A$) and $A$ ($a$) is a copy (garbage) of the input $a$ ($A$) and so is the output $C_{out}$ ($C_{in}$) with respect to the input $C_{in}$ ($C_{out}$).

When an addition is computed (forward calculation), the internal bits $C_{int}$ are used for carry transmission from one stage to the next during the majority operations, while the input $C_{int'}$ of the undo block rebuild the initial input $c_{in}$ during the undo operations ($C_{out} = c_{in}$). When a difference is computed (backward calculation), the inverse process occurs leading to the calculation of the difference.

For 1 bit calculation, $C_{int}$ would be directly connected to $C_{int'}$ whereas if the adder size $n \geq 2$, the most significant bits addition is performed by using only two Feynman gates; one is used to compute the bit sum XOR ($\oplus$) and the other to sum-up the carry to the final result. Each extra bit addition is realized by cascading supplementary do-undo blocks, linked together by connecting $C_{int}$ to $c_{in}$ and $C_{out}$ to $C_{int'}$ as presented in the full schematic of the 4 bits adder in Fig.3.

Let us notice that simplification presented in Fig.4 has been used for the real design of the unmajority block.

III. DESIGN AND REALIZATION

This 4 bits Cuccaro adder has been designed using the Cadence® computer-aided design environment software. Each electrical simulation has been performed using its Cadence Spectre® simulator (see Fig.7 and Fig.8 as examples).

The chip processing has been realized at ONSemiconductor through the Europractice / IMEC consortium in 350 nm standard CMOS technology. The transistor lengths used are $L = 350 \text{ nm}$, both for n-type and p-type transistors, while widths are respectively $W_n = 500 \text{ nm}$ and $W_p = 1500 \text{ nm}$. The chip contains a total of 160 transistors (80 n-type and 80 p-type).

The Cadence Layout of the studied adder is shown in Fig.5 while Fig.6 presents a photograph of the realized chip.

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The Cadence Layout of the studied adder is shown in Fig.5 while Fig.6 presents a photograph of the realized chip.
We can easily recognize the dual inputs and outputs at each left and right hand sides, and the two dual carries $C_{in}$ and $C_{out}$ at the top side while in the middle part are the wider wires used for the substrate and the N-wells biasing (typically, $V_{SS} = -1.2 \text{ V}$ and $V_{DD} = 1.2 \text{ V}$).

IV. SIMULATION RESULTS

Starting with the simulation, two different types of signal have been used: "traditional” square pulses and quasi-adiabatic triangle pulses.

A. Input signal definitions

The maximal signal voltages used, both for simulation and for measurements are $V_+ = 1 \text{ V}$ for logic "1” and $V_- = -1 \text{ V}$ for logic "0”.

If the first type of signal (square pulse) is applied, for the unchanged bits, the voltages corresponding to the desired logic levels remain constant while others are fastly switched.

If the second signal type (adiabatic) is applied, a triangular pulse ranges from $\frac{V_+ + V_-}{2}$ to the desired logic level at each calculation step. Thus, in case of the second signal type, all signals (i.e. both the unchanged bits and the changing ones) temporarily come to the equilibrium voltage, half-way logic "1” and logic "0”.

The two signal types can be seen in Fig. 7 presenting the minuend $S$ and subtrahends $A$ of nine subsequent subtractions $S - A$.

B. Simulated outputs

Let us stress that the pulses of Fig. 7 are representative of the input signals previously defined. Fig. 8 presents the corresponding difference $b$ and garbage $a$ calculated outputs when using the Cuccaro adder in reverse mode (subtractor), for both the square and triangle adiabatic pulse shapes. In Fig. 8, we see that rectangular pulses do not all have the same amplitude, that they are also presenting large transition peaks while for the triangle adiabatic signals, the pulses are regular, without any transition peak.

Let us also notice that when the size $n$ of the numbers to be computed is larger than two, some exceptional errors may occur when square signals are used while this doesn’t happen when adiabatic signals are applied (Fig. 8).

One reason for that comes directly from the transmission gate that is build of two complementary transistors in parallel. Due to the steep slope generated during the transition of the square pulse, if a delay occurs between the control signals on the transistor gates and the signals transmitted through the transistor channels, the gates do not shut down the signal at the very moment of the transition and an opposite signal that would have been cut down is transmitted anyway, occasionally causing calculation errors.

If the adiabatic signals are used, the smooth transitions allow a delay between the signals. In effect, during the transition steps, the two transistors are working below their threshold voltages, drastically lowering the amplitude of the output signal (see also Fig. 10 and comments). As the example of Fig. 9 shows, a phase difference – expressed as a percentage of

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**Fig. 7. Cadence Spectre® simulation: comparison of rectangular and adiabatic input pulses of same frequency.**

Top line is the least significant bits $S_0$ and $A_0$ whereas the bottom one corresponds to the most significant bits $S_3$ and $A_3$.

**Fig. 8. Cadence Spectre® simulation: comparison of the output signals when using rectangular and adiabatic input pulses, respectively.** According to Spectre® electrical simulation software, using rectangular pulses introduces several errors in computation. None of these errors occur when using triangular pulses. Top line is the least significant bits $b_0$ and $a_0$ bit whereas the bottom one corresponds to the most significant ones $b_3$ and $a_3$. 

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the period – as large as a quarter of a period $PrCT = \pm 25\%$ (plain lines) does not introduce any calculation errors, even if the final shape of the signal is deformed and if some small variations of the output potential (inferior to 40 mV) appear when the transmission gate is closed. In fact, this maximal percentage will be dependant of the ratio between the maximum pulse voltage and the pseudo-threshold voltage of the transmission-gates used. When a bigger phase appears, the output signal is no more zeroed before the command pulse.

A positive (respectively negative) phase indicates a delay of the input (respectively command) signal with respect to the command (respectively input) pulse.

It is also easy to verify that dual signals (e.g. $a$ and $\bar{a}$) are closely synchronized by symmetry of the dual circuits – each signal and its complementary having passed through the same number of transmission gates.

As a consequence, even if we need to pay attention to the design of the circuit in order to limit the phase introduced between the different signals used in the calculation, the triangular adiabatic shape signals allow to drastically reduce the number of level restorers and pulse resynchronizers.

V. TESTS AND MEASUREMENTS

Several extensive measurement steps have been performed on the Cuccaro circuit, both in forward (adder) and in reverse (subtractor) calculations.

Constant voltage computations are primary presented allowing to bring information principally on impedance and voltage bias impact on calculation. In a second hand, we present the adiabatic generator used for the experimental application of the input pulses. We then present measurements of the outputs obtained both in direct and reverse calculations.

A. Preliminary DC measurements

The voltages applied: substrate and NWELL biases, input constant biases, have been measured using a Flucke 175 multimeter in DC mode.

A large part of the truth table has been first successfully checked and some measurements have been done for some particular conditions:

- In direct calculation (adder), 32 different conditions: $c_{in} = 0$, $b = 1010$ and $a$ varying, as well as $c_{in} = 0$, $a = 0101$ and $b$ varying.
- In reverse calculation (subtractor), 16 different test vectors: ($C_{out} = 0$, $A = 0101$ and $S$ varying).

From these measurements, the maximum voltage drop found for the adder is 30 mV whereas the maximum voltage drop for subtractor was found to be 6 mV over a constant input bias of 1 V. In any cases, the output drops are inferior to 3 % of the input signal biases which is the order of magnitude of the measurement uncertainties.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{adiabatic_pulses.eps}
\caption{Cadence Spectre© simulation of a complementary pass transistor gate: comparison between adiabatic and square pulses when a dephasing of $PrCT = \pm 25\%$ (bold) and 25% of the period is applied between the command pulses and the signal to be transmitted. The dashed lines correspond to a phase equal to 0 %. The frequency used is 5 MHz.}
\end{figure}

Further in the calculation steps, the signals may become symmetrical again, when two phased signals are used successively as the command and the transmitted signals. This then leads to a narrowing of the final pulse width (as seen in Fig.9) but does not impact significantly its maximal amplitude that is related to the impedance of the circuits. In Fig.9 (dash lines), the simulated output signal obtained from a single transmission gate when the phase equals zero ($PrCT = 0\%$) is larger than the output signals obtained from a complex circuit such as a subtractor (Fig.8), where a phase is introduced by the propagation of the internal carries $C_{int}$ and $\overline{C_{int}}$. In effect, in the Cuccaro adder, the $c_{in}$ signal when going through the whole computation, has to be transmitted twice at each stage of the calculation. The greater the bit size $n$, the greater the number of stages, the greater the phase introduced, the narrower the pulse. When using a square signal, this may produce wrong pulses whereas using an adiabatic shape, the signal will be deformed, narrowed, but the calculation will remain possible with very few loss in amplitude and still guaranteeing a correct logic “1” or correct logic “0”. This was checked up to $n = 6$ bits.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{input_voltages.eps}
\caption{Measurements of the input voltage dependance of bit $b_2$ as a function of the output voltage bias of bit $S_2$, for the reverse (subtractor) calculation: $b = S - A = 0110 - 0101 = 0001$.}
\end{figure}

\footnote{The transmitted signal is reduced when the command signal amplitude is smaller than the threshold voltage of the pass-transistor gate, causing an asymmetry of the transmitted signal.}
in reverse mode (i.e. as subtractor). In fact, same curves are obtained for each output whether in direct mode (adder) or reverse one (subtractor), except for the most significant bit $A_3$ (respectively $a_3$) that is made by a direct connection to its corresponding input $a_3$ (respectively $A_3$) (see Fig. 4). For all measurement performed, the voltages $V_{DD}$ and $V_{SS} = V_{GND}$ are kept constant to 1.2 V and $-1.2$ V respectively.

We see from Fig. 10 that a minimal voltage input as small as 420 mV can still be used (without security marge) to perform calculations with this Cuccaro chip. This is also coherent with the general shape of simulated output pulses in Fig. 3 and with the experimental measurements in Fig. 12 and Fig. 13 that present the same transition when applying a linear input variation (triangular adiabatic pulse). But in these last cases, one also needs to account for the impact of the phases introduced by the internal carry signals that not only narrow the pulse width but also slightly increase the minimal voltage input possibly used to perform a calculation. We then arbitrarily fix the limit of ±500 mV as the minimum logical amplitude acceptable to define a logic “1” (above 500 mV) or a logic “0” (below −500 mV). This choice is experimentally driven by the fact that within these two defined domains, the calculated signals obtained have always been found conform to what was expected.

The pseudo-threshold voltage $V_{PT}$ of the Cuccaro adder is found to be about 350 mV, which is also close to the simulated value of the transmission-gate pseudo-threshold voltage. Between about 420 mV down to $V_{PT} = 350$ mV a transition occurs that corresponds to the linear mode of the transmission gate transistors. Below $V_{PT}$, the transmission gates are blocked and, as expected, no signal is transmitted.

B. Generation of the adiabatic signal

In order to apply the necessary adiabatic pulses, we designed an electronic module allowing to generate 4 types of triangle pulses:

1) A logic “1” ranging from $V_{DD} + V_2$ up to $+1$ V and back,

2) A logic “0” ranging from $V_2 + V_3$ down to $-1$ V and back,

3) A switching signal ranging between $+1$ V and $-1$ V,

4) The dual signal of the switching signal that is the symmetrical signal from the mean value $V_{DD} + V_3$.

These four signals are all build up from one single triangular pulse ranging between $+1$ V and $-1$ V by using a single buffer (switching pulse), an inverter followed by a buffer (switching pulse) or using a precision full-wave rectifier followed either by a buffer or by an inverter and a buffer (logic “1” and logic “0” respectively). Fig. 11 presents the full schematic of the electronic module used to generate the experimental adiabatic pulses. It is a very simple thus very accurate circuit presenting very low output impedances. The operational amplifiers are polarized at ±3 V while the input triangular signal has an amplitude of 2 V. This allows the operational amplifiers to work in the linear part of their characteristics and gives very accurate and reproducible pulses.

C. Adiabatic AC measurements

AC measurements have been performed using a commercial Tektronix TDS 210 digital oscilloscope. The output charge is its ×1 probe that can be modeled by a 1 MΩ resistor in parallel with a 32 pF capacitance. By using the voltage divider technique, the experimental output impedance of the Cuccaro adder (or subtractor) is found to be $Z_{out} \simeq 5250$ Ω $\pm 5$ % whereas it is found to be close to 8 kΩ by Cadence Spectre simulation. Fig. 12 and Fig. 13 show typical experimental measurements realized respectively in forward (adder) and backward (subtractor) computation.

Experimental Measurements in forward computation –

Input $a_0$ and output $S_3$

Input $a_0$ and output $\overline{S_3}$

Fig. 12. Oscilloscope snap-view of the 4th dual bits ($S_3$, $\overline{S_3}$) as a function of the dual switching input bits ($a_0$, $\overline{a_0}$). Calculation of $S = a_0 + b + 1 + 011X + 0110 = 11X \overline{X}$ (with $c_{in} = 1$), measured when using adiabatic pulses.

In Fig. 12 the dual output ($S_3$, $\overline{S_3}$) is given as the example of a constant logic “1” adiabatic output and compared to the switching input $a_0$. For clarity, the dual signal $\overline{S_3}$ that is a logic “0” is also compared to the same input $a_0$ – the complementary $\overline{a_0}$ being the exact opposite signal of $a_0$ ($\overline{a_0} = \overline{a_0}$).

As previously found in Fig. 10, the impact of the threshold voltage can be seen on both dual output signals in Fig. 12 at the rise front whereas at the descending one, its impact is superimposed to capacitance effect coming from the probe.

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1 We will use the term pseudo-threshold voltage when related to a gate in order to differentiate this value from the transistor threshold voltages.
that slows down the decrease of the signal to the mean value \( V_{+} + V_{-} \), here equal to zero, while all the transmission gates of the chip are reaching their open circuit functionality.

**Experimental Measurements in backward computation –**

<table>
<thead>
<tr>
<th>Input ( A_0 ) and output ( b_3 )</th>
<th>Input ( A_0 ) and output ( b_3 )</th>
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<tbody>
<tr>
<td>CH1 412mV CH2 412mV M 1.00ms</td>
<td>CH1 412mV CH2 412mV M 1.00ms</td>
</tr>
<tr>
<td>Regular triangle: dual output ((A_0, \overline{A_0}))</td>
<td>Irregular triangle: dual input ((\overline{b_3}, \overline{b_3}))</td>
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Fig. 13. Oscilloscope snap-view of the 4th dual switching bits \( (b_3, \overline{b_3}) \) as a function of the the dual switching bits \( (A_0, \overline{A_0}) = (X, \overline{X}) \), obtained by reverse (subtractor) calculation of \( S - A = b = 0110 - 010X = 00\overline{X}X \) with \( C_{out} = 0 \) and measured when using adiabatic pulses.

As shown in Fig.13, very similar results are obtained when performing a subtraction using the same chip in reverse direction. Here, the output calculated \( (b_3, \overline{b_3}) \) signal, shown as typical example, is a constant dual zero also compared to the same input signal \( A_0 \).

As the carry propagates twice through the whole circuit, a maximal delay is obtained between the two extreme output bit pulses (e.g. \( a_0 \) and \( b_3 \)). The maximum delay \( \tau = 91 \mu s \) is measured between outputs \( a_0 \) and \( b_3 \) during a subtraction (Fig.14). This delay is obtained between two pulses at a frequency 120 Hz, when probes are connected to the outputs. According to these measurements, it is possible to cascade more than 23 Cuccaro gates before the output delay reaches 25% of the period such that calculation errors may appear.

As a last experiment, we cascaded an adder with a subtractor. This experiment allowed to verify that two sequential calculations can be performed and that the addition can really be uncomputed giving back the initial input signal as in the example given in Fig.15.

A large part of the identity truth table (different from the first experiment) has been checked with no errors detected. Also, no significant extra phase nor attenuation occurred during this computation steps, giving the indication that a lot of reversible chips may be cascaded.

Nevertheless, the output signal having been transmitted through several transmission gates is not the exact input signal, but a calculated one and inherit of the same characteristics than another calculated output.

**VI. CONCLUSION**

We presented real calculations performed in a 350 nm CMOS technology reversible (quantum) adder used in both direction: forward adder and backward subtractor.

The superiority of making use of adiabatic triangle pulses has been demonstrated and explained, showing that using adiabatic pulses on pass-transistor based reversible (quantum) chips is a suitable solution to perform linear calculations.

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