# The complex Jacobi iterative method for non-paraxial beam propagation in nonlinear optical waveguides

Khai Q. Le and Peter Bienstman

*Photonics Research Group, Department of Information Technology, Ghent University, Sint-Pietersnieuwstraat 41, 9000 Ghent, Belgium*

khai.le@intec.ugent.be

The recently introduced beam propagation method using complex Jacobi iteration adapted for modeling of non-paraxial beam propagation in nonlinear optical waveguides is presented in this paper. The beam propagation equation is based on our recently proposed modified Padé(1,1) approximant operator. The resulting approach is very efficient and well-suited for large structures with long propagation paths.

## 1. Introduction

An external excitation of a nonlinear waveguide was demonstrated to produce spatial multisoliton emission from the waveguide, in which a sequence of bright solitons could be emitted for sufficient input power [1]. For waveguides with cladding made by nonlinear material where the refractive index depends on field intensity, it is difficult to obtain analytical solutions and thus, computational efforts are necessary for the design and modeling of these kinds of nonlinear optical waveguide devices. There are several methods developed to simulate the optical propagation in nonlinear waveguides in which the beam propagation method (BPM) has become one of the most powerful and efficient techniques.

A great number of BPM versions including the finite element-based and the finite difference-based BPM have been developed for these kinds of problems [2-3]. Those methods normally used the traditional direct matrix inversion (DMI) to find a solution of a propagation equation between two successive propagation steps. However, for large structures with long propagation paths and varying boundaries along propagation direction, the DMI technique is numerically very intensive.

To overcome this problem, recently we proposed the iterative BPM where at each propagation step the beam propagation equation is recast in terms of a Helmholtz equation with a source term and then solved efficiently by the recently introduced complex Jacobi iteration (CJI) [4]. There, the effective absorption coefficient (or medium loss) appearing in the Helmholtz equation is very high. This is a condition leading to rapid convergence of the CJI method.

Furthermore, by introducing an extra calculation step the CJI method has been successfully employed to simulate photonic components consisting of materials with nonlinear Kerr effects [5]. Therefore, by doing the same procedure for dealing with nonlinear effects at each propagation step, our recently proposed iterative BPM is extended and is very competitive for modeling optical propagation in nonlinear optical waveguides.

## 2. Formulation

Efforts to improve the limitations of the paraxial approximation or Fresnel equation in the beam propagation method have so far made use of wide-angle (or non-paraxial) formulations. Different treatments of non-paraxial BPM have been developed. There exist rational approximants of the square root operator, the exponential of the square root operator, real Padé(m,n) approximant operators [6]. The Padé-approximant-based non-paraxial BPM has become one of the most commonly used techniques for modeling optical waveguide structures. However, real Padé(m,n) approximant operators incorrectly propagate evanescent waves. To circumvent this problem, recently we proposed the so-called modified Padé(m,n) approximant operators [7]. The resulting operators not only give evanescent waves a desired damping, but also allow more accurate approximations to the Helmholtz equation than real Padé(m,n) approximant operators. In this work, rather than using real Padé approximant operators as the existing approaches the beam propagation the equation for the analysis of non-paraxial propagation in nonlinear waveguides is based on the modified Padé(1,1) approximant operator as follows: [4]

$$(1+\xi_1 P)\Psi^{m+1} = (1+\xi_2 P)\Psi^m \tag{1}$$

where $P = \nabla_\perp^2 + k_0^2(n^2 - n_{ref}^2) = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2(n^2 - n_{ref}^2)$, $k = k_0 n_{ref}$, $n_{ref}$ is the reference refractive index, $\xi_1 = \frac{1}{4k^2(1+\beta^2)} - \frac{i\Delta z}{4k}$, $\xi_2 = \frac{1}{4k^2(1+\beta^2)} + \frac{i\Delta z}{4k}$, $\beta$ is a damping parameter, $\Delta z$ is the propagation step size and n is the intensity dependent refractive index as follows [1]:

$$n^2 = n_l^2 + \gamma_l |\Psi|^2, \quad l = c, s, f. \tag{2}$$

with the subscripts c,s,f referring to the cladding, substrate and film, respectively.

By dividing both sides of Eq. (1) by $\xi_1$, we obtained

$$[\nabla_\perp^2 + k_0^2(n^2 - n_{ref}^2) + \frac{1}{\xi_1}]\Psi^{m+1} = (\frac{\xi_2}{\xi_1} P + \frac{1}{\xi_1})\Psi^m \tag{3}$$

or

$$[\nabla_\perp^2 + k_0^2(n^2 - n_{ref}^2) + \frac{1}{\xi_1}]\Psi^{m+1} = source\ term\,(ST). \tag{4}$$

It is clearly seen that the beam propagation equation can be recast as a Helmholtz equation with a source term in an effective medium with loss (or effective absorption) determined by the imaginary part of $\frac{1}{\xi_1} = 4k_0^2 n_{ref}^2 \frac{ik\Delta z}{[1 + k_0^2 n_{ref}^2 \beta^2/2]^2 + k^2 \Delta z^2}$. The effective absorption results in propagation equation based on the conventional and modified Padé(1,1) approximant operator is depicted in Fig. 1. It can be seen from the figure that the smaller propagation step size, the higher effective absorption in propagation equation based on the modified Padé(1,1) approximant operator is obtained. In addition, the amount of this loss is normally very high for typical choice of $k\Delta$ . This is a condition that favors rapid convergence for the CJI method. Thus, the CJI solution of the propagation equation based on the modified operator will converge faster than those based on the conventional operator.

Furthermore, by adding an extra calculation step for dealing with nonlinear effects as well-done for a different method in [5] and using PMLs to absorb incident radiation without any additional parasitic reflections, regardless of wavelength, incident angle or polarization as boundary conditions, the CJI method is thus a promising technique to solve the nonlinear beam propagation equation quickly and effectively.

## 3. Numerical results

We consider a nonlinear optical waveguide with a linear core bounded by linear and nonlinear claddings as shown in Fig. 2, where the refractive index $n_s = n_c = .55, n_f = .57$, $\gamma_{\_} = ).01$, $\gamma_{\_} = \gamma_{\_} = )$, W=5 $\mu$ ı, Y=50 $\mu$ ı, d=1 $\mu$ ı and the optical wavelength $\lambda = .3\mu$ ı. The fundamental TE mode as excitation is launched into the waveguide. The beam propagation in linear optical waveguide is shown in Fig. 3 (a), while the evaluation of input beam in nonlinear optical waveguide is depicted in Fig. 3(b). From the figure, a soliton is emitted through the film cladding interface into the nonlinear cladding and propagates away from it.

The calculated results are certainly in very good agreement with those obtained by the other authors [1,2] because we are dealing with the same equation. However, what we are focusing on is a performance of the CJI method in comparison with the DMI method. For a small propagation step size, we found that the amount of effective absorption is very high. This is condition that favors rapid convergence of the CJI method. With a very strict propagation error tolerance of $10^{-7}$, the CJI method for solution of the conventional Padé(1,1) approximant-based propagation equation required about 0.035 seconds to obtain a solution between two successive propagation steps, and 0.021 seconds for those based on the modified operator. It is attributed to the higher effective absorption as shown in Fig. 1. Whereas the direct matrix inversion performed the propagation in 0.365 seconds, around ten times slower than the CJI method. Definitely, for large structures with a long path length required for a very small propagation step size that results in frequent matrix inversions, the DMI technique is numerically very intensive. In contrast, for typical choices of $k\Delta z$ the CJI technique offers rapid convergence and shorter runtimes.

## 4. Conclusions

The recently introduced BPM adapted for modeling of non-paraxial beam propagation in nonlinear optical waveguides has been presented. By typically choice of propagation step size, the iterative method offers rapid convergence and is very competitive for demanding problems in comparison with the traditional direct matrix inversion for nonlinear BPM.

## Acknowledgement

Parts of this work were performed within the context of the Belgian IAP project Photonics@Be.

## References

1. E. M. Wright, G. I. Stegeman, C. T. Seaton, J. V. Moloney, and A. D. Boardman, "Multisoliton emission from a nonlinear waveguide," *Phys. Rev. A* vol. 34, pp. 4442-4444, 1986.
2. T. Yasui, M. Koshiba, and Y. Tsuji, "A wide-angle finite element beam propagation method with perfectly matched layers for nonlinear optical waveguides," *J. Lightwave Technol.*, vol. 17, pp. 1909-1915, 1999.
3. J. de-Oliva-Rubio, and I. Molina-Fernandez, "Fast semivectorial nonlinear finite-difference beam propagation method," *Microwave Opt. Technol. Lett.*, vol. 40, pp. 73-77, 2004.

4. Khai Q. Le, R. Godoy-Rubio, P. Bienstman and G. R. Hadley, "The complex Jacobi iterative method for three-dimensional wide-anglebeam propagation," *Opt. Express*, vol. 16, pp. 17021-17030, 2008.
5. P. Vandersteegen, B. Maes, P. Bienstman, and R. Baets, "*Opt. and Quantum. Electron.,*" vol. 38, pp. 35-44, 2006.
6. G. R. Hadley, "Wide-angle beam propagation using Padé approximant operators," *Opt. Lett.*, vol. 17, pp. 1426-1428, 1992.
7. Khai Q. Le, "Complex Padé approximant operators for wide-angle beam propagation, " *Opt. Commun*., vol. 282, pp. 1252-1254, 2009.
8. G. R. Hadley, "A complex Jacobi iterative method for the indefinite Helmholtz equation," *J. Comp. Phys*., vol. 203, pp. 358-370, 2005.

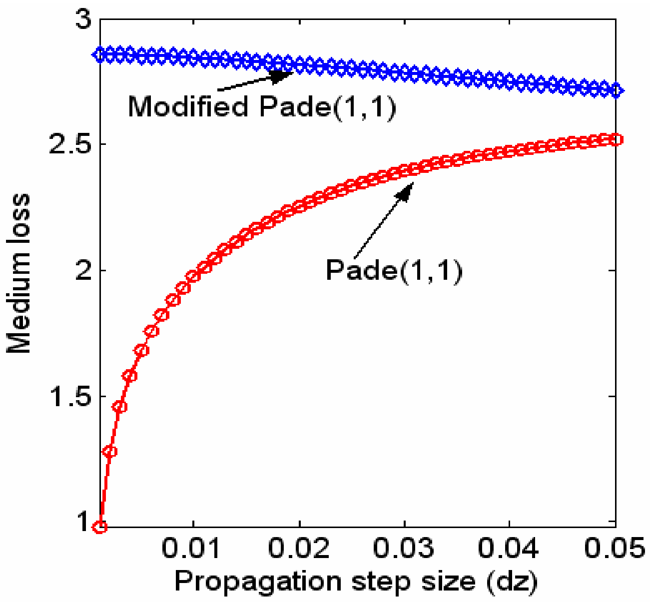


Fig. 1. Amount of effective absorption in propagation equation with respect to propagation step resolution.

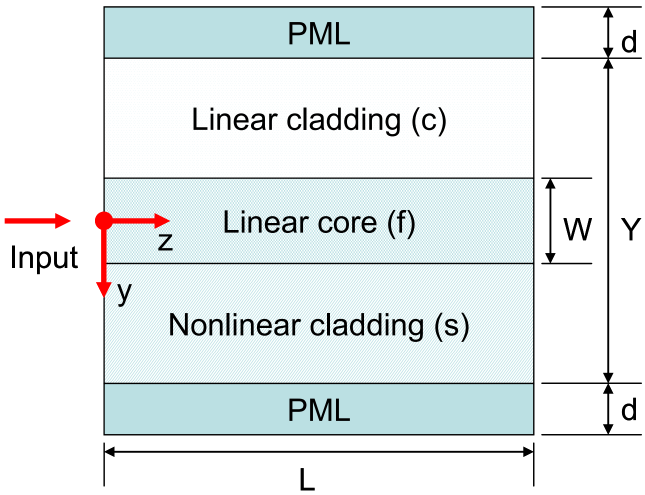


Fig. 2. Nonlinear optical waveguide with a linear core bounded by linear and nonlinear claddings.

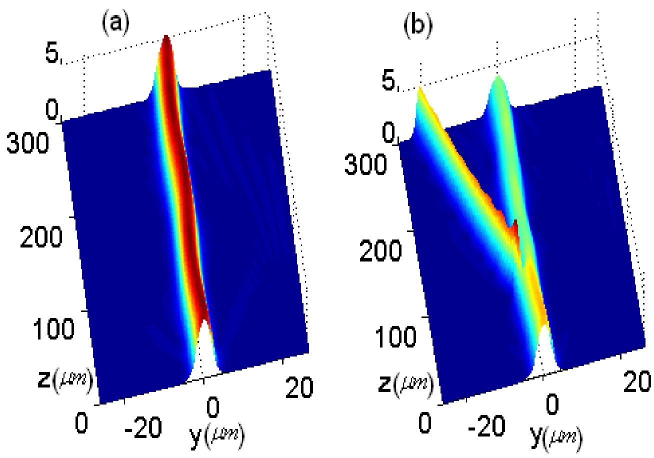


Fig. 3. (a) TE mode propagating in linear waveguide and (b) soliton emission in nonlinear waveguide.