Improving the Robustness of Vector Fitting to Outliers in the Data

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Abstract. This letter introduces a new algorithm for broadband macro-modeling of passive electronic components from frequency response data. It modifies the Vector Fitting algorithm in such a way that the L_1 norm of the approximation error is minimized, rather than the L_2 norm. It is shown that this approach is more robust towards outliers in the data.

1 Introduction

Robust broadband macromodeling techniques are of crucial importance for efficient time domain and frequency domain simulation of high-speed interconnect structures. The standard Vector Fitting (VF) algorithm starts from an S-parameter frequency response, sampled at a discrete set of frequencies. It computes a macromodel by minimizing a weighted iterative cost function in the L_2 sense [1]. In real-life situations, the quality of the L_2 fitting model may be degraded due to outliers in the data. An outlier is a value in the data that deviates strongly from the other values, and is usually caused by measurement or instrumentation errors. This paper proposes a modified Vector Fitting algorithm that minimizes the L_1 norm of the complex fitting error instead [2]. Numerical results illustrate that the new approach is more robust with respect to outliers [3].

2 Vector Fitting Technique

2.1 Modeling Algorithm

Given a discrete set of S-parameter data samples $\{s_k = j\omega_k, H(s_k)\}_{k=0}^K$, a rational macromodel with numerator $N^t(s)$ and denominator $D^t(s)$ can be computed

iteratively ($t = 1, \dots, T$) by successively solving least squares problems [1, 4]

$$\arg \min J = \sum_{k=0}^K |w(s_k)|^2 \left| \sigma^t(s_k) H(s_k) - (\sigma H)^t(s_k) \right|^2 \quad (1)$$

As in [5], both $(\sigma H)^t(s)$ and $\sigma^t(s)$ belong to a linear span of P rational basis functions $\Psi(s, a_p^{t-1}) = (s - a_p^{t-1})^{-1}$ that are based on the previously identified poles a_p^{t-1} . Note that the initial poles a_p^0 are chosen by a heuristical scheme [1]

$$(\sigma H)^t(s) = \frac{N^t(s)}{D^{t-1}(s)} = c_0^t + \sum_{p=1}^P c_p^t \Psi(s, a_p^{t-1}) \quad (2)$$

$$\sigma^t(s) = \frac{D^t(s)}{D^{t-1}(s)} = 1 + \sum_{p=1}^P \tilde{c}_p^t \Psi(s, a_p^{t-1}) \quad (3)$$

In this letter, it is shown that the L_1 norm can be minimized by selecting the user-defined frequency-dependent weighting factor $w(s)$ in (1) as

$$w(s) = \frac{\sqrt{|H(s) - H^{t-1}(s)|}}{H(s) - H^{t-1}(s)} \quad (4)$$

where $H^{t-1}(s) = N^{t-1}/D^{t-1}$ denotes the model response at iteration step $t - 1$.

2.2 Proof Outline

First, define the auxiliary function $f(s)$ as

$$f(s) = \frac{|D^t(s)H(s) - N^t(s)|^2}{|D^{t-1}(s)H(s) - N^{t-1}(s)|^2} \quad (5)$$

Applying the weighting factor $w(s)$ in (4) to the cost function J in (1) yields

$$\arg \min J = \sum_{k=0}^K |H(s_k) - H^{t-1}(s_k)| f(s_k) \quad (6)$$

Hence, upon convergence of the iterative scheme ($D^{t-1} \rightarrow D^t$ and $N^{t-1} \rightarrow N^t$), it follows that $f(s) \rightarrow 1$. Therefore, it is clear that cost function J in (6) effectively minimizes the L_1 norm of the complex fitting error $\|H(s) - H^{t-1}(s)\|_1$.

3 Example : Quarter Wavelength Filter

The reflection coefficient S_{11} of a 2-port microwave quarter wavelength filter is computed over the frequency range [1 GHz - 12 GHz]. Suppose that, due to

inaccuracies in the data acquisition, the S-parameter response contains 6 outlying data samples which are marked by black arrows. All data samples are modelled by a rational 28-pole strictly proper transfer function using the proposed fitting methodology (L_1 norm) and the standard Vector Fitting algorithm (L_2 norm).

The resulting macromodels are shown as a solid red curve in Figs. 1 and 2, and the data samples are marked as blue dots. It is clear from Fig. 1 that the L_1 norm approximation yields an overall accurate result, and is not much affected by the presence of the outliers. On the other hand, the outliers lead to a strong degradation of the model quality for the L_2 norm approximation, as shown in Fig. 2. This observation is confirmed by Fig. 3 where the maximum absolute fitting error of both fitting models is shown as a function of the frequency.

4 Conclusion

An iterative algorithm is proposed for L_1 norm identification of broadband macromodels from S-parameter data. It is shown that the method is more robust when the frequency response contains outliers. The effectiveness of the approach is illustrated by applying it to a quarter wavelength filter.

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References

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Fig 1 : Magnitude of S_{11} : model L_1 (red) and data (blue)

Fig 2 : Magnitude of S_{11} : model L_2 (red) and data (blue)

Fig 3 : Absolute fitting error of model L_1 (green) and model L_2 (blue)